Heuristic for the Capacitated *p*-Median Problem under a CVRP Approach

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Abstract

The Capacitated *p*-Median Problem (CPMP) is an important topic within the Logistics and Supply Chain Management fields. Because the CPMP is of NP-hard complexity, heuristic algorithms are the best suitable methods to provide feasible (or near optimal) solutions for this problem. In this paper an algorithm is proposed to provide solutions for the CPMP. This approach considers the formulation of the VRP (Vehicle Routing Problem) as basis for the CPMP. The results provide an insight about the implications of this assumption.

Keywords
Capacitated *p*-Median Problem, Heuristic Methods, Vehicle Routing Problem

1. Introduction

The *p*-median problem (PMP) is an important topic within the field of logistics and supply chain management [7]. In general, the PMP is focused on finding the best location of p facilities to minimize the distance between n demand nodes (customers) and the nearest of the selected facilities [2]. For the capacitated *p*-median problem (CPMP) the total demand of the set of customers assigned to a facility must not exceed its capacity [3].

The CPMP has been approached with different solving methods (particularly metaheuristics) as it is considered an NP-hard problem which is unpractical to be solved with exact methods [3, 1, 4]. Table 1 presents a general review of significant works reported in the literature for the CPMP. As presented, highly competitive solutions can be achieved with advanced techniques as Clustering Search (CS). However other techniques may lead to maximum errors up to 15% [5] and 50% [4].

Similarly to the CPMP the capacitated Vehicle Routing Problem (CVRP) consists on finding the best p routes of minimum distance to cover de demand of n customers. Each of these routes is covered by a single vehicle and all vehicles are of the same capacity.

In this work a solution approach for the CPMP is presented by taking as a basis the model for the CVRP. The solution approach considers geometric assumptions and random search operators to find suitable solutions of maximum coverage for the CPMP. The advances of the present work are presented as follows: in Section 2 a general background of the CPMP and the CVRP is presented while in Section 3 the description of the solution approach is described. Then, in Section 4 the results on a large CPMP instance are presented and discussed. Finally, in Section 5 our future work is presented.

Table 1. Review of Solving Methods for the CPMP: *n* = number of clients, *p* = number of facilities.

Work	Method	Instances	Performance
[1]	Clustering Search (CS) that integrates: Simulated Annealing (SA), Iterative Clustering (IC), an Analyzer Module (AM) and a Local Searcher (LS)	OR-Library: 10 CPMPs with n=50 and p=5, 10 CPMPs with n=100 and p=10; SJC: 6 CPMPs with n=[100, 402] and p=[10, 40]	Maximum Error < 0.5%
[3]	Genetic Algorithms (GAs)	OR-Library: 2 CPMPs with n=50 and p=5, 3 CPMPs with n=100 and p=10; SJC: 6 CPMPs with n = [100, 402] and p=[10, 40]	Maximum Error < 1.0%
[5]	Genetic Algorithms (GAs)	OR-Library: 10 CPMPs with n=50 and p=5, 10 CPMPs with n=100 and p=10	Maximum Error < 15.0%
[4]	Two-Phase Polynomial Heuristic Algorithm: Clustering and Variable Neighborhood Search (VNS)	Modified TSP-LIB: 5 CPMPs with n = 3038 and p=[600, 1000]; SJC: 7 CPMPs with n=[100, 402] and p = [10, 30]; TA (own instances): 7 CPMPs with n=[25, 100] and p = [4, 6]; Real Instances (own instances): 7 CPMPs from subsets of 13221 customers of a food distributor in Brazil	Maximum Error (by comparing with best known results and performance of other heuristics): < 50.0%

2. The CVRP and CPMP

Transportation Planning (TP) is defined as the process of making decisions concerning resource requirements of transportation considering their importance in achieving strategic goals in the short and long term. In general, TP is involved in the assessment, design and scheduling of transportation facilities and it is based on specific planning models for each context.

