Revenue Management Model with Customer Behavior: An Application for Road Transportation Industry

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Abstract

In this paper a revenue management model with customer behavior approach is applied to a problem in road transportation industry. Revenue management problems are studied extensively for airline and tourism industries which have conditions such as perishable capacity booked prior to use, different fix prices for different classes, and fluctuation of availability of classes. Although revenue management is applicable to road transportation industry, few studies can be found in literature. The problem is to find an optimal decision policy to decide which subset of fare classes to offer in each point in time while considering customer behavior such as buy-up and buy-down. Well known discrete-time dynamic programming model with Bellman equations is used to solve the cases of full truck load and less than truck load in the road transport industry. Efficient sets are defined and used to reduce the size of solution space and to determine the optimal decision policy. Hypothetical examples illustrated that for a high demand environment this model can increase the expected revenue for road transportation industry.

Keywords
Revenue Management, Discrete-time Dynamic Programming, Road Transportation

1. Introduction

Revenue management methods are used by airline and tourism industries since 1970. To apply these methods, it is necessary to have a stable demand, , perishable capacity, booking capacity before use , reusable inventory, different customer segments and the ability to estimate the future demand. Segmentation, forecasting, budgeting, pricing and capacity control are main research areas of revenue management.

Littlewood's (1972) paper about passenger control and forecasting is the pioneering work which dramatically changes the airline industry. For single-leg revenue management problems, the models EMSR-a and EMSR-b were developed by Belobaba (1987) and explained in Belobaba (1987) and Belobaba (1989). Sönmez and Esnaf (2015) used EMSR-a and EMSR-b methods in Turkish railway passenger transportation and improved the expected revenue.

Talluri and van Ryzin (2004a) discussed the most of the problems in revenue management applications. Their article proposes a model for the revenue management problem with customer behavior (Talluri & van Ryzin, 2004b). Gallego, et al. (2004) developed a network revenue management model when multiple products are demanded. Liu and van Ryzin (2008) extended the customer behavior model by including network structure to the problem. Fiala (2012) developed a three-layer model for revenue management problems with customer behavior. Özkan, et al. (2015) showed how optimal policy changes in different environments. Sierag, et al. (2015) included overbooking and cancellations to the basic model with customer behaviors. Koenig and Neissner (2015) searched for risk aware optimal policies.

Demand forecasting is another research area in revenue management. Azadeh, et al. (2015) developed a non-parametric approach for demand forecasting.
Discrete-time models are used in revenue management models because of the complex nature of continuous-time models. Arslan and Frenk (2015) used a continuous-time model with Poisson arrivals to overcome this complexity. Krishner and Rediak (2015) also used a continuous-time network revenue management model where the demand is rejected if a threshold criteria is not satisfied.
There are researches about logistics industry but these researches are related to air cargo problem. Huang and Lu (2015) considered an air cargo problem as a network revenue management problem and developed a network model. The paper of Wang et.al (2015) which is one of the few examples about sea transportation involves with seasonal container deliveries. With high demand environment, road transportation is a suitable industry for revenue management researches. The vehicles usually serve two different freight services: full-truck-load (FTL) and less-than-truck-load (LTL). In this paper, the discrete-time dynamic programming model developed by Talluri and van Ryzin (2004b) is applied to road transportation industry in order to increase expected revenues. When similar studies in the literature are examined, papers using the discrete-time dynamic programming model for road transportation problem are few.
The remainder of the paper is organized as follows. The problem is introduced and a model is proposed in the second section. In the third section some concepts are defined like efficient sets which reduce the solution space dramatically. In the fourth section of the study two hypothetical examples are presented, solved and solutions are summarized. Finally, in section 4 we summarized the results and discussed future work.

