New exact method to solve multi-capacitated location problem using set partitioning formulation

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ABSTRACT
In this paper, we present one generalization of the famous capacitated p-median location problem, called budget constraint multi-capacitated location problem (MCLP). This generalization is characterized by allowing each facility to be used with different capacity levels. We consider n customers, m facilities and l capacity levels, we note that the solution shape of MCLP can be represented as a set of disjoint clusters, each cluster is composed of one facility and a subset of customers. When creating clusters, some constraints must be met, namely the level selection and capacity. In this work, we present the new formulation of the MCLP based on set partitioning, then we suggest an adapted solving method, which will be called NFF (Nearest Facility First). The NFF approach is used in two ways: as a heuristic by taking only the first solution found or exact method when waiting finish the execution. Computational results are presented at the end using instances that we have created under some criteria of difficulties or adapted from those of p-median problems available in literature. The NFF method provides very good results for low and medium difficulty instances, but it is less effective for the more complex ones. To remedy this problem, the method will be supplemented by column generation approach.

KEYWORDS
Location, p-median, set partitioning, heuristic, exact approach, column generation.

1. INTRODUCTION

Locating facilities is one of the main problems when it comes to making strategic or tactical decisions. The objective of this kind of problems is usually to minimize a cost function that can include the cost for the assignment as well as the opening cost of facilities. In many location variants, the facilities' opening cost is initially fixed by a budget constraint; in such case, only the assignment cost is minimized. (Klose, 2000)

Facility location has been the subject of a large number of publications in the fields of supply chain optimization and operational research. The p-median is the most famous location problem that we can find hugely in literature, the Capacitated P-Median location Problem CPMP is a variant of this well-known problem subject the capacity.

Given a bipartite graph G(V, U, E) where V and U are respectively the sets of customers and facilities' nodes and E is the set of edges. V and U are each independent set such that every edge connects a vertex in U to one in V.

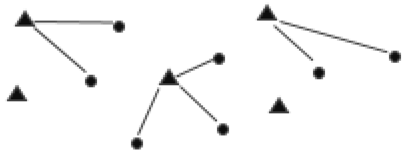

Figure 1: Graphical representation of location problem

The location problem graph is composed of many connected sub-graphs; each one is also made up of either one facility solely (triangle) or a facility with a partition of customers (points).

Let E = {*ij*: $i \in$ V, j ∈ U} and c_{ij} the assignment cost of the customer *i* to the facility *j*, the customer demand d_i and the facility capacity u_j are associated respectively with node $i \in$ V and node j ∈ U.

Let x_{ij} be the binary variable associated with the edge $ij \in$ E (x_{ij} = 1 if customer *i* is assigned to facility *j*, 0 otherwise) and let y_j be the binary variable associated with the median node $j \in$ E (y_j = 1 if facility *j* is used, 0 otherwise). The integer linear mathematical formulation is:

$$Min \sum_{i \in N} \sum_{j \in M} d_i c_{ij} x_{ij} \qquad (1)$$

$$\sum_{j \in M} x_{ij} = 1 , \quad i \in N \qquad (2)$$

$$\sum_{i \in N} d_i x_{ij} \leq u_j y_j \quad , \quad j \in M \qquad (3)$$

$$x_{ij} \leq y_j , \quad i \in N , \text{j} \in M \qquad (4)$$

$$\sum_{j \in M} y_j = \quad p, \quad \text{j} \in M \qquad (5)$$

$$x_{ij} \in \{0,1\}, i \in N, \text{j} \in M \qquad (6)$$

$$y_j \in \{0,1\} , \text{j} \in M \qquad (7)$$

The objective (1) is only to minimize the assignment costs, which may occur as transport costs by a unit of distance, in this variant we are not looking to minimize the opening costs of the medians. The demand constraints (2) force each customer to be assigned to one and only one median. Constraints (3) impose that the capacity of a median must not be exceeded, we assume that all facilities have the same capacity ***u***. Constraints (4) are used to restrict customers to be assigned to a closed facility. It can be noted that in the capacitated p-median location problem, this constraint becomes redundant with the capacity constraints; however, the experience shows that this constraint represents a valid high inequality to reduce the execution time. Constraint (5) is to specify that the number of medians must be equal to ***p***. (6) and (7) are the integrality constraints.