Within the context of TP the Capacitated Vehicle Routing Problem (CVRP) is an important element. As shown in Fig.1 the VRP determines a set of routes that begin and end at a specific point (e.g., a distribution center). These routes must cover a finite number of points and meet their demand requirements. Each route must be covered by a single vehicle (of finite capacity) and only one vehicle can serve a customer. The designed routes must be of minimum cost (distance, time, emissions, etc.) [6]

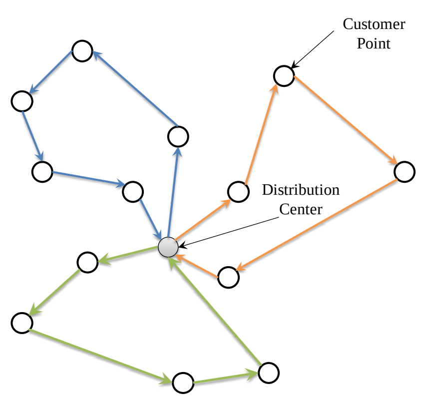

Figure 1. Example of a distribution network under the CVRP

Another problem of interest in TP is the Capacitated *p*-Median Problem (CPMP). As shown in Fig. 2 the CPMP consists on finding the most suitable locations of distribution centers to provide maximum coverage for a set or cluster of points. Each distribution center has finite capacity and must meet the demands of the associated points [2].

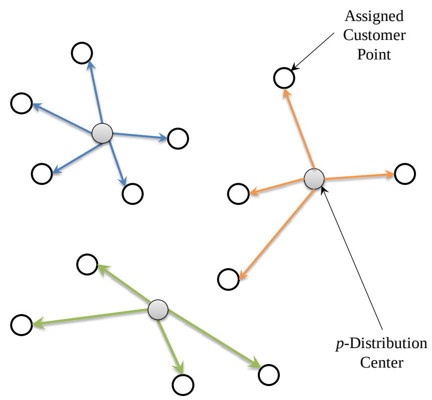

Figure 2. Example of a distribution network under the CPMP

3. The CVRP-CPMP Heuristic Approach

In this work we present an heuristic approach to provide feasible solutions for the CPMP based on the CVRP. Fig. 3 shows how the CVRP model can be adapted to a CPMP model. Then the general diagram of the heuristic developed to solve the adapted CPMP model is shown in Fig. 4.