2. Discrete-time Dynamic Programming Model

A discrete-time dynamic programming model developed by Talluri and van Ryzin (2004b) is used here and their model assumes that time is discrete and each discrete period is the smallest time unit that only one demand can occur. Time is denoted by t, $t = 0$ is the time of service $and\ t = T$ is the current time showing that T periods are left to the service time. The demand probability is shown by λ and it is considered same for all time periods t. There are n fare classes, $N = \{1, \ldots, n\}$ is the set of all fare classes. For every $j \in N$ fare class, revenue shown by r_i is obtained. Fare classes are indexed such that their revenues increase while their indices decrease, $r_1 \geq r_2 \geq \cdots \geq r_n \geq 0$.
For each time period t , the company must offer a subset $S \subseteq N$ of fair classes. When the set of fair classes S is offered, the probability of purchasing a fair class j is denoted by $P_j(S)$. When $j \notin S$ then $P_j(S) = 0$. $j = 0$ denote the no-purchase case while $P_0(S)$ is the no-purchase probability. Hence the probability of purchasing a fare class j is $\lambda P_j(S)$ and the probability of no-purchase is $\lambda P_0(S) + (1 - \lambda)$.
For each $S \subseteq N$ with $j \in S$, it is assumed that $P_j(S) \geq 0$ and $\sum_{j \in S} P_j(S) + P_0(S) = 1$ for the sake of satisfying probability properties.
Let C denote the total number of vehicles or total capacity. x denotes the number of available vehicles. Let $V_t(x)$ denote the maximum expected revenue. Then the Bellman equation for $V_t(x)$ is defined by the following equation:

$$V_t(x) = \max_{S \subseteq N} \left\{ \sum_{j \in S} \lambda P_j(S) \left(r_j + V_{t-1}(x-1) \right) + (\lambda P_0(S) + 1 - \lambda) V_{t-1}(x) \right\}$$

$$V_t(x) = \max_{S \subseteq N} \left\{ \sum_{j \in S} \lambda P_j(S) \left(r_j - \Delta V_{t-1}(x) \right) \right\} + V_{t-1}(x), \tag{1}$$

$\Delta V_{t-1}(x) = V_{t-1}(x) - V_{t-1}(x-1)$ is the marginal cost of capacity. The boundary conditions are:

$$V_t(0) = 0, \quad t = 1, \ldots, T$$

and

$$V_0(x) = 0, \quad x = 1, \ldots, C$$

2.1 Optimization

Equation (1) can be written briefly:

$$V_t(x) = \max_{S \subseteq N}\{\lambda(R(S) - Q(S)\Delta V_{t-1}(x))\} + V_{t-1}(x) \tag{2}$$

Here

$$Q(S) = \sum_{j \in S} P_j(S) = 1 - P_0(S)$$

$$R(S) = \sum_{j \in S} P_j(s) r_j$$

$Q(S)$ is the total purchase probability and $R(S)$ is the total expected revenue when the set S is offered.

2.1.1 Efficient Sets

To find the optimal solution, 2^n subsets must be evaluated. Talluri and van Ryzin (2004b) proved that the evaluation of efficient sets is enough to find an optimal strategy as given below.

Definition 1: A set T *is* inefficient, if there exists $\alpha(S), \forall S \subseteq N$ and $\sum_{S \subseteq N} \alpha(S) = 1$ such that

$$Q(T) \geq \sum_{S \subseteq N} \alpha(S) Q(S)$$

and

$$R(T) < \sum_{S \subseteq N} \alpha(S) R(S)$$

and otherwise inefficient.

Property 1: Inefficient sets are never an optimal solution to DP model.

Property 2: If efficient sets are ordered such that $Q(S_1) \leq Q(S_2) \leq \cdots \leq Q(S_m)$, *then* $R(S_1) \leq R(S_2) \leq \cdots \leq R(S_m)$

Theorem 1: An optimal policy for the model is to choose k^* sets from $\{S_k: k = 1, \dots, m\}$ ordered efficient sets which maximize the model. For a specific t, the biggest optimal index, k^*, increases with the remaining capacity x and for a specific x, decreases with the remaining time t (Talluri & van Ryzin, 2004b).