The search for p-median nodes in a network or graph is a classical location problem. The purpose is to locate p facilities (medians) in order to minimize the assignment cost of customers to facilities. The CPMP defines for each median one capacity that characterizes the service provided. The total demands for all customers assigned to one facility cannot exceed its service capacity.

In various industry field the service costs increase with its capacity used, the application of any single CPMP problem present a waste in terms of resources, the capacity of the service presented can hugely exceed the customer demands. In order to generalize the CPMP for more complex situations faced in industry, MCLP appeared for the first time in (EL AMRANI, BENADADA, & GENDRON, 2016) defines several capacity levels for each facility. Thus, a facility can be open at only one capacity level. The total demands of the assigned customers define the capacity level to use. Each opened level has a corresponding cost.

Several applications in the industry use the MCLP concept such as telecommunications, energy management, and many others. This can explain the fact that this is one of the most important known problems having impact on the strategic decisions.

The aim of this problem is to optimize the related transport cost of assigning customers to facilities. Each customer has a fixed d emand served by a single open facility. The facilities can be used in one of many pre-set levels of capacity; no one can be open for more than one level at the same time. By assigning customers to facilities, we have to check that the total demands of customers served by each facility is less than its level capacity used. The sum of facility opening costs is bounded by a limit budget.

Unlike the p-median problem, widely discussed in the literature, the MCLP is a new problem that we did not find any existing study, only in (EL AMRANI, BENADADA, & GENDRON, 2016). MCLP problem is NP-complete problem because it represents a generalization of CPMP (Garey MR, 1979). Variants of the latter appeared in (Roberto Baldacci, 2001), (Luiz AN Lorena, 2003), (Edson LF Senne, 2004), (Maurizio Boccia, 2007) and (Sittipong Dantrakula, 2014), dynamic location problems in (Behmardi & Lee, 2008), (Gama & Captivo, 1998), (DIAS, CAPTIVO, & IMACO, 2006) and network problems in (FRANTZESKAKIS & GANDY, 1989), (Current, Ratick, & ReVelle, 1997), (Ebery, Krishnamoorthy, Ernst, & Boland, 2000), and (Shu, Ma, & Li, 2010). To solve the CPMP several approaches have been proposed: (Maurizio Boccia, 2007) used a cutting planes algorithm based on Fenchel cuts (Edson LF Senne, 2004) (Alberto Ceselli, 2005) and (Ceselli, 2003) proposed the application of Branch & Price and branch & Bound met hods based on Lagrangian relaxation, (Luiz aN Lorena, 2003) proposed a resolution with column generation and (Gama & Captivo, 1998), (Holmberg, 1998) and (Klose, 2000) used other methods.

In this study of proposing new formulation of MCLP based on set partitioning, we will test the new model by using resolution methods, namely the Branch and Cut used by CPLEX solver and a new exact resolution approach called NFF.

The NFF proposed approach to solve MCLP consists on assigning each customer to the nearest possible facility, to do that we will sort all facilities by the assignment cost. At the first iteration, all customers are assigned to the nearest facility. Then we compare the sum of the demands assigned to each facility by its capacity level used. If this iteration provides one solution, so the solution found is optimal, otherwise we allow customers to be assigned to the next nearest facilities until arriving at a feasible solution. The NFF alg orithm can be used in two ways, when generating clusters, the formulation applied start to provide some very good solutions (according to the NFF philosophy). If we intend to look for the optimal solution we let the program finish the execution, otherwise some earlier so lutions can be enough as a heuristic approach in very reasonable execution time.

This paper is organized as follows. In section two, we discuss the new formulation of the MCLP based on set partitioning, the third one is devoted to the solving methods, namely the Branch and cut used by CPLEX and a new suitable exact approach, called NFF. Computational results are presented in the second last section before the conclusion.

2. FORMULATION

The MCLP problem is a location problem with capacity where facilities can be used at several levels, each level is characterized by a certain capacity to respect and the facility can open only one level at once. In the mathematical formulation, we will need t o create additional variables and notations and modify the constraints of the p revious problem such that each facility must respect the maximum capacity of the selected level.