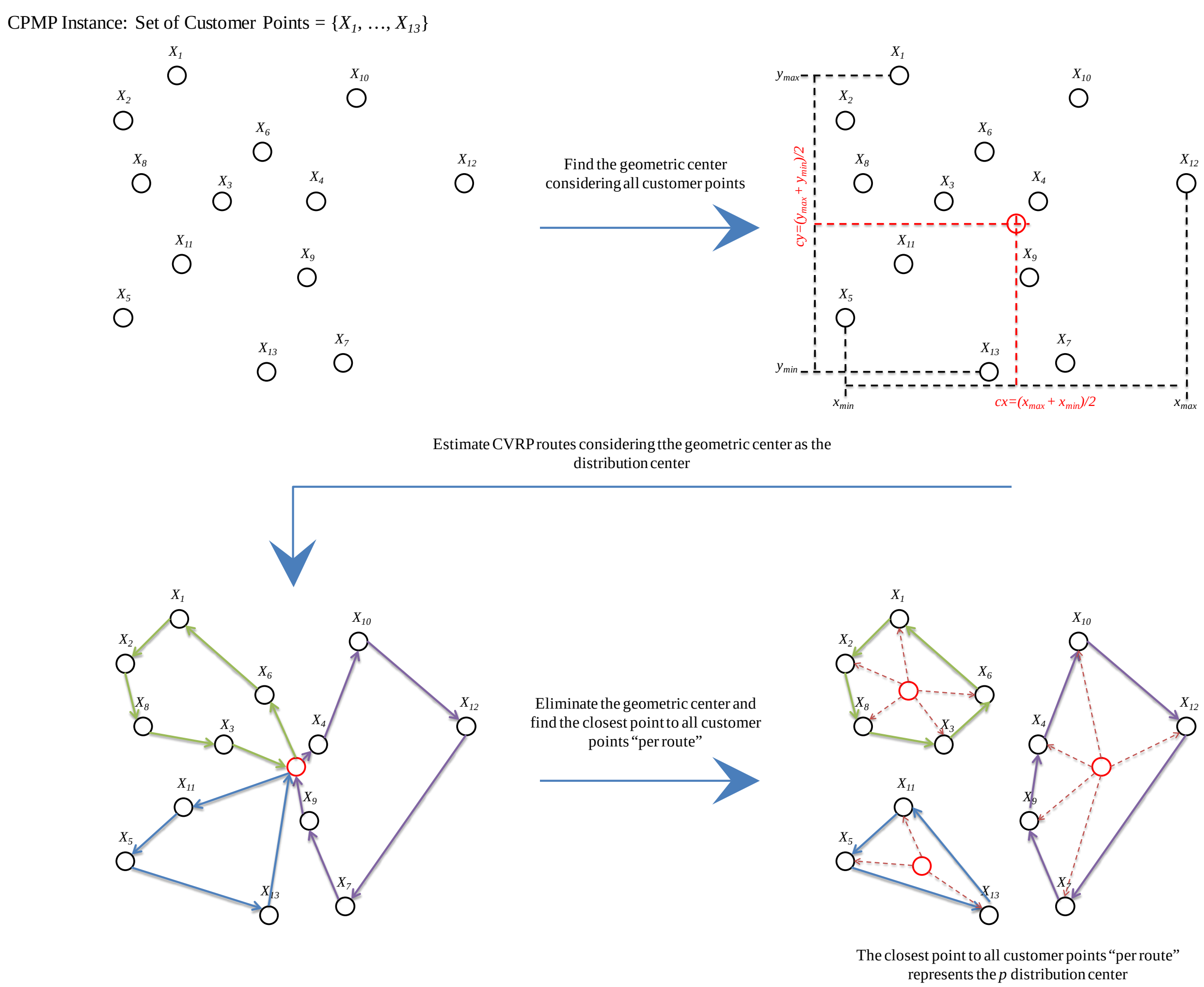

Figure 3. Adaptation steps for the CVRP-CPMP model

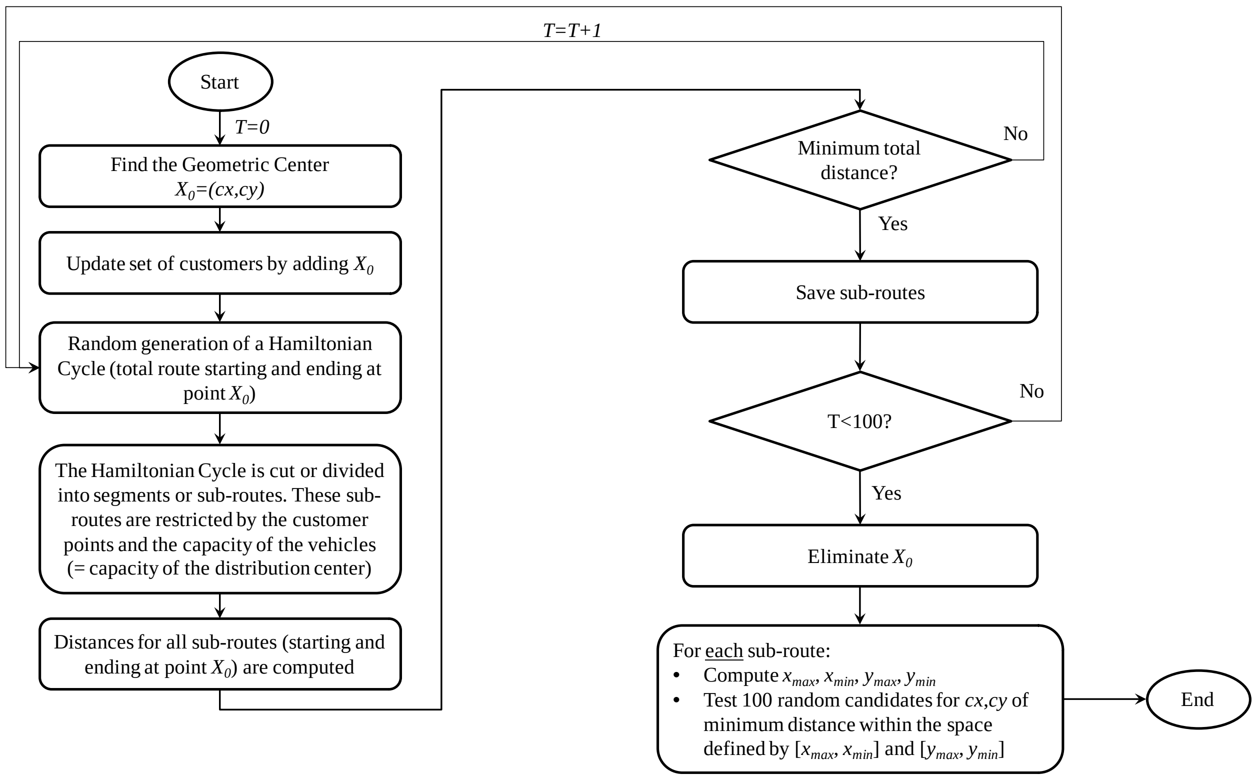

Figure 4. Proposed heuristic for the CVRP-CPMP model

4. Results on Large CPMP Instances

In order to test the proposed algorithm the CPMP instance DONI1 was considered. This instance consists of 1000 client points (n=1000) to be assigned to 6 distribution centers (i.e., p=6). The best-known solution for this instance leads to a total euclidean distance of 3021.41. Fig. 5 (a) presents the CPMP solution generated by the proposed algorithm which led to a total euclidean distance of 5564.10. This result is different from the best-known solution by approximately 84%.

