

# **An optimized algorithm for 2D images comparing based on Hausdorff Distance**

**Amina Kharbach, Amar Merdani, Benaissa Bellach, Mohamed Rahmoun**

Physics and Engineering Department, LSE2I, ENSAO  
University Mohammed Ist  
Oujda, Morocco

[a.kharbach@ump.ma](mailto:a.kharbach@ump.ma), [merdani.amar@yahoo.fr](mailto:merdani.amar@yahoo.fr), [benaisa.bellach@gmail.com](mailto:benaisa.bellach@gmail.com),  
[moham.rahmoun@gmail.com](mailto:moham.rahmoun@gmail.com)

**Mouad Elomari**

Applied Mathematics and Information Systems Department, MATSI, ESTO  
University Mohammed Ist  
Oujda, Morocco

[elomari.mouad@gmail.com](mailto:elomari.mouad@gmail.com)

## **Abstract**

The ability to compare two or more images, or finding duplicate images in a large collection, is a very tricky matter. For this reason, all scientific research in this field are aimed at studying how to describe and achieve what is best for quantifying the difference between two images because it's often needed in automated visual inspection. In this paper, we propose an improved algorithm for 2D images comparing based on Hausdorff Distance which is used to measure the degree of similarity or dissimilarity between two objects to make matching more efficiently. In this case we used Genetic Algorithm which is a powerful and attractive procedure for function optimization, but the solution generated by this technique do not guarantee to be the global optimal. A follow-up optimization scheme such as the line search method is applied, which is capable of finding the minimum value of a unimodal function over a finite search interval. The experimental results show that the proposed method is capable of matching 2D shape with higher speed and precision.

## **Keywords**

Hausdorff Distance; Genetic algorithm; Line search; Shape matching.

## **1. Introduction**

All scientific research are aimed at studying how to describe and achieve what is best. One of the main problems is to match a set of image or compare their information. Shape matching designed to measure the similarity between two given shapes using some distance measure. It plays an important role in many of today's applications such recognition, classification, alignment and registration; citation of S.Belongie (2002), J.Wang (2012), X.Huang (2006). Matching has been approached in a number of ways, including the generalized Hough transform (D.H. Ballard 1981), geometric hashing (G. Stockman 1987), the alignment method (D. Huttenlocher 1987), Fourier descriptors (S. Loncaric 1998), wavelet transform (C. Jacobs 1995) and neural networks (S. Gold 1995).

Traditional template matching based on relevant criteria (translation, rotation) requires extremely long calculation. Therein several solutions were proposed to optimize algorithms for measuring the resemblance between images which are an essential ingredient in shape matching (J-X.Du 2005, X.Zhu 2008).

Although the term similarity is often used, it corresponds to the notion of distance. The dissimilarity must be small when two shapes contain similar regions, and the measure should not penalize for regions that do not match. In our work we used Hausdorff distance which represents one of the important tool that applies to detect difference between shapes (D-G. Sim 1999, D.P. Huttenlocher 1993).

Among the various algorithm of optimization, we have chosen the Genetic Algorithm that is introduced in the 1970s by John Holland and it is based on techniques derived from the genetic and evolutionary mechanisms of nature: crossover, mutation, selection. It has been successfully applied to a variety of optimization problems (K.F. Man 1996) and is used to quickly find the optimal solution of the object matching. However, the solution generated by this technique do not guarantee to be the global optimal. In order to remedy this problem, we used linear search method (A.Kumari 2012) to improve the accuracy of the object matching. It finds a particular value in a list that consists of checking every one of its elements, one at a time and in sequence until the desired one is found.

The rest of the paper is composed as follows. Section 2 describes the operation of Hausdorff distance, lists its properties and justifies its use. Section 3 present genetic algorithm method, its strategy and stages of implementation, section 4 describes linear search method and shows its efficiently by proposed algorithm, in section 5 we present our results, we conclude in Section 6.

## 2. Hausdorff Distance

Felix Hausdorff German mathematicien has developed a distance that returns a large value if the two compared images is very different and small if they look alike. It can be defined by:  
Given two finite sets  $A = \{a_1, \dots, a_p\}$  and  $B = \{b_1, \dots, b_q\}$ . Hausdorff distance between A and B is defined as follows:

$$H(A,B) = \max( h(A,B) , h(B,A) ) \quad (1)$$

Where  $h(A,B)$  ranks each point of A based on its nearest point of B and uses the most mismatched point. It is given by:

$$h(A, B) = \max_{a \in A} \min_{b \in B} \| a - b \| \quad (2)$$

$$h(B, A) = \max_{b \in B} \min_{a \in A} \| b - a \| \quad (3)$$

Where  $\| \cdot \|$  defined a certain distance range of scattered point sets, the article uses the Euclidean distance.