The mathematical formulation is as follows:

$$Min \sum_{i \in N} \sum_{j \in M} d_i c_{ij} x_{ij} \qquad (8)$$

$$\sum_{j \in M} x_{ij} = 1, \qquad i \in N \qquad (9)$$

$$\sum_{i \in N} d_i x_{ij} \leq \sum_{k \in K} u^k y_j^k, \quad j \in M \quad (10)$$

$$\sum_{k \in K} y_j^k \leq 1, \quad j \in M \qquad (11)$$

$$x_{ij} \le \sum_{k\in K} y_j^k, \qquad i \in N, j \in M \quad (12)$$

$$\sum_{j\in M}\sum_{k\in K} f^k y_j^k \le B \qquad (13)$$

$$x_{ij} \in \{0,1\}, \quad \in N, \quad j \in M$$

$$y_j{}^k \in \{0,1\}, j\in M, k \in K$$

Where $\boldsymbol{K}$ denotes the set of levels, $\boldsymbol{u^k}$ represent capacity of level $\boldsymbol{k}$, $\boldsymbol{f^k}$ is the opening cost associated with level $\boldsymbol{k}$, **B** is the limit budget on the total opening costs.
$\boldsymbol{y_j^k}$ is decision binary variable that is one if and only if the facility $\boldsymbol{j}$ is opened and used at the level $\boldsymbol{k}$.

This model generalizes the capacitated p-median problem including the capacity levels concept. This new data appears in the capacity constraints (10) and at the budget constraint (13), because the facilities have different opening costs. (12) are also additional constraints which represent valid inequalities that cut the feasible region. (10) are capacity constraints and (11) force the facility to be open at one level at most.

In this section, we choose to re-formulate our MCLP as a set partitioning problem. Given a bipartite graph G(V, U, E) where V and U are respectively the sets of customers and facilities' nodes and E is the set of edges. V and U are independent sets such that every edge connects a vertex in U to one in V.

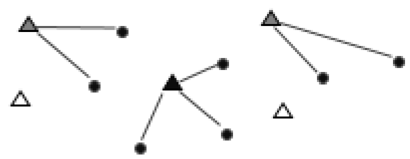

Figure 2 : Graphical representation of MCLP problem

The MCLP problem graph is composed of many connected sub-graphs, the facility (triangle) can be operated in many levels: white triangle for closed facility, grey one for the level one or black for a level with the biggest capacity, the capacity level is chosen according to customers' demands. We can easily see that the final solution will be in form of a set of pair (facility, clusters subset), this pair will be called "cluster". All of these clusters will build the feasible region of this new model. This new formulation bellow will be called MCLP based on set-partitioning formulation.

$$Min \sum_{j=1}^{m}\sum_{p=1}^{l} c_p^j x_p^j \qquad (14)$$

$$\sum_{j=1}^{m}\sum_{p=1}^{l} a_{ip} x_p^j = 1, \quad i \in N \qquad (15)$$

$$\sum_{p=1}^{l} b_{jp} x_p^j \le 1, \quad j \in M \qquad (16)$$

$$\sum_{j=1}^{m}\sum_{p=1}^{l} f_p x_p^j \le B \qquad (17)$$

$$x_p^j \in \{0,1\}, p\in \{1,2,..,l\}, j \in M$$

$S = \{S_1, S_2, \dots, S_l\}$ indicates the set of subset of N;
$S \subseteq \boldsymbol{\mathcal{P}}(N)$.
$\boldsymbol{c_p^j}$ is the total assignment cost of the customer subset $\boldsymbol{S_p}$ to the facility $\boldsymbol{j}$.

$$A = [\boldsymbol{a_{ip}}]_{n\times l} \ with \ \boldsymbol{a_{ip}} = \begin{cases} 1 & si \ \boldsymbol{i} \in S_p \\ 0 & sinon \end{cases}$$

Satisfying $\sum_{i\in N} \boldsymbol{d_i a_{ip}} \leq Max_k\{\boldsymbol{u^k}\}$

$$B = [\boldsymbol{b_{jp}}]_{m\times l} \ with \ \boldsymbol{b_{jp}} = \begin{cases} 1 \ if \ \boldsymbol{j} \ is \ assigned \ to \ S_p \\ 0 \ \ otherwise \end{cases}$$

$\boldsymbol{f_p} = Min_k\left\{\boldsymbol{f^k}\middle|\sum_{i\in N} \boldsymbol{d_i a_{ip}} \leq \boldsymbol{u^k}\right\}$ and $\boldsymbol{c_p^j} = \sum_{i\in S_p} \boldsymbol{d_i c_{ij}}$

$\boldsymbol{x_p^j}$ is a decision binary variable that is equal to one if and only if cluster build by facility **j** and customer subset $\boldsymbol{S_p}$ belongs to the solution.