This theorem states that for the ordered sets the optimum set index is increasing with the remaining capacity and decreasing with the remaining time.

2.1.1.1 Determination of Efficient Sets

After selecting the first efficient set as $S_0 = \emptyset$, with "greatest marginal revenue" method efficient sets can be determined. S_i being the ith efficient set, $(i+1)th$ efficient set, S_{i+1}, is the one with $Q(S) \geq Q(S_i)$ and $R(S) \geq R(S_i)$, which maximizes the marginal revenue ratio (Talluri & van Ryzin,2004b).

$$\frac{R(S) - R(S_i)}{Q(S) - Q(S_i)}.$$

After finding the efficient sets, optimal policy can be found with the help of Theorem 1. For each efficient set S_k, let p_k be the protection level of efficient set S_k. If efficient sets are ordered and indexed such that each set in the sequence is a subset of the next set, protection levels show which sets to offer. If the remaining capacity is less than the protection level p_k, the sets before S_k in the sequence are closed. Protection levels are defined as follows:

$$p_k = max\{x: R_k - Q_k \Delta V_{t-1}(x) > R_{k+1} - Q_{k+1}\Delta V_{t-1}(x)\}, \qquad k = 1,2,\dots,m-1$$

It's assumed that $p_0 = 0$ and $p_m = C$ (Talluri and van Ryzin, 2004b).

3. Hypothetical Examples

Two hypothetical examples are used to show the behavior of the model in two well-known conditions in road transportation industry. In the first situation all loads are the full-truck-load; second example is related to the less-than-truck-load case.

3.1 Hypothetical Example 1

It is assumed that an international freight company offers three fare classes for a known region. These fare classes are express (ex), normal (n) and flexible (fl) fare classes. With normal fare class, the company delivers the cargo in 7-8 days. When express fare class is chosen, the delivery duration reduces to 6 days. Express fare class is the most expensive fare class. The flexible fare class guarantees the delivery up to 10 days and cheaper than other fare classes.

Table 1: Fare Classes and Their Properties

Fare Class	Deadline	Revenue
Express	6 days	300 €
Normal	7-8 days	200 €
Flexible	Up to 10 days	150 €

Customers of the company come from five different segments. In table 2, the customer preferences and the probabilities of arriving customers being in each segment is given:

Table 2: Customer segments and their properties for example 1

Customer Segment	Probability	Express Choice	Normal Choice	Flexible Choice
Seg. 1	0,1	Yes	No	No
Seg. 2	0,2	Yes	Yes	No
Seg. 3	0,3	No	Yes	No
Seg. 4	0,2	No	Yes	Yes
Seg. 5	0,2	No	No	Yes

Table 3 shows the choice probabilities according to given information. For an independent demand model, if fare class k is closed than all the demand for this fare class is lost. In choice-based model, when fare class k is closed customers may choose to buy from other classes (customers buy-up or buy-down).

Table 3: Choice probabilities of each fare class sets, *Q(S)*, *R(S)* and efficiency

S	$P_{Ex}(S)$	$P_N(S)$	$P_{Fl}(S)$	$P_0(S)$	$Q(S)$	$R(S)$	Efficient?
$\{\emptyset\}$	0	0	0	1	0	0	Yes
$\{Ex\}$	0,3	0	0	0,7	0,3	90	Yes
$\{N\}$	0	0,7	0	0,3	0,7	140	No
$\{Fl\}$	0	0	0,4	0,6	0,4	60	No
$\{Ex, N\}$	0,1	0,7	0	0,2	0,8	170	Yes
$\{Ex, Fl\}$	0,3	0	0,4	0,3	0,7	150	No
$\{N, Fl\}$	0	0,5	0,4	0,1	0,9	160	No
$\{Ex, N, Fl\}$	0,1	0,5	0,4	0	1	190	Yes

For the first example, the sets $\{\emptyset\}, \{Ex\}, \{Ex, N\}$ and $\{Ex, N, Fl\}$ are efficient sets. Efficient sets are denoted in Table 3. The protection levels are calculated and shown in Table 4.

