Location Routing Problem with Transportation Mode Options

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Abstract

The location routing problem (LRP) entails two problems: the Facility Location Problem (FLP) and the Vehicle Routing Problem with Time Windows (VRPTW). Both are NP-hard problems and thus have been studied individually. The simple approach to solving the LRP is to solve each problem sequentially. The sequential method solves the FLP first and then solves the VRPTW. This method is relatively easy.

There is only one transportation mode for the original LRP. The transportation cost of the mode is calculated by travel distance. In the real world, however, we combine transportation mode options so as to reduce the cost. Our objective is to extend the original LRP, so as to handle transportation mode options. We propose an extended model of the LRP, along with a solving algorithm. Our method is based on Mixed Integer Programming (MIP), which has a pre-process and post-process to combine the VRPTW. Computational experiments showed that our method reduced delivery cost by 12% compared to the sequential method.

Keywords

Location Routing Problem, Facility Location Problem, Vehicle Routing Problem with Time Windows, Heuristics, Mixed Integer Programming, Transportation mode options

1. Introduction

Recently, specialty shops in Japan that meet consumer needs have witnessed growing sales. For example, sports shops are seeing growing sales due to wider public awareness of health and fitness. Consequently, there will be a shortage of facilities used to deliver products to these shops. We should therefore redesign the locations of such facilities. The problem of determining the facility locations and the shops to which products are to be delivered from each location is called the Facility Location Problem (FLP).

Our criteria for selecting facility locations are the operating cost of facilities and the transportation cost. The operating cost is calculated as the sum of unit prices of the selected facilities. There are two types of transportation cost to be calculated. One is the vehicle rental cost that is calculated by the travel distance of vehicles; the other is the freight cost that is calculated by the freight weight and shipping distance.

The type of calculation is decided by a transportation mode, such as route delivery, direct delivery or courier delivery. In route delivery mode, we deliver products to some shops by using a vehicle and the vehicle rental cost is charged. We should solve the Vehicle Routing Problem with Time Windows (VRPTW) so as to decide the vehicle routes. In direct delivery mode, we also deliver products to a shop by using a vehicle and the vehicle rental cost is charged. In courier delivery mode, we deliver products to a shop by a courier service and the freight cost is charged.

As mentioned above, the types of delivery cost calculation are decided by transportation modes. So, we should select facility locations and transportation modes so as to solve our FLP. The FLP with the VRPTW is a well-known problem called the LRP. Both are NP-hard problems and thus have been studied individually. The simple approach to solving the LRP is to solve each problem sequentially. The sequential method solves the FLP first and then solves

the VRPTW. This method is relatively easy. In this method, however, the locations of facilities and the shops to which products are to be delivered from each location are decided without vehicle routes, so the VRPTW might result in higher cost. For example, in cases where shops have short time windows, a vehicle might not be able to deliver products to more than two shops due to overlapping time windows. If we could consider time windows in the first step, the shops to which products are delivered from each location might be changed so as to minimize the number of vehicles necessary. This means that the vehicle rental cost might also be reduced.

From the perspective of extending models, Melechovsky, J., Prins, C. and Calvo, R. W. (2005) proposed one LRP along with a non-linear costs model that can be solved by metaheuristics. Nagy, G. and Salhi, S. (2007) surveyed the state of the art regarding LRP. In this paper, they showed variations of the LRP, such as the Stochastic LRP and Dynamic LRP.

Many researchers have tackled the challenge of solving the LRP. Wu, Low and Bai (2002) proposed the LRP can be solved by heuristic, Yu. *et al.* (2010) proposed a simulated annealing heuristic and Escobar, John W. *et al*. (2014) proposed a granular variable tabu neighborhood search. Gündiiz, H. I. (2011) proposed one LRP along with time windows that can be solved by tabu search heuristics. Belenguer, J. M., Benavent, E. *et al.* (2011) proposed a branch-and-cut method for LRP and solved small test sets. Mousavi, S. M. and Reza T. (2013) solved a LRP with cross-docking, by a hybrid simulated annealing algorithm. Koç, Çağrı, *et al*. (2016) extended the LRP to have multiple fleet size. Bae, H. and Ilkyeong M. (2016) solved multi-depot VRP with time windows considering delivery and installation vehicles.