It may be noted that in this formulation the constraints (8), (9), (13) are respectively equivalent to (14), (15), (17) and the constraints (9), (10), (11) appeared explicitly in the initial formulation become implicitly considered. Constraint (16), which is not declared in the first model explicitly, prevents us to have, in the feasible solution, the same facility in more than one cluster.

If **S** is the set of all subsets of **N**, the formulation can give an optimal solution to the MCLP. However, the number of subsets may be very huge, and an exact resolution with a solver such as CPLEX becomes impossible for a reasonable time. Therefore, we must think of a way to solve this problem with a reduced computational time. To circumvent this situation, we propose in the next section a method to prohibit the assignment of customers to facilities supposed too far, limiting the number of produced clusters.

3. SOLVING METHODS

To proceed with solving the problem, we will suggest to test the new formulation with an exact method well known, that of Branch and Cut, although this method presents difficulties for large instanced, it will serve to validate our new modeling. The adapted method for the present formulation will be detailed afterwards.

3.1. Branch and cut

The Branch and cut is a combination of two algorithms into one, namely the Branch and Bound and cutting plans.

Algorithm

1- At each node of the resolution tree, a linear relaxation is solved by the method of cuts.
2- If the solution is feasible (or if it is above a known solution), the summit declared pruned.
3- If all nodes are pruned, STOP, the best solution found is optimal.
4- Otherwise, choose a non-pruned node, select a fractional variable xi, and consider two sub-problems by fixing xi at 1 and xi at 0 (Branching).
5- Solving every problem by generating new unsatisfied constraints (cutting). Go to step 3.

The branch and cut algorithm is more efficient because it is reliable and more fast than the branch and bound.

3.2. Nearest Facility First

In this section, we present the resolution method, which consists of banning the creation of clusters with a too high total assignment cost.

The first step in solving this problem is the generation of clusters. In the first place, we classify, for each customer, the facilities in ascending order of assignment costs. Then we assign to each customer the least expensive facility. After this initial assignment, and for all iterations that follow, we compare for each facility the sum of customer demands assigned with the different capacity levels. If the total of these demands exceeds the highest level, then the cluster is rejected by unfeasibility. Otherwise, we create the clusters with the lowest level satisfying all demands. Thus, the opening cost of the cluster is that of the selected level, and the assignment cost of the cluster is the sum of all assignment.

The solution found in this step, if it exists, can only be optimal, because it presents an ideal case, where each customer is assigned to the nearest facility, which means the least costly. Otherwise, we regenerate the clusters by adding the second closest facility for a customer in terms of assignment cost, and so on until achieve a feasible solution or arrive at a number of clusters considered too high for reasonable computational time. If necessary, we use the column generation method.

To explain more the logic of the method, we detail the steps in the following algorithm:

Algorithm

Generate_ clusters

If the set CL (clusters' set) does not exceed a maximum number (5000 clusters)

1- Search all assignments already made or clusters already created, each facility with its customers associated.

Let's n the number of customers, for each pair (facility, part of customers).

2- If the sum of all demands exceeds the maximum capacity
   a- The cluster with ***n*** customers is rejected by unfeasibility.
   b- If ***n***>*0* we create ***n*** clusters each one with ***n-1*** customers. Otherwise STOP.
   c- Come back to (2-)

Otherwise

i = 0;

a- If the sum of demands does not exceed the capacity (***i***), we add this cluster in the set L with the capacity (***i***) and the corresponding opening cost and total assignment costs.
b- Otherwise ***i←i+1***, come back to (a-) ;
c- Remove duplicates and STOP.