The Hausdorff distance is very useful as a dissimilarity measure between graphical objects, it returns zero if both sets are identical, moreover any object transformation (translation, rotation, scaling...) can be taking into consideration by searching for the minimum of the directed Hausdorff distance between original object and transformed one. It is well known that the Hausdorff distance is a metric that is satisfying identity, uniqueness, and strong triangle inequality, with:

- Non-negativity means  $d(A,B) \geq 0$ .
- Identity property says that  $d(A,B) = 0$  implies  $A=B$ .
- Symmetry is that  $d(A,B) = d(B,A)$ .
- Triangle inequality is  $d(A,C) \leq d(A,B) + d(B,C)$ .

These properties can be used for a fast data base search, thanks to its, a big parts of a transformation parameters space can be excluded from computation when, for example, we are seeking for a minimum of the Hausdorff distance between one object and geometrical transformation of another.

### 3. Optimization model based on Hausdorff distance in shape matching

In this article, we mainly consider three parameters  $(x, y, \theta)$  that represent respectively translation and rotation transformation applied to the object. As shown in Figure 1, The key of shape matching is to find an optimal set of those parameters  $(x, y, \theta)$  for higher degree of matching.

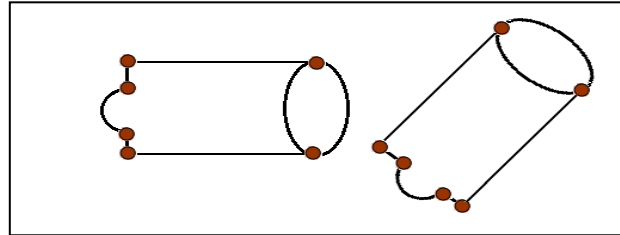


Figure 1. Experiment with the object O and model M

Two main steps are included to determine parameters of the best match between shapes.

- Establishment of a optimisation model
- Solving the optimization model.

The direct comparison of the degree of similarity of the two large-size object edge image requires a very large amount of calculation, in this case we deal with edge feature points instead of the edge point to calculate Hausdorff distance that reduces the computational complexity.

Let  $O$  is the object edge feature set of points,  $M$  is a feature point set of model edge,  $t = (x, y, \theta)$  as a model translation and rotation parameters. Problem searched using Hausdorff distance can be expressed as follows:

$$\min H(t(M), O) \quad (4)$$

Usually the transformation  $t$  can be given by:

$$t(M) = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} * M \pm \begin{bmatrix} x \\ y \end{bmatrix} \quad (5)$$

### 4. Solving parameters of Matching

The existence of multiple local minima in the search space by using equation (4) requires a lot of computing time. Here we first use the genetic algorithm to accelerate the process of matching, then we use linear search method to further improve search accuracy.

#### 1.1 Genetic Algorithms

A genetic algorithm is a method for solving optimization problems based on a natural selection process that mimics biological evolution. Unlike, Classical Algorithm that generates a single point at each iteration, Genetic Algorithm generates a population of points, the best point in the population approaches an optimal solution.

To use it, three problems are to be solved, constructing fitness function, coding type, and choosing suitable operators:

- A principle of coding : data coding quality condition the success of genetic algorithm, in our case we use gray code that has as a following propriety : Between two elements  $n$  and  $n+1$  one bit differs unlike the