This paper is organized as follows: Section 2 introduces the required notation, defines the problem, and proposes a mixed integer linear optimization model; Section 3 presents the facility location sub-problem solved as a mixed integer linear program, which has a pre-process and a post-process to combine the VRPTW; Section 4 presents the computational results; and Section 5 gives some concluding remarks.

2. Original Location Routing Problem

In this paper, the LRP is defined as directed graph $G = (V, A)$. V is a node set that consists of a subset of potential depot sites and a subset of customers. A is a set of arcs linking any two nodes of node set V. The original LRP is described as follows:

Notation

y_d	Binary variable for depot site selection, which is 1 if site *d* is selected and 0 otherwise
z_{di}	Binary variable for customer allocation, which is 1 if customer *i* is supplied from depot *d* and 0 otherwise
x_{ijk}	Binary variable for route selection, which is 1 if node *j* is visited after node *i* by vehicle *k* and 0 otherwise
T_i	Arrival time at customer *i*
w_i	Waiting time at customer *i*
F_d	Initial cost of depot *d*
c_{ij}	Transportation cost of node *i* to node *j*
t_{ij}	Travelling time of node *i* to node *j*
q_i	Demand of customer *i*
Q_i	Capacity of depot *d*
D	Set of potential depot sites
I	Set of customers
K	Set of vehicles

Objective Function

$$\textbf{min cost} = \sum_{d \in D} F_d \cdot y_d + \sum_{k \in K} \sum_{(i,j) \in A} c_{ij} \cdot x_{ijk} \tag{1}$$

Constraints

$$\sum_{i \in I} q_i \cdot z_{di} \leq Q_d \tag{2}$$

$$\sum_{k \in K} \sum_{i \in V} x_{ijk} = 1 \qquad (j \in I) \tag{3}$$

$$\sum_{i \in V} x_{ijk} - \sum_{j \in V} x_{jik} = 0 \qquad (k \in K, i \in V) \tag{4}$$

$$\sum_{d \in D} \sum_{j \in V} x_{dik} \quad \leq 1 \qquad (k \in K) \tag{5}$$

$$\sum_{k \in K} \sum_{i \in S} \sum_{j \in V \setminus S} x_{ijk} \quad \geq 1 \qquad (S \in I) \tag{6}$$

$$\sum_{i \in V} (x_{dsk} + x_{sik}) \leq z_{di} + 1 \qquad (d \in D, i \in I, k \in K) \tag{7}$$

$$\sum_{i \in V} \sum_{j \in I} q_j \cdot x_{ijk} \leq C \qquad (k \in K) \tag{8}$$

$$T_i + w_i \geq a_i \qquad (i \in I) \tag{9}$$

$$T_i \leq b_i \qquad (i \in I) \tag{10}$$

$$-M\left(1 - x_{ijk}\right) - (T_j - T_i - w_i - t_{ij}) \leq 0 \qquad (i \in I, j \in J, k \in K) \tag{11}$$

$$M\left(1 - x_{ijk}\right) - (T_j - T_i - w_i - t_{ij}) \geq 0 \qquad (i \in I, j \in J, k \in K) \tag{12}$$

This model has been tackled by many researchers. We modified the original LRP so as to approximate the actual cost. The transportation cost of our model consists of the vehicle rental cost and the freight cost. The vehicle rental cost is the calculated sum of vehicle multiple unit prices, and defined as a step function of travel distance. The freight cost is the freight charge by weight, and defined as a step function of travel distance. To calculate transportation cost, Eq. (1) is modified as follows:

$$\textbf{min} \text{ cost} = \sum_{d \in D} F_d \cdot y_d + \sum_{k \in K} \sum_{p \in P} cr_{k,p} \cdot xr_{k,p} + \sum_{d \in D} \sum_{i \in I} cd_{d,i} \cdot qf_i \cdot zd_{d,i} \tag{13}$$

Where,

$xr_{k,p}$ Binary variable for travel distance level of vehicle, which is 1 if vehicle *k* travels more than level *p* distance and 0 otherwise

$zd_{d,i}$ Binary variable for courier delivery, which is 1 if customer *i* is supplied by courier delivery from depot *d* and 0 otherwise

$cr_{k,p}$ Vehicle unit price of level *p* in vehicle *k*

$cd_{d,i}$ Unit freight charge from depot *d* to customer *i*

qf_i Freight weight of demand of customer *i*

$xr_{k,p}$ is determined as follows:

$$if\ max_{k,p-1} \leq td_k \leq max_{k,p}\ then\ r_{k,p} = 1 \tag{14}$$

Our model consists of Eqs. (2)-(14).

