Repairable 3-Out-Of-4: Cold Standby System Availability

Mohamed Grida
Industrial Engineering Department
Faculty of Engineering, Zagazig University
Zagazig, Egypt.
mogrida@zu.edu.eg

Abdelnaser Zaid
Faculty of Computers and Informatics
Zagazig University, Egypt
nasserhr@zu.edu.eg

Ghada Kholief
Engineering Sector
Egyptian Radio and Television Union
Cairo, Egypt
eng.ghadakholief@yahoo.com

Abstract

Systems operating in risky environments strive for guaranteeing the highest possible availability. This paper addresses the effect of redundancy and components' economy of scale on achieving a high level of availability. An availability estimation model for a 3-out-4 cold standby system was developed and compared with 6-out-8 system.

Keywords
3-out-of-4: cold standby system, Markov Model, Standby system, steady-state availability

1. Introduction

Engineering systems are usually repairable. When they fail to function as required, a repair process commences. Therefore, a system may not be available throughout its operating life, and its availability is measured as the fraction of the time it is available for functioning.

The desired level of availability can be obtained by providing sufficient redundancies, reducing the failure probability, and reducing the repair time. For repairable systems, availability is a more appropriate performance indicator than reliability, because it encapsulates both of reliability and maintainability [1]. As a measure, availability depends on what types of downtimes to include in the analysis. Therefore, there are different definitions of availability [2]. Among these definitions, operational availability is the most common, which is usually defined as the long-term fraction of the time that an item is available [3]. The operational availability is based on actual events that happened to the system.

Redundancy is an effective tool to improve the availability of a system by adding redundant component. The status of this extra component determines the type the system redundancy as shown in Figure 1 [4].

1.1 K-Out-Of-N System

It is referred to as M-Out-Of-N systems "MOON" or majority voting systems. This setting is considered a parallel redundancy setting, which requires at least (k) components to operate successfully out of the (n) total parallel components to describe the system state as a successfully operational one.

1.2 Cold Standby

In this configuration, the redundant units are kept in a dormant mode with a zero failure rate, while the active units encounter a higher failure rate of (λ).

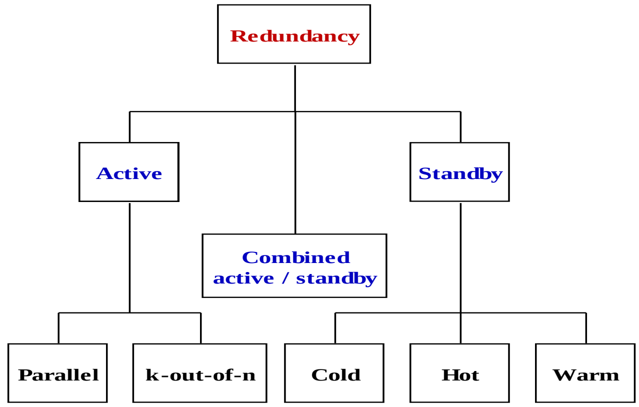


Figure 1: Different Type of Redundancy

2. Literature Review

The literature addressed the importance of redundancy to improve the system availability. Aven utilized Markov theory to formulate the availability of redundant standby systems and performed a simulation using MIRIAM to evaluate the formulae [5]. Wang and Kuo developed the steady-state availability of a series system with mixed standby components [6]. Wang and Loman examined the availability of an (N-1)-out-of-N/M parallel system with M cold standby units and one active unit [7]. Mishra and Jainb obtained steady state availability of main K-out-of-N:F secondary subsystems. If more than k units of main subsystem fail then the main subsystem shut off the secondary subsystem [8]. Smidt-Destombes et al.(2004) and Wang et al.(2016) studied the availability of a k-out-of-N system considering the trade-off among spare part inventory, repair capacity, and maintenance policy [9, 10]. El-Damcese and El-Sodany used Markov model to analyze the reliability and availability of a K-out-of-N:G system with three types of failure [11]. One year later, they utilized Markov model to develop the availability of K-out-of-N:G warm standby parallel repairable system [12]. Haggag analyzed the availability and the profitability of a redundant repairable 3-out-of-4 system under different preventive maintenance policies [13]. Jin et al. estimated the availability of K-out-of-N:G hot standby systems considering redundancy sharing [14]. Arabi and Jahromi optimized the availability a series system with multiple load sharing subsystems [15]. Suleiman et al. measured the effectiveness of a complex repairable series-parallel system involving four types of failure [16]. Juang et al. developed a knowledge system for the availability design of series- parallel systems using object-oriented program technique [17]. Chuan Ke et al. analyzed a repairable K-out-of-(M + W) retrial system with M identical primary components, W warm standby components and one repair facility [18]. Wang et al. compared between four different system configurations with warm standby components and standby switching failures based on their reliability and availability [19]. Jain and Rani examined the availability of warm standby repairable system considering switch failure and delay of reboot [20]. Jain et al. utilized Markov model to study the performance of a machining system with warm spares and heterogeneous servers considering switch failure [21]. Gupta et al. employed Markov birth-death process to analyze the performance of ash handling unit of a steam thermal power plant [22]. Kumar et al. developed a stochastic model to analyze the performance of a two-unit cold standby [23].