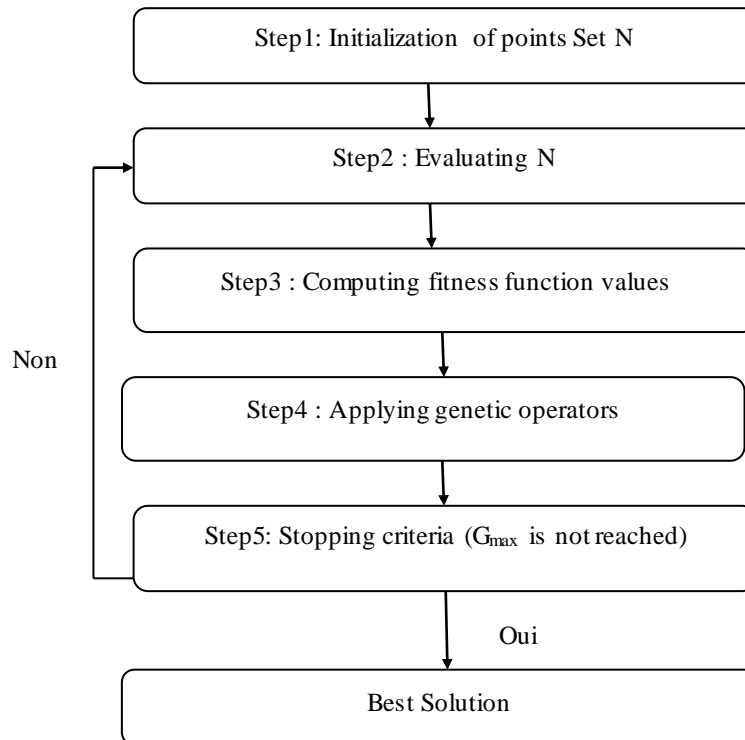
binary coding that often use hamming distance as a measure of dissimilarity and this coding shows its limits.

- Initial population generation mechanism: The choice of this set is important because it can make convergence more or less quick to global optimum.
- The fitness function: Aims to provide a meaningful, measurable, and comparable value given a set of genes (set of pixels in our case). We proposed this following function where we add the number one in order to avoid null value in the denominator:

$$F(x,y,\theta)=1/(1+H(t(M),O)) \quad (6)$$

- Genetic operators: Generally, genetic algorithm is characterized by three types of operators: selection, crossover, mutation that their objective is to achieve a better solution. In this paper, we assume that population size ( $N = 50$ ), the crossover rate ( $pc = 1.0$ ), mutation rate ( $pm$ ) and ( $G_{max}$ ) the maximum iterative step (see table 1).
- Initialization and stopping criteria : the convergence condition can be of various kinds, in our case we have combined the minimum rate wish we hope to attain and a computation time not be exceeded.

The algorithm can be resumed by this:



## 1.2 Linear Search

Results obtained using genetic algorithm in the first stage will be treated by a linear search method which presents a feasible solution for the decision variables as well as a reasonable computational CPU time . The main advantage of this method is its ability to locate the near global optimal solutions for the fitness function with its strong criteria of global convergence. In our work we propose this algorithm as follows:

---

**Algorithm1: Linear search ()**

---

1. Set the initial search step  $g$ , shrink ratio step  $e$ , solver precision  $h$
2. Set the initial variable value  $v_i$  ( $i = 1, 2, \dots, n$ ),  $n$  is the total number of variables.
3. While (search step size not less than precision  $h$ )
  - do ( $i = 1$  to  $n$ )
    - {
    - 4. Calculate the objective function value  $H(v_i + g)$ ,  $H(v_i - g)$ ,  $H(v_i)$ 
      - if  $H(v_i + g) < H(v_i)$ 
        - let  $v_i = v_i + g$
      - else if  $H(v_i - g) < H(v_i)$ 
        - let  $v_i = v_i - g$
    - }
5. If (No more superior than the previous solutions may be obtained)

←  $g$

$g / e$

---

In the iteration start-up phase, a one-dimensional search selects a large step to speed up the convergence rate. Search results gradually tend to be the optimal solution, thereafter, search step is gradually reduced until it meets the search accuracy requirements. When  $e = 0.618$ , the search process for the golden section search; when  $e = 0.5$ , the search process for binary search.

#### 4. The experimental results

In this article, we use matlab language to implement the methods described above. Input images resolution is 219x214. Genetic Algorithms values and linear search parameters are as shown in Table 1, wherein the  $(x, y)$  range is  $[-256, 255]$  and  $\theta$  range is  $[-\pi, \pi]$ . Encoding is using gray code; initialization of a gene is randomly generated. To verify the affectivity of our algorithm, a lot of simulated experiments are performed. What follows is an example of the simulations. Firstly, figure 2 presents feature point sets of  $M$  and  $O$  respectively.