3. Mixed Integer Programming based Algorithm for LRP

Mixed Integer Programming (MIP) is a very strong method of optimizing objective functions under constrains. However, MIP can only be applied when the model is described as linear equations. Our model is more complex because Eq. (14) is difficult to model by MIP. We divided the original problem into linear and non-linear equations, in an attempt to solve the problem. First, we solve non-linear equations about the VRPTW by heuristics to generate all candidate routes. Second, we model transportation mode selection by linear equations using the result of the first step. In this step, MIP is used in selecting facility locations and routes from candidate routes. Finally, we solve non-linear equations again. In this case, the shops to which products are to be delivered from each location are fixed.

We thus solve this problem in three steps (Fig. 1). The first step is the pre-processing step. We generate candidate vehicle routes by solving the VRP at all potential depot sites. The second step concerns selecting the facility locations and routes by MIP. The third and last step is the post-processing step. In this step, we fixed the routings.

3.1 First Step: Pre-processing by VRP

In this step, we generate all candidate vehicle routes. We assume that all customers are supplied from only one depot site. We solve the VRP for each potential depot site. Figure 2 shows an example of the first step. In this case, there are two potential depot sites (depot *X* and *Y*) and four customers (customer *A*, *B*, *C* and *D*). The right side of Fig. 2 shows the VRP result of depot *Y*.

The VRP of each potential depot site contains Eqs. (3)-(6) and (9)-(12). We solve the VRP by greedy based heuristics. As we need to solve many VRPs, speedy solving is important.