EndIf

EndGenerate_ clusters

0- Initially there are a set N of customers, a set M of facilities and a empty set CL.
1- For each customer, sort the facilities in ascending order in terms of assignment.
2- Assign to each customer the nearest facility.
   Repeat |N|*(|M|-1) times
   3- Generate the clusters.
   4- If solution is found, it can only be optimal, STOP.
      (the algorithm stops by optimality).
   5- Otherwise, add the best assignment (customers, facility) and come back to (4).
      EndRepeat
      (if no solution found)
   Repeat |CL| times, and let ***p*** = 1
6- Develop the cluster ***p*** with all possible combinations.
7- If one solution is found, STOP. Otherwise, p←***p+1***, come back to (6-)

Otherwise we move to a resolution with columns Generation.

EndAlgorithm

This algorithm always finishes by find the solution, if such a solution exists.

The following diagram describe the sequence of the algorithm:

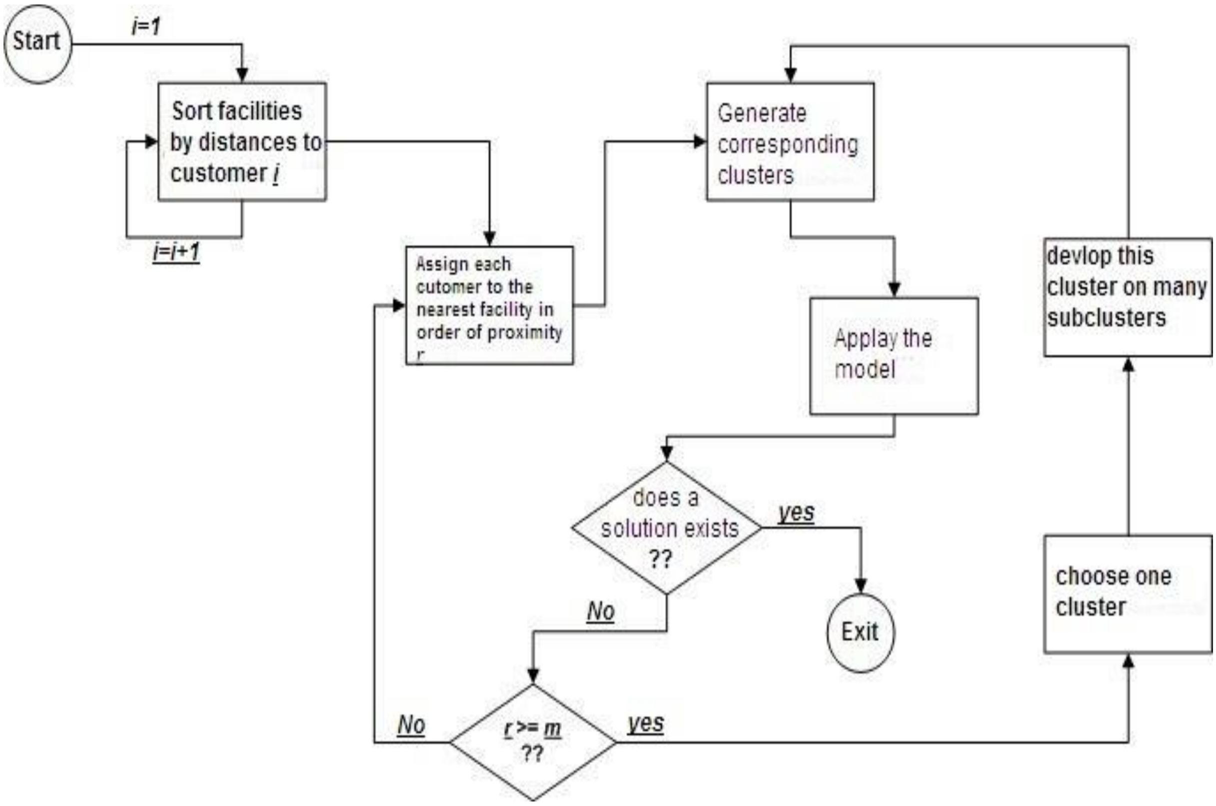

Figure 3 : NFF Algorithm Diagram

4. COMPUTATIONAL RESULTS

We did not find any reference in the literature on MCLP. This model therefore has, as we know, no existing instances for testing and comparison. For this, we will in the context of this work create instances using semi-random values based on a justified and appropriate choice. We will also use some p-median instances to supplement the computational tests, since the MCLP is a generalization of the latter.