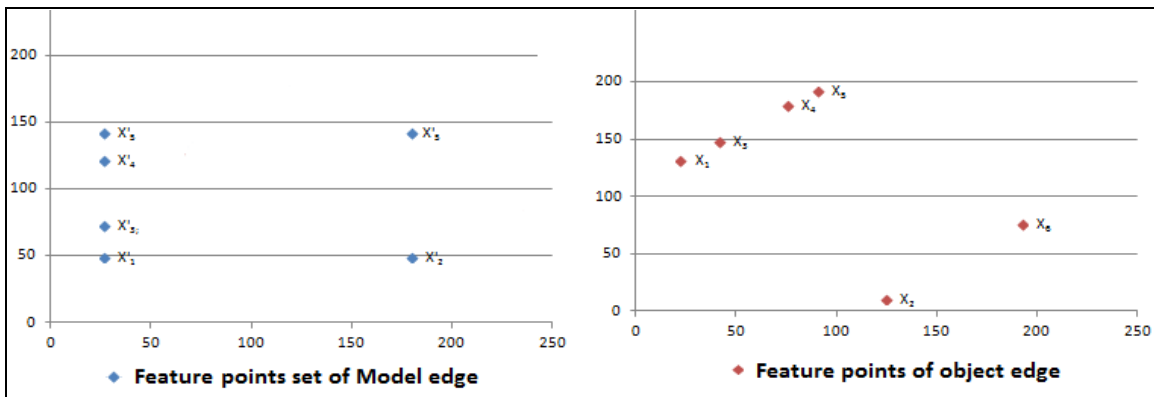


Figure 2. Feature point sets of  $M$  and  $O$

Our aim is to find optimal set of transformation parameters. In this experiment, the parameters of algorithm are set as:  $N=50$ . After running our algorithm, the best chromosome is obtained and its corresponding affine transformation parameters are presented in table 2. To give an intuitive observation, we plot  $T(M)$  together with  $O$  in figure 3, it is easy seen that each matchable point approach its pairing point very near.

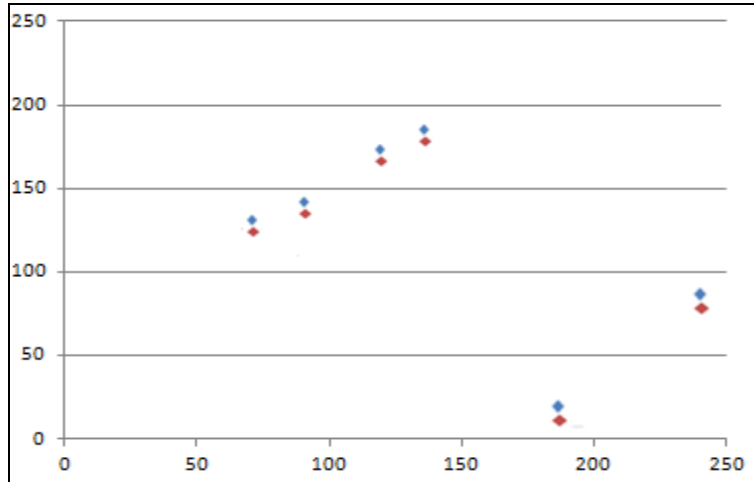


Figure 3. Genetic algorithm implementation result

Using the linear search method the matching result is completely correct( figure 4).

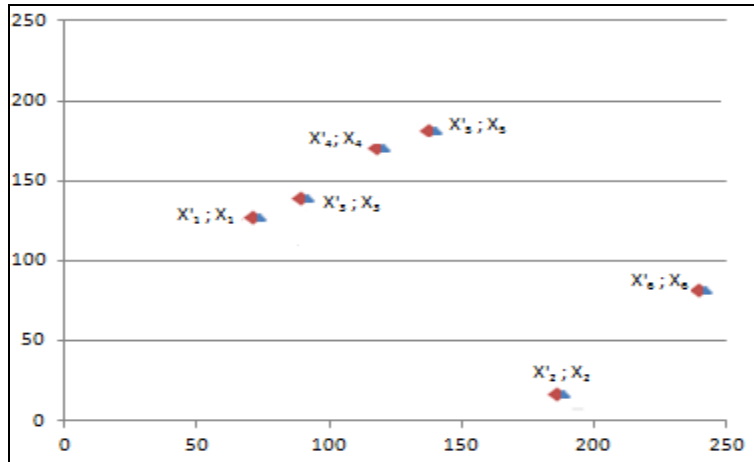


Figure 4. Linear search implementation result

Our algorithm has been 20 independent operations; figures 5 and 6 present respectively each iteration of the genetic search process and linear search results for two images mentioned above.