As seen in table 4, when the capacity is below 11, S_1 has the highest expected revenue ($\boldsymbol{R_k - Q_k \Delta V_{t-1}(x)}$), so $S_1 = \{Ex\}$ is labeled as the optimum set. If the capacity is between 12 and 19, the expected revenue of S_2 becomes the highest, the set $S_2 = \{Ex, N\}$ becomes optimum and in this range of remaining capacity, both Express and Normal services are offered. When the capacity is 20, since S_3 has the highest expected revenue value, Flexible class is opened when all services are offered.

Hypothetical Example 2 shows a revenue management problem with less-than-truck load (LTL) case:

3.2 Hypothetical Example 2

The first example will be generalized to the case where LTL is possible. In the road transportation industry, capacity is measured by two variables; volume and weight. The volume is limited to the vehicle's volume while the weight is limited by the legislations. With this two-dimensional capacity case, the model can be updated as follows:

$$V_t(x,y) = \max_{S \subseteq N} \left\{ \sum_{j \in S} \lambda P_j(S) \left(r_j + V_{t-1}(x-a, y-b) \right) + (\lambda P_0(S) + 1 - \lambda) V_{t-1}(x,y) \right\}$$

$$V_t(0,0) = 0, \quad t = 1, \dots, T$$

$$V_0(x,y) = 0, \quad x = 1, \dots, X \;\; y = 1, \dots, Y$$

$$a \leq x$$

$$b \leq y$$

a and b are the random variables which shows the volume and weight of the cargo. X is the total capacity with respect to volume and Y is the total capacity with respect to weight.

Table 4: Protection levels for the first example

x	$\Delta V_{t-1}(x)$	$R_k - Q_k \Delta V_{t-1}(x)$			$k_t^*(x)$
		k=1	k=2	k=3	
1	299,85	0,05	-69,88	-109,85	1
2	298,59	0,42	-68,87	-108,59	1
3	293,65	1,91	-64,92	-103,65	1
4	281,47	5,56	-55,18	-91,47	1
5	260,34	11,90	-38,28	-70,34	1
6	232,93	20,12	-16,35	-42,93	1
7	205,44	28,37	5,65	-15,44	1
8	183,70	34,89	23,04	6,30	1
9	170,04	38,99	33,97	19,96	1
10	163,27	41,02	39,38	26,73	1
11	160,69	41,79	41,45	29,31	1
12	159,97	42,01	42,02	30,03	2
13	159,09	42,27	42,73	30,91	2
14	156,09	43,17	45,13	33,91	2
15	149,07	45,28	50,74	40,93	2
16	137,05	48,89	60,36	52,95	2
17	121,96	53,41	72,43	68,04	2
18	108,58	57,43	83,14	81,42	2
19	100,85	59,75	89,32	89,15	2
20	81,37	65,59	104,90	108,63	3

Expressing the capacity in two dimensional, continuous variables increase the complexity of the problem. To restate the problem with discrete variables some assumptions has to be made similar to the air-cargo model of Huang, et al. (2015).

In LTL case, two-dimensional capacity measure is reduced to one-dimensional measure by using a special unit called lademeter. A trailer is 13,6 lademeters. A lademeter is a one-meter-long part of a truck or load of 1250 kg. If considering volume results give a higher lademeter value then volume is used to find lademeter, otherwise weight is used. When lademeter is used as the capacity unit, the model is simplified as follows:

$$V_t(x) = \max_{S \subseteq N} \left\{ \sum_{j \in S} \lambda P_j(S) \left(r_j + V_{t-1}(x-c) \right) + (\lambda P_0(S) + 1 - \lambda) V_{t-1}(x) \right\}$$

$$V_t(0,0) = 0, \quad t = 1, \dots, T$$

$$V_0(x,y) = 0, \quad x = 1, \dots, C$$

$$c \leq x$$

c is the lademeter measurement of the cargo and C is the total capacity according to lademeter.