On the other hand, the literature addressed non-identical component. Khatab et al. evaluated the stationary availability of K-out-of-N:G systems with non-identical components subject to repair priorities using a multi-dimensional Markov model and Monte Carlo simulation [24]. Wu et al. developed analytical availability models for K-out-of- N:G warm standby repairable systems with many non-identical components [25]. Zhang et al. utilized Markov model to study a K-out-of-(M+N):G warm standby system model with non-identical components [26]. Kumar et al. employed a semi-Markovian approach to analyze the performance of a two non-identical unit redundant system [27]. EL-Sherbeny developed a formula for calculating the steady state availability of two-unit cold standby system with non-identical components [28].

3. Availability of Standby System

3.1 System Description

For simplicity, we started with a two elements system, one operational and one on standby; if either element fails, it will be repaired. However, if both elements fail, they will be repaired sequentially. Therefore, the system has three states as shown in Figure 2.

States:

S_0 – both elements up

S_1 – one element up, one element down

S_2 – both elements down

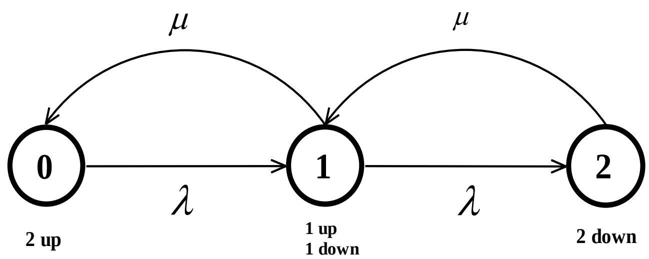


Figure 2: Markov Transition Diagram for Standby System.

3.2 Assumption

Each element has a constant operating failure rate λ , and a constant repair rate μ .

3.3 System state

The transition of the system from a state to another is best described by the transition matrix given below:

Present \ Future	S_0	S_1	S_2
S_0	$1-\lambda\Delta t$	$\lambda\Delta t$	0
S_1	$\mu\Delta t$	$1-\lambda\Delta t-\mu\Delta t$	$\lambda\Delta t$
S_2	0	$\mu\Delta t$	$1-\mu\Delta t$

Table 1: Transition Probability Matrix.

After Δt, the system may be in state S_0 based on two scenarios: either being at state S_0 and sticking to it or moving to it from a previous state S1. Therefore the probability of having the system instate S_0 can be written as follows:

$$P_0(t+\Delta t) = P_0(t)(1-\lambda\Delta t) + P_1(t)\mu\Delta t \qquad (1)$$

Where $P_1(t)$ is the probability of having the system in state S_1

Similarly, the next two equations describe the transition to state S_1, S_2.

$$P_1(t+\Delta t) = P_0(t)\lambda\Delta t + P_1(t)(1-\lambda\Delta t)(1-\mu\Delta t) + P_2(t)\mu\Delta t \qquad (2)$$

$$P_2(t+\Delta t) = P_1(t)\lambda\Delta t + P_2(t)(1-\mu\Delta t) \qquad (3)$$

From Equation (1),

$$\frac{P_0(t+\Delta t) - P_0(t)}{\Delta t} = -\lambda P_0(t) + \mu P_1(t) \qquad (4)$$

Taking the limit as $\Delta t \to 0$ we get,

$$\lim_{\Delta t\to 0} \frac{P_0(t+\Delta t) - P_0(t)}{\Delta t} = \frac{dP_0(t)}{dt} \qquad (5)$$