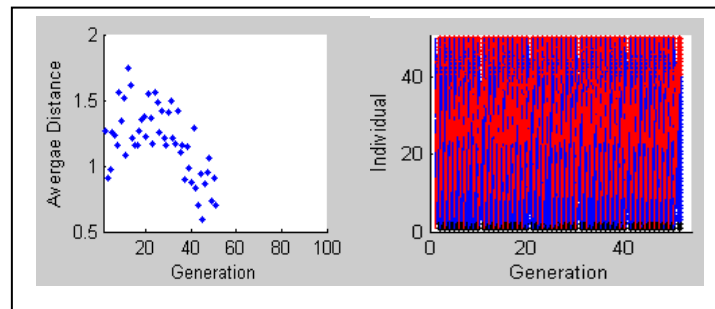


Figure 5. Illustrative diagram of average distance and evaluation pixels according to two images M and O

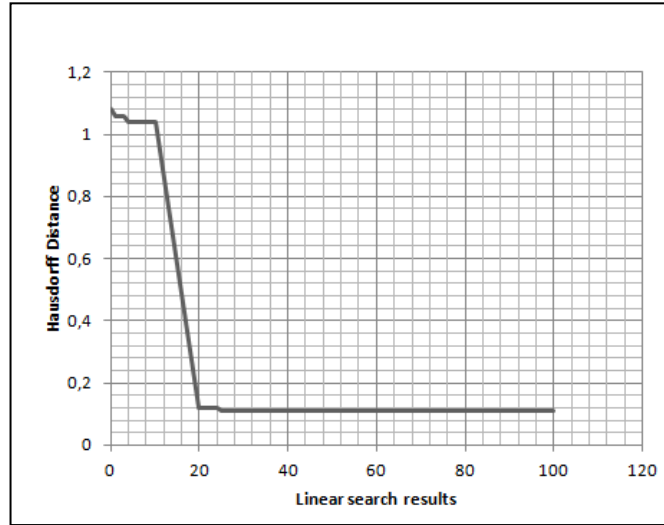


Figure 6. Iterative search and linear search results

Calculation errors, the measured average and running time of the algorithm of two search results are shown in table 2, You can see that the average genetic algorithm stop running in the 84th, linear search iteration stops after 23 steps. It must be pointed out that our optimization matching method based on genetic algorithm can achieve good result for shapes matching, but it will cost much more computational time to obtain a better result; the other used method (Linear search) compute much more fast. So, our proposition will be a good choice to optimize shape matching.

TABLE 1. EXPERIMENTAL PARAMETERS

Parameters	
<i>Genetic search</i>	<i>Linear search</i>
N = 50	$\sigma = 0.1$
$G_{\max} = 200$	$h = 0.01$
$p_c = 1.0$	$e = 0.618$
$p_M = (0.2, 0.995)$	
L = 10	

TABLE 2. AVERAGE NUMBER OF SELECTED, CPU TIME AND OPERATING RESULTS

	<i>Average number</i>	<i>CPU(s)</i>	<i>(x,y,d)</i>
<i>genetic algorithm</i>	84	4.3	(-13.000,-10.500,2.552544)
<i>Linear search</i>	23	0.4	(-13.206,-10.550,2.542287)

## 5. Conclusion

This paper presents an effective shape matching method based on Hausdorff distance. Our method can high computing efficiency and good matching results because introducing of GA which improve the optimal solution search capability and linear such which improve the search accuracy. Our future research will focus in using windows hausdorff distance instead of Direct one. Apply this method in registration.