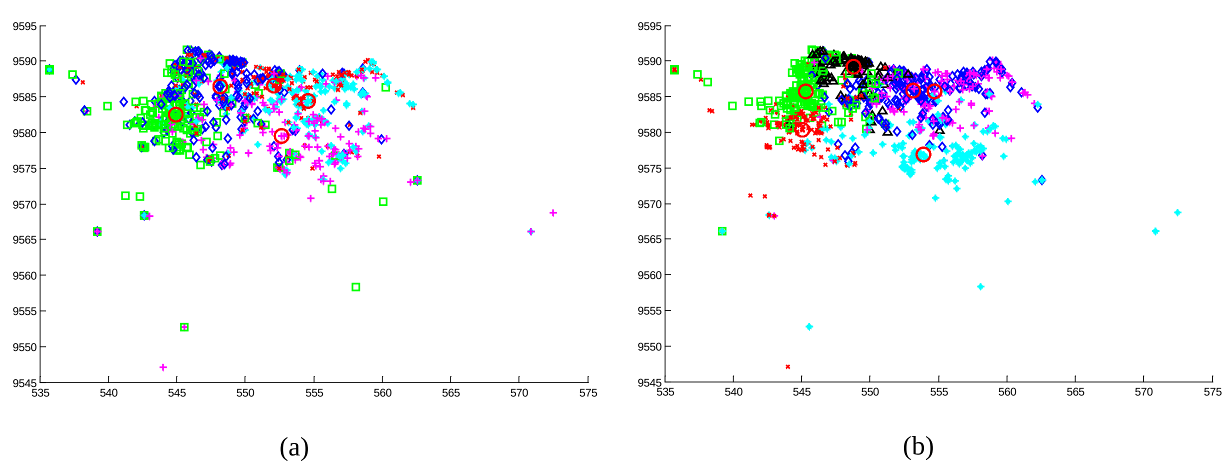

Figure 5. Results of the proposed heuristic for the CVRP-CPMP model: (a) without available-capacity restriction, (b) with available-capacity restriction

As presented in Fig. 5(a) there are 5 distribution centers instead of 6 and the clusters are not well defined. This is caused by the CVRP mechanism of determining the minimum set of sub-routes (in this case, it determined a number of sub-routes smaller than the optimal number of distribution centers). Also, it was found that the quality of the Hamiltonian Cycle is important for the CPMP clustering process.

In order to improve on this situation an available-capacity restriction was set for each vehicle: the capacity of each vehicle was reduced by 10%. This increased the number of sub-routes to match the optimal number of distribution centers. Also, the Hamiltonian Cycle was improved by single exchange operators. Fig. 5(b) shows the result obtained on the DONI1 instance considering these changes in the proposed heuristic. This solution led to 6 distribution centers with a total euclidean distance of 4269.2 and more defined clusters. This solution is different from the best-known solution by approximately 41%.

5. Future work

In this work we explored on an heuristic approach to provide feasible solutions for the CPMP which is an important topic in Logistics. This approach, based on the CVRP model, generated unsuitable solutions for the CPMP. The client-center allocation process, which is restricted by the capacity of the vehicles – centers, led to different number of resources (vehicles, centers) on the CPMP-CVRP. This increased the difference ratio from best-known solutions.

For this approach is important to adjust the capacity restrictions and improve the quality of the total route (Hamiltonian Cycle) that is considered as basis for the CVRP solution. These changes led to improved performance of the proposed approach.

Although these results are not as competitive as those obtained with other methods, the improved integration of CVRP-CPMP can provide more complete solutions for routing and covering scenarios. In order to accomplish this goal the future work is focused on the following points:

- Integrate the CPMP restrictions within the formulation of the CVRP.
- Develop a more efficient partitioning of the total route into sub-routes considering the CPMP restrictions.

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