Then

$$\frac{dP_0(t)}{dt} = -\lambda P_0(t) + \mu P_1(t) \qquad (6)$$

It is obvious that the time derivative of the probability that the system being in state S_0 is equal to the inflow to state S_0 minus the outflow from S_0. Similarly, Equations (2, 3) can be transformed as follow:

$$\frac{dP_1(t)}{dt} = \lambda P_0(t) - (\lambda + \mu)P_1(t) + \mu P_2(t) \quad (7)$$

$$\frac{dP_2(t)}{dt} = \lambda P_1(t) - \mu P_2(t) \quad (8)$$

Since the steady-state availability A_{SS} calculated in limit as $t \to \infty$
Then probability is not function of time,

$$\lim_{t\to\infty} \frac{dP_0(t)}{dt} = -\lambda P_0(\infty) + \mu P_1(\infty) = 0 \quad (9)$$

$$\lim_{t\to\infty} \frac{dP_1(t)}{dt} = \lambda P_0(\infty) - (\mu + \lambda)P_1(\infty) + \mu P_2(\infty) = 0 \quad (10)$$

$$\lim_{t\to\infty} \frac{dP_2(t)}{dt} = \lambda P_1(\infty) - \mu P_2(\infty) \quad (11)$$

From Equation (9),

$$P_1(\infty) = \frac{\lambda P_0(\infty)}{\mu} \quad (12)$$

From Equation (11),

$$P_2(\infty) = \frac{\lambda P_1(\infty)}{\mu} = \frac{\lambda^2}{\mu^2} P_0(\infty) \quad (13)$$

Since

$$P_0(\infty) + P_1(\infty) + P_2(\infty) = 1 \quad (14)$$

Then

$$P_0(\infty) + \frac{\lambda}{\mu} P_0(\infty) + \frac{\lambda^2}{\mu^2} P_0(\infty) = 1 \quad (15)$$

$$[1 + \frac{\lambda}{\mu} + \frac{\lambda^2}{\mu^2}]P_0(\infty) = 1 \quad (16)$$

$$P_0(\infty) = \frac{1}{[1 + \frac{\lambda}{\mu} + \frac{\lambda^2}{2\mu^2}]} = \frac{\mu^2}{\mu^2 + \lambda\mu + \lambda^2} \quad (17)$$

$$P_1(\infty) = \frac{\mu\lambda}{\mu^2 + \lambda\mu + \lambda^2} \quad (18)$$

S*teady-State availability* A_{SS} is the long term probability that system is in either state 0 or 1, then

$$A_{SS} = P_0(\infty) + P_1(\infty) \quad (19)$$

$$A_{SS} = \frac{\mu(\mu + \lambda)}{\mu^2 + \mu\lambda + \lambda^2} \quad (20)$$

4. Availability of 3-Out-Of-4: Cold Standby System

4.1 System Description

The system we concern is composed of four identical components; they can be in operation, failure or standby.

4.2 Assumptions

We assumed:

- Each component has a constant operating failure rate λ , and a constant repair rate μ .
- Components do not fail simultaneously.
- Components are repaired sequentially.
- When a component fails, it is instantaneously replaced by one of the standbys if there is one.

4.3 System States

Using Markov transition diagram to analyze system states as *shown in* Figure 3 and applying the time derivative of state probabilities obtained in Equation (6), we can write the following expressions as the state probabilities of the system.

$$\frac{dP_0(t)}{dt} = -3\lambda P_0 + \mu P_1 \quad (21)$$

$$\frac{dP_1(t)}{dt} = 3\lambda P_0 - (3\lambda + \mu)P_1 + \mu P_2 \quad (22)$$

$$\frac{dP_2(t)}{dt} = 3\lambda P_1 - (2\lambda + \mu)P_2 + \mu P_3 \quad (23)$$

$$\frac{dP_3(t)}{dt} = 2\lambda P_2 - (\lambda + \mu)P_3 + \mu P_4 \quad (24)$$

$$\frac{dP_4(t)}{dt} = \lambda P_3 - \mu P_4 \quad (25)$$

Under steady state, the time derivatives of state probability are zero then,

$$P_1 = \frac{3\lambda}{\mu} P_0 \quad (26)$$

$$P_2 = \frac{9\lambda^2}{\mu^2} P_0 \quad (27)$$

$$P_3 = \frac{18\lambda^3}{\mu^3} P_0 \quad (28)$$

$$P_4 = \frac{18\lambda^4}{\mu^4} P_0 \quad (29)$$

Combining Equations (26, 27, 28, 29) and condition

$$\sum_{i=0}^{4} P_{(i)}(t) = 1, \quad (30)$$

$$P_0 = \frac{\mu^4}{\mu^4 + 3\lambda\mu^3 + 9\lambda^2\mu^2 + 18\lambda^3\mu + 18\lambda^4} \quad (31)$$