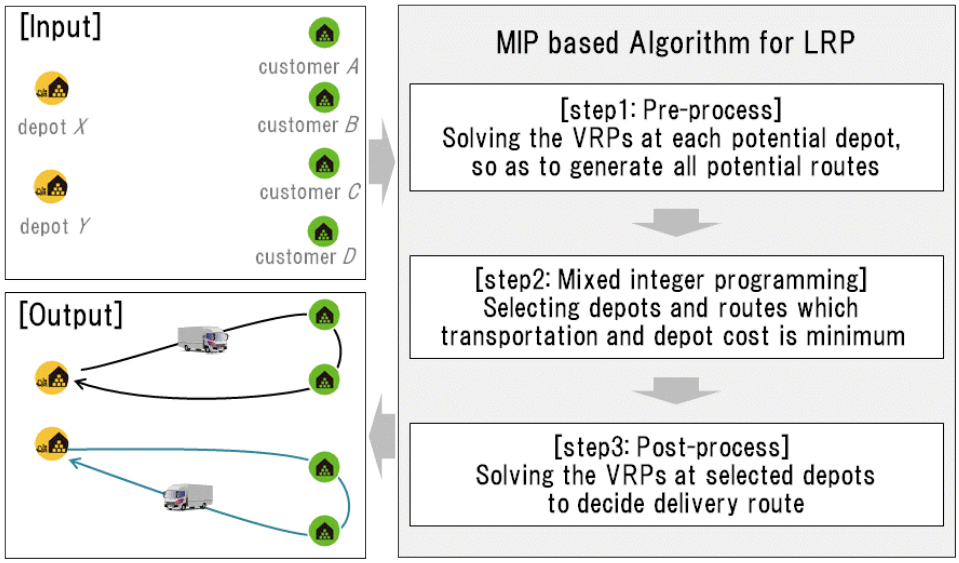

Figure 1. Overview of our method

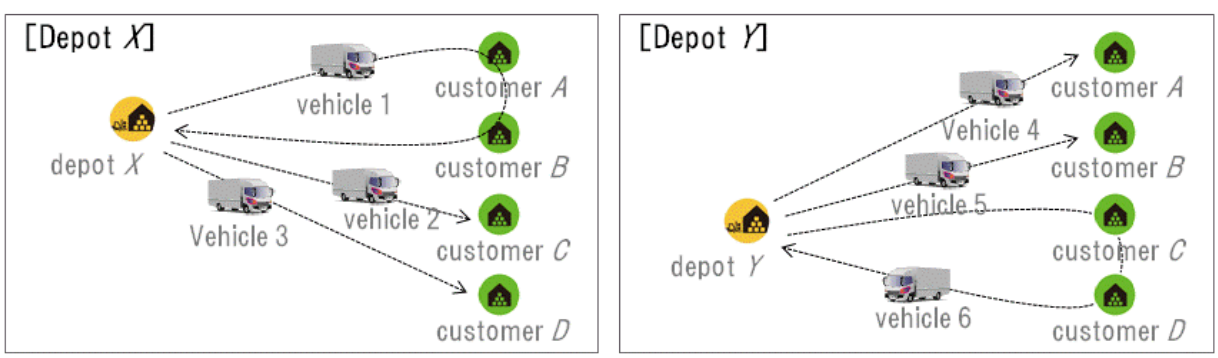

Figure 2. Example of step 1 (VRP of each potential depot)

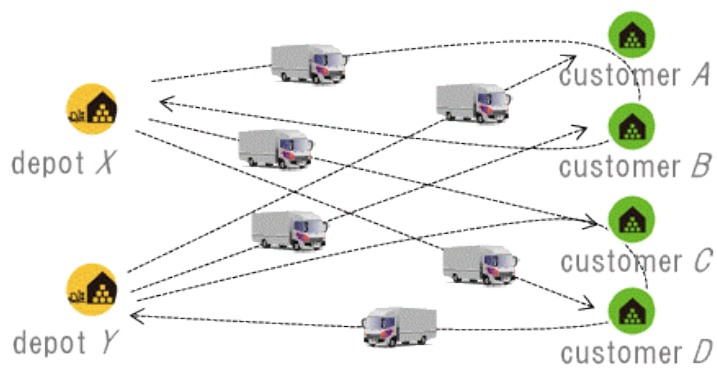

Figure 3. Example of step 2 (potential depots and routes)

3.2 Second Step: Selection of Facility Locations and Routes

In this step, we select the facility locations and routes (Fig. 3). Candidate routes are generated in step 1. Each travel distance of vehicle k is determined and vehicle unit prices are calculated. We use these prices to calculate the objective function.

We modify the FLC as follows:

Objective Function

$$\textbf{min}\ \text{cost} = \sum_{d \in D} F_d \cdot y_d + \sum_{k \in K} VP_k \cdot zv_k + \sum_{d \in D} \sum_{i \in I} cd_{d,i} \cdot qf_i \cdot zd_{d,i} \tag{15}$$

zv_k Binary variable for vehicle selection, which is 1 if vehicle k is selected and 0 otherwise
VP_k Vehicle unit price of vehicle k, calculated in step1

Constraints

$$z_{d,i} \leq \sum_{k \in K | d \in R_k \cap i \in R_k} zv_k + zd_{d,i} \qquad (d \in D, i \in I) \tag{16}$$

$$\sum_{d \in D} z_{d,i} = 1 \qquad (i \in I) \tag{17}$$

R_k Set of nodes which is contained in route of vehicle k

The constraints of our model are shown in Eqs. (16)-(17) and Eq. (2). The objective function is Eq. (15). We solve this FLP by using a mixed integer programming solver, such as the Gurobi Optimizer. Some customers might be located along multiple selected routes. Therefore, we should reroute the vehicles concerned.

3.3 Third Step: Post-processing by VRP

In the last step, we solve the VRP of each selected depot, individually. When all customers are located along only one route, this step can be omitted.

4. Computational Experiments

In this section, we present computational experiments showing the effect of integration. We implemented our method and the sequential method by C++. The sequential method first solves the FLP so as to select depots and allocations, and then solves the VRP. All results were obtained on a PC equipped with a 3.4-GHz Intel Core i7 CPU and 16.0 GB of main memory. We used two test sets containing modified real world data.

Test set 1 is used to verify our method of selecting transportation modes. There are two fixed depots. Our method decides the shops to which products are to be delivered from each depot, and the transportation mode. Test set 2 is used to verify our method of selecting facility locations and transportation modes. There are 15 potential depot sites. Our method determines the facility locations and transportation modes.