We turn the two algorithms using the CPLEX solver. We use the version 7 of Java and version 12 of Cplex and we run the program on a machine i7-2600 CPU @ 3.40 GHz.

Our data set consists of five instance classes which have five levels of difficulty (easy, medium, difficult, very difficult and complex). The difficulty of these instances is based on the size of the problem, which is often measured by the number of customers. In contrast, the number of facilities and the number of levels used have a low impact on the size of the problem. Each difficulty level contains several test instances. These instances also represent difficulty sub-levels. They are characterized by the dispersion of the points (customers compared to plants) and also by the amount of available resources. Experience shows that the difficulty of the problem varies proportionally to the variance of customer-facility distances and customer demands. At the same time, it varies inversely with the budget allocated to the opening of facilities, and to their capacity levels. Thus, increasing the difficulty, while keeping the feasibility, we multiply the number of iterations needed to find the optimal solution.

This algorithm is composed of two parts: the first concerns the reformulation of the mathematical model and the adaptation of data in the form of clusters by manipulation with JAVA. In the second we solved the new model using the CPLEX solver.

The parameters of these instances are described in the following table:

DL: Difficulty level (E: easy, M: medium, D: difficult et VD: very difficult)
NC: Number of Customer
NF: Number of Facility
NL: Number of capacity Levels
NCL: Number of Clusters used to find solution.

For simplification reasons for large size problems, we add at each iteration, a set of clusters instead of adding one. This means that the number of clusters with which we find the feasible solution is not minimum, and the first solution found when running is not necessarily optimal, but very near the optimal one, so we have to wait in such case finishing the calculation.

The following table lists the various instances used and the computational results found.

Table 1 : Computational Results

Instance	NC	NF	NL	Number of variables	Branch & Cut		NFF					
							NCL	First Solution found (heuristic)			Optimal Solution	
					Obj	CPU (s)		Obj	CPU(s)	GAP	Obj	CPU(s)
F1	10	3	2	36	103	0,04	14	103	0.15	0%	103	0.15
F2	10	5	2	60	586	1,26	473	586	0.20	0%	586	0.20
F3	20	5	3	115	387	3,04	489	387	0.15	0%	387	0.15
F4	30	8	3	264	334	4,75	734	334	0.16	0%	334	0.16
M1	50	4	3	212	8826	0,07	69	8826	0.55	0%	8826	0.55
M2	50	6	4	324	2612	2,40	1014	2612	1.52	0%	2612	1.52
M3	70	6	4	444	7079	10,55	1116	7079	2.43	0%	7079	2.43
D1	100	10	5	1 050	12618	0,124	55	12618	0.45	0%	12618	0.45
D2	100	15	5	1 575	1587	0,38	259	1587	0.52	0%	1587	0.52
D3	200	15	8	3 120	90312	0,592	112845	112763	8,02	19.91%	90312	19.36
TD1	300	25	10	7 750	34175	0.842	1214	34584	1.14	1.18%	34175	1.95
TD2	300	30	10	9 300	25037	0.967	2034	25146	1.26	0.43%	25037	2.05
TD3	402	30	12	12 420	42233	0.827	3248	43011	2.35	1.80%	42233	4.45
TD4	402	40	12	16 560	39805	1.872	10451	42614	1,02	6,59%	39805	12.43
C1	500	50	4	25 200	25452	11,87	4365	26314	5.28	3.27%	25452	49.66
C2	1000	100	4	100 400	46719	143,53	4813	47624	12.69	1.90%	46719	72.46
C3	3038	600	10	1 828 800	-	-	3128	68421	52.2	12.39%	59942	89,35
C4	3038	700	10	2 133 600	-	-	2305	45126	46.52	-	-	-
C5	3038	1000	10	3 048 000	-	-	1345	97542	20.14	-	-	-

According to the computational results, the NFF approach is based on the concept of attacking the simplest first; we start by assigning each customer to the nearest facility, if a solution is found it will be optimal, if necessary we test the next facility. And so on until arrive at the solution.