The production levels for all x and all t are given in Table 5 for the second example. For a constant t, the protection levels increase with x and for constant x the protection levels decrease with t.

The effects of control policy and chosen policy are determined by a simulation. The simulation generates demanded capacities from a given interval of acceptable lademeters. The demanded capacity is generated from a uniform probability density function. The probability of an occurrence of a demand in a given period t is $\lambda = 0{,}98$. The simulation is run for 100 times. The results are compared with the "all demands are accepted by occurrence order (ADAOO)" policy.

The results are summarized in Table 6.

Table 5: Production levels for example 2

k*	**t**																				
		20	**19**	**18**	**17**	**16**	**15**	**14**	**13**	**12**	**11**	**10**	**9**	**8**	**7**	**6**	**5**	**4**	**3**	**2**	**1**
x	**1**	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	3
	3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	3	3
	4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	2	3	3	3
	5	1	1	1	1	1	1	1	1	1	1	1	1	1	2	2	2	3	3	3	3
	6	1	1	1	1	1	1	1	1	1	1	1	2	2	2	2	3	3	3	3	3
	7	1	1	1	1	1	1	1	1	1	2	2	2	2	2	3	3	3	3	3	3
	8	1	1	1	1	1	1	1	1	2	2	2	2	2	3	3	3	3	3	3	3
	9	1	1	1	1	1	1	2	2	2	2	2	2	3	3	3	3	3	3	3	3
	10	1	1	1	1	2	2	2	2	2	2	3	3	3	3	3	3	3	3	3	3
	11	1	1	2	2	2	2	2	2	2	3	3	3	3	3	3	3	3	3	3	3
	12	2	2	2	2	2	2	2	2	3	3	3	3	3	3	3	3	3	3	3	3
	13	2	2	2	2	2	2	2	3	3	3	3	3	3	3	3	3	3	3	3	3
	14	2	2	2	2	2	2	3	3	3	3	3	3	3	3	3	3	3	3	3	3
	15	2	2	2	2	2	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
	16	2	2	2	2	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
	17	2	2	2	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
	18	2	2	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
	19	2	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
	20	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3

As seen in table 6, the model produces better results for high demands but same results for low demands. When demand is generated in a highly variant environment, ADAOO produces better results.

Table 6: Simulation Results

Demand interval (lademeter)	**ADAOO Mean Revenue**	**DP Mean Revenue**	**Difference**
1-10	3716,5	3716,5	%0
1-20	3733,0	3711,5	-%1
10-20	2891,0	3185,5	%10

4. Conclusion

In this paper, the discrete-time dynamic programming model developed by Talluri and van Ryzin (2004b) is used for the application of the road transportation industry. In the road transportation industry, when demand is large, such that each demand requires nearly a whole truck or more, the revenue can be increased using an optimal policy determined by solving the discrete-time dynamic programming model. The customer behaviors are highly variable when different fare products are offered because road transportation industry mostly delivers raw materials to a production plant with strict stock policies and manufacturing schedules. Therefore choice based model is preferable than independent demand model when considering customer behavior in revenue management problems for this industry. However only two hypothetic examples are employed, the results show that revenues can be increased by 10% after using optimal policy. These results prove that the discrete-time dynamic programming-based model with Bellmann equations and efficient sets is applicable to road transportation industry and its usage is highly recommended.

For the future research, the model can be extended by considering network case as a network revenue management problem. Another interesting extension is using two-dimensional capacity vector while taking cargo legislations into account. In order to calculate revenues for each demand, a preliminary vehicle routing problem can be solved. Moreover one could consider using time-dependent probabilities for demand.

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