4.1 Test set 1: To verify transportation mode selection

We compared our method to the combination method, which enumerates all shop allocations and calculation costs over a limited range. In this test set, there are 80 shops and two fixed depots.

The depot on the west side is called the West depot; the depot on the east side is called the East depot. From the perspective of cost reduction, the West depot should deliver products to shops on the west side and the East depot should deliver products to shops on the east side. However, products can be delivered from either depot to shops located between the West and East depots. Therefore, we generated all allocation patterns, selected a transportation method, and calculated the transportation cost.

Table 1 shows the cost results. The difference between Case 1 and Case 2 is the location of a fixed depot. The results show that our method calculates transportation cost within accuracy of 1% up. As real world input data includes some errors, these results reflect the efficiency of our method.

Table 1. Computational result of test set 1

All values are normalized by combination method.

	Case 1		Case 2	
	Comb. method	Our method	Comb. method	Our method
Vehicle rental cost	0.334	0.406	0.726	0.735
Freight cost	0.666	0.596	0.274	0.273
Total delivery cost	1.000	1.002	1.000	1.008
Number of vehicles	1.000	1.375	1.000	1.000

4.2 Test set 2: To verify facility locations and transportation mode selection

Test set 2 contains 15 potential depot sites and 80 customers in Japan. We calculated three cases where two depots were selected in the first case, four depots in the second case, and six depots in the third case.

Table 1 shows the number of vehicles and the delivery cost of each case.

Table 2. Computational result of test set 2

All values are normalized by previous method.

	Case 1		Case 2		Case 3	
	Sequential method	Our method	Sequential method	Our method	Sequential method	Our method
Vehicle rental cost	0.760	0.604	0.735	0.588	0.656	0.707
Freight cost	0.240	0.281	0.265	0.259	0.344	0.208
Total delivery cost	1.000	0.885	1.000	0.847	1.000	0.914
Number of vehicles	1.000	0.923	1.000	0.929	1.000	1.154
Vehicle rental cost per vehicle	1.000	0.861	1.000	0.800	1.000	0.934

Figure 4 shows the selected depots and customer allocation in Case 1. The figures on the left show allocation by the sequential method; those on the right show allocation by our method. These figures show that the sequential method allocates so as to equal the distance to the farthest customer. The travel distances of vehicles tend to be long. In contrast, our method allocates so that the travel distance of vehicles tends to be short. As a result, our method reduced the route delivery cost per vehicle by 14% compared to the sequential method, and reduced the total delivery cost by 12%.

Figures 5 and 6 show the selected depots and customer allocation in Cases 2 and 3, respectively. Both cases show the same features as Case 1. Our method reduced the route delivery cost per vehicle by 20% in Case 2 and by 7% in Case 3.

As the distance to the farthest customer is relatively shorter than the vehicle travel distance, the rate of reduction is smaller than in Case 2.

These results confirmed that our method efficiently selects depots and routes in order to reduce delivery cost.