## References

- S. Belongie, J. Malik, J. Puzicha, Shape matching and object recognition using shape contexts, *IEEE transactions on pattern analysis and machine intelligence*, vol. 24, no. 24, April 2002.
- J. Wang, X. Bai, X. Youa, W. Liua, L. Latecki, Shape matching and classification using height functions, *Pattern Recognition Letters*, vol. 33, no 2, pp. 134-143, 2012.
- X. Huang, N. Paragios, D. Metaxas, Shape Registration in Implicit Spaces using Information Theory and Free Form Deformations, *IEEE transactions on pattern analysis and machine intelligence*, vol. 28, no 8, pp. 1303-1318, 2006.
- Ballard, Dana H. Generalizing the Hough transform to detect arbitrary shapes. *Pattern recognition*, vol. 13, no 2, pp. 111-122, 1981.
- G. Stockman, George. Object recognition and localization via pose clustering. *Computer Vision, Graphics, and Image Processing*, vol. 40, no 3, pp. 361-387, 1987.
- D. Huttenlocher and S. Ullman. Object recognition using alignment. *The International Conference on Computer Vision*, vol. 87, pp. 102-111, 1987.
- S. Loncaric. A survey of shape analysis techniques. *Pattern Recognition*, vol. 31, no 8, pp. 983-1001, 1998.
- C. Jacobs, A. Finkelstein, and D. Salesin. Fast multiresolution image querying. *Computer Graphics Proceedings*, ACM, pp. 277-286, 1995.
- S. Gold, Steven Allen. Matching and learning structural and spatial representations with neural networks. 1996.
- J-X. Du, D-S. Huang, J. Zhang, X-F. Wang. Shape matching using fuzzy discrete particle swarm optimization. *IEEE Swarm Intelligence Symposium*, pp. 405-408, 2005.
- X. Zhu. Shape Matching Based on Ant Colony Optimization. *Advanced Intelligent Computing Theories and Applications. With Aspects of Contemporary Intelligent Computing Techniques communications in Computer and Information Science*, vol. 15, pp. 101-108, 2008.
- D-G. Sim, O-K. Kwon, R-H. Park. Object Matching Algorithms Using Robust Hausdorff Distance Measures. *IEEE transactions on image processing*, vol. 8, no. 3, March 1999.
- D.P. Huttenlocher, G.A. Klanderman, W.J. Rucklidge. Comparing Images Using the Hausdorff Distance. *IEEE transactions on pattern analysis and machine intelligence*, vol 15, no. 9, September 1993.
- K. F. Man, K. S. Tang, S. Kwong, Genetic Algorithms: Concepts and Applications. *IEEE transactions on industrial electronics*, vol. 43, no. 5, October 1996.
- A. Kumari, R. Tripathi, M. Pal, S. Chakraborty. Linear Search Versus Binary Search: A Statistical Comparison For Binomial Inputs. *International Journal of Computer Science, Engineering and Applications (IJCSA) Vol. 2, No. 2, April 2012.*

## Biography

**Amina Kharbach** (a.kharbach@ump.ac.ma) graduated from ENSAO (National Higher School for Applied Science, Oujda) with a degree of state engineer in telecommunication and network in 2008. She is currently a Ph.D. candidate at LSE2I-Laboratory/ Mohammed First University of Oujda under the supervision of Pr. Mohammed Rahmoun and Pr. Benaissa Bellach. Her research activities focus on the local dissimilarity map functionality, by building a measurement allowing registration between images. She has served as a reviewer for several international conferences and journals.

**Amar Merdani** (merdani.amar@yahoo.fr) received his B.Sc. degree in Electronics and computing from Moulay Ismail University (2003, Morocco), and his M.Sc. in Electronics and telecommunications from Mohammed First University of Oujda (2010, Morocco). He is currently a Ph.D. candidate at LSE2I-Laboratory/ Mohammed First University of Oujda under the supervision of Pr. Mohammed Rahmoun. His main field of research interest is segmentation of medical image and measuring the quality of the result obtained by a segmentation algorithm.

**Benaissa Bellach** (benaissa.bellach@gmail.com) received the M.Sc. degree in Electrical Engineering and Automation at the University of Science and Technology of Montpellier (1999, France) and Ph.D. degrees in Instrumentation and Computer Image from the University of Burgundy in 2000 and 2003, respectively. He is a professor at the National Higher School for Applied Science, Oujda, Mohammed First University, Morocco. He was working with the A.T.E.R CReSTIC Research Centre (STIC), the University of Reims Champagne-Ardenne. His



research interests include: the development of the signal decomposition methods called EMD (Empirical Decomposition to) with noise application of 1D and 2D signals, registration, binarisation.

**Mohammed Rahmoun** (moha1rahmoun@gmail.com) is the head of Electronics Department, Computer Science and Telecommunications, he is the Coordinator of the die Electronics Engineering and Computer Engineering. He is the director of laboratory of electronic systems, computer and image (LSE2I) in National School of Applied Sciences, Mohammed first university, Oujda, Morocco. His current research interests include Digital Signal Processing, Intelligent Systems and Industrial Process Control.