The NFF provides two solutions approaches:

- Approximate solution by taking the first solution appeared while executing program and this has the advantage of reduced computational time. This can be used when we need only very good (not optimal) solution and the execution time is more important than the solution quality.
- Optimal solution when we let the program finish the instructions until the end. This is used when execution time is not very important.

We notice from the table of results that the approach works very well with a very reasonable execution time for easy instances in terms of dispersion points and in terms of resource availability, but it is less effective for difficult instances. The NFF heuristic approach continue to provide solutions even for complex instances, but unfortunately, these solutions are not optimal. It is for this reason that we will harness the power of our method in such instances and we will complete the approach by the column generation method, if the method is not successful after the generation of 5000 clusters.

In these results, we have defined the Gab for only the NFF heuristic approach, because the exact approach always gives optimal solutions and the GAB should normally always equal to zero.

5. CONCLUSION

In this paper, we introduced the multi-capacitated location problem with budget constraint. We have proposed a mathematical formulation based on set partitioning. We propose Branch and Cut to validate the new formulation, and finally the NFF method that gives the optimal solution for all instances in which the difficulty level is medium or difficult. For the difficult and complex instances, we decide to use the heuristic version of the NFF method, which is more adequate in such cases. An alternative for such instances would be to use the column generation method.

6. BIBLIOGRAPHY

ALBERTO CESELLI, G. R. A BRANCH-AND-PRICE ALGORITHM FOR THE CAPACITATED P-MEDIAN PROBLEM. (2005).

Behmardi, B., & Lee, S. Dynamic Multi-commodity Capacitated Facility Location Problem. *Industrial Engineering Research Conference*, (pp. 1914-1919). Corvallis. (2008).

Ceselli, A. Two exact algorithms for capacitated p-median problem. (2003).

Current, J., Ratick, S., & ReVelle, C. Dynamic facility location when the total number of facilities is. *European Journal of Operational Research 110*, 597-609. (1997).

DIAS, J., CAPTIVO, M. E., & IMACO, J. . Capacitated dynamic location problems with opening, closure and. *IMA Journal of Management Mathematics Advance Access published*, 1-32. (2006).

Ebery, J., Krishnamoorthy, M., Ernst, A., & Boland, N. The capacitated multiple allocation hub location problem:. *European Journal of Operational Research 120*, 614-631. (2000).

Edson L.F. Senne, L. A. A branch-and-price approach to p-median location poblems. (2004).

EL AMRANI, M., BENADADA, Y., & GENDRON, B. Generalization of capacitated p-median location problem : modeling and resolution. *International IEEE Conference of Logistics Operations Managment* (pp. 1-6). FES: IEEE, SCOPUS. (2016).

FRANTZESKAKIS, M., & GANDY, W. THE USE OF STATE SPACE RELAXATION FOR THE DYNAMIC. *Annals of Operations Research 18*, 189-212. (1989).

Gama, F. S., & Captivo, M. E. A heuristic approach for the discrete dynamic. *Location Science 6*, 211-223. (1998).

Garey MR, J. D. Computers and intractability : a guide to the theory of NP completeness. (1979).

Holmberg, K. Exact solution methods for uncapacitated location problems with. *European Journal of Operational Research 114 (1999)*, 127-140. (1998).

Klose, A. A Lagrangean relax-and-cut approach for the two-stage. *European Journal of Operational Research 126*, 408-421. (2000).

Luiz A.N. Lorena, E. L. A column generation approach to capacitated p-median problems. (2003). http://www.lac.inpe.br/~lorena/instancias.html.

Maurizio Boccia, A. S. A Cut and Branch Approach for the Capacitated p-Median Problem Based on Fenchel Cutting Planes. (2007).

Roberto Baldacci, E. H. A new method for solving capacitated location problems based on a set partiotioning approach. (2001).

Shu, J., Ma, Q., & Li, S. Integrated location and two-echelon inventory network. *Annals of Operations Research*, 233-247. (2010).

Sittipong Dantrakula, C. L. Applied p-median and p-center algorithms for facility location problems. (2014).

Thanh, P. N., Bostel, N., & ton, O. P. A dynamic model for facility location in the design of complex. *International Journal Production Economics 113*, 678–693. (2008).

7. BIOGRAPHY

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