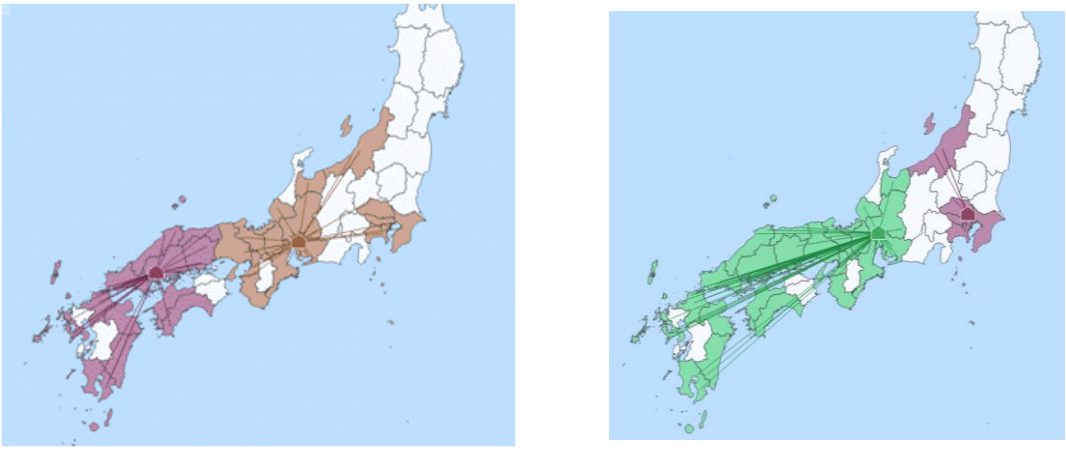

Figure 4. Customer allocation in Case 1

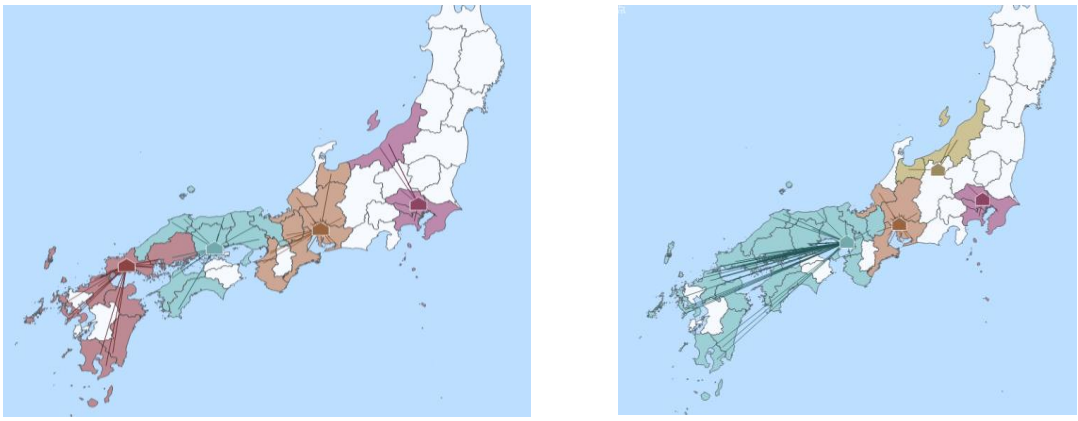

Figure 5. Customer allocation in Case 2

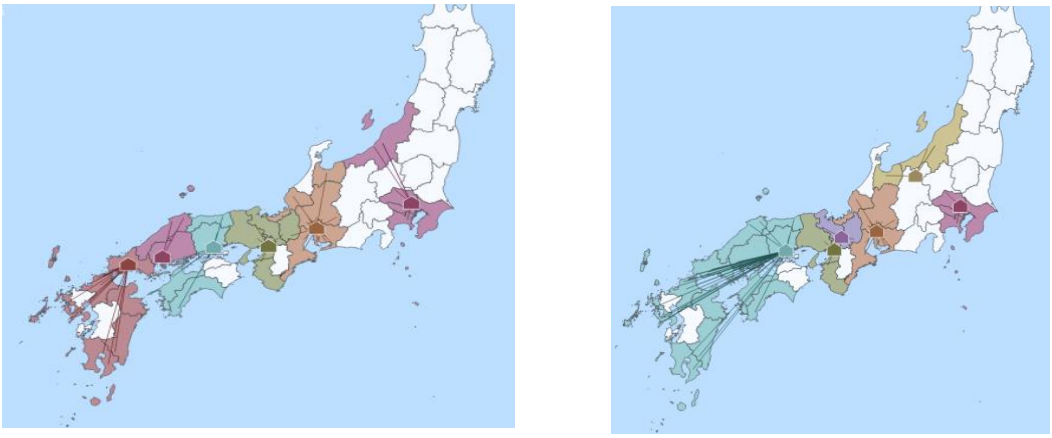

Figure 6. Customer allocation in Case 3

Source: National Land numerical information, Ministry of Land, Infrastructure, Transport and Tourism of Japan

5. Conclusions

In this paper, we presented an extended model of the LRP that can handle several transportation modes. And we proposed a mixed integer programming based algorithm for the LRP. The algorithm has a pre-process and a post-process so as to integrate the VRP into the FLP.

Through our preliminary experiments with sample data, we observed that the model could yield cost reduction due to the proper selection of transportation modes. Computational experiments showed that our method reduced delivery cost by 12% compared to the sequential method.

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