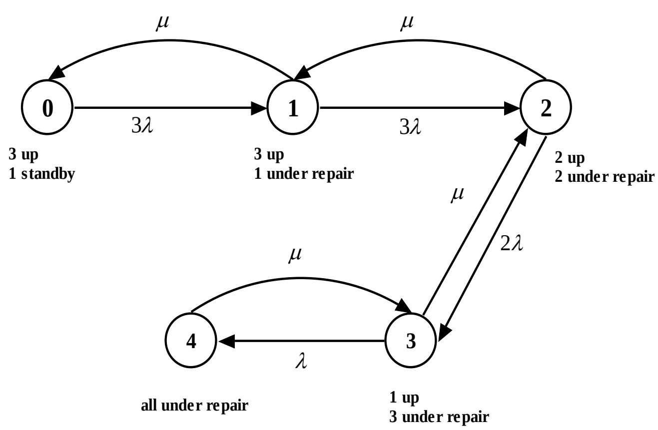


Figure 3: Markov Transition Diagram for 3-Out-Of-4: Cold Standby System.

The system is in operation when it is in either the state (0) or the state (1). Therefore, the general form to calculate the stationary availability of the system is obtained *using* Equation (19), as

$$A_{SS} = \frac{\mu^4 + 3\lambda\mu^3}{\mu^4 + 3\lambda\mu^3 + 9\lambda^2\mu^2 + 18\lambda^3\mu + 18\lambda^4)} \quad (32)$$

5. Availability of 6-Out-Of-8: Cold Standby System

5.1 System Description

In this system, we assumed that each of the previous system components is replaced with two new components. Each new component has half the capacity of the original one. Other than the capacity, the new components are similar to the old ones. Therefore, the above system is decomposed into a system of eight identical components with the same failure and repair rates.

5.2 System States

Using Markov transition diagram to analyze the system as s*hown in* Figure 4 and following the same procedure given in the above systems, we can calculate the probabilities function as follows*:*

$$P_1 = \frac{6\lambda}{\mu} P_0 \quad (33)$$

$$P_2 = \frac{36\lambda^2}{\mu^2} P_0 \quad (34)$$

$$P_3 = \frac{216\lambda^3}{\mu^3} P_0 \quad (35)$$

$$P_4 = \frac{1080\lambda^4}{\mu^4} P_0 \quad (36)$$

$$P_5 = \frac{4320\lambda^5}{\mu^5} P_0 \quad (37)$$

$$P6 = \frac{12960\lambda^6}{\mu^6} P_0 \quad (38)$$

$$P_7 = \frac{25920\lambda^7}{\mu^7} P_0 \quad (39)$$

$$P8 = \frac{25920\lambda^8}{\mu^8} P_0 \quad (40)$$

$$P_0 = \frac{\mu^8}{\mu^8 + 6\lambda\mu^7 + 36\lambda^2\mu^6 + 216\lambda^3\mu^5 + 1080\lambda^4\mu^4 + 4320\lambda^5\mu^3 + 12960\lambda^6\mu^2 + 25920\lambda^7\mu + 25920\lambda^8} \quad (41)$$

The system is considering operating in the state P_o, the state P_1, and the state P_2. Therefore, the general form of steady-state availability:

$$A_{SS} = P_0(\infty) + P_1(\infty) + P_2(\infty) \quad (42)$$

Using equations (33, 34, and 41), equation (42) can be rewritten as:

$$A_{SS} = \left(\frac{\mu^8 + 6\lambda\mu^7 + 36\lambda^2\mu^6}{\mu^8 + 6\lambda\mu^7 + 36\lambda^2\mu^6 + 216\lambda^3\mu^5 + 1080\lambda^4\mu^4 + 4320\lambda^5\mu^3 + 12960\lambda^6\mu^2 + 25920\lambda^7\mu + 25920\lambda^8} \right) \quad (43)$$

6. Numerical Analysis and Discussion

In order to analyze the effect of splitting the capacity of the components on the system availability, a new term (ρ) is defined as the ratio of the repair rate to the component failure rate:

$$\rho = \mu/\lambda \quad (44)$$

6.1 1-out –of -2 cold standby system

From Equations (20, 44)

$$A_{SS} = \frac{\rho^2 + \rho}{\rho^2 + \rho + 1} \quad (45)$$

6.2 3-out-of-4 cold standby system

From Equations (32, 44)

$$A_{SS} = \frac{\rho^4 + 3\rho^3}{(\rho^4 + 3\rho^3 + 9\rho^2 + 18\rho + 18)} \quad (46)$$

6.3 6-out-of-8 cold standby system

From Equations (43, 44)

$$A_{SS} = \frac{\rho^8 + 6\rho^7 + 36\rho^6}{\rho^8 + 6\rho^7 + 36\rho^6 + 4320\,\rho^3 + 12960\,\rho^2 + 25920\,\rho + 25920} \quad (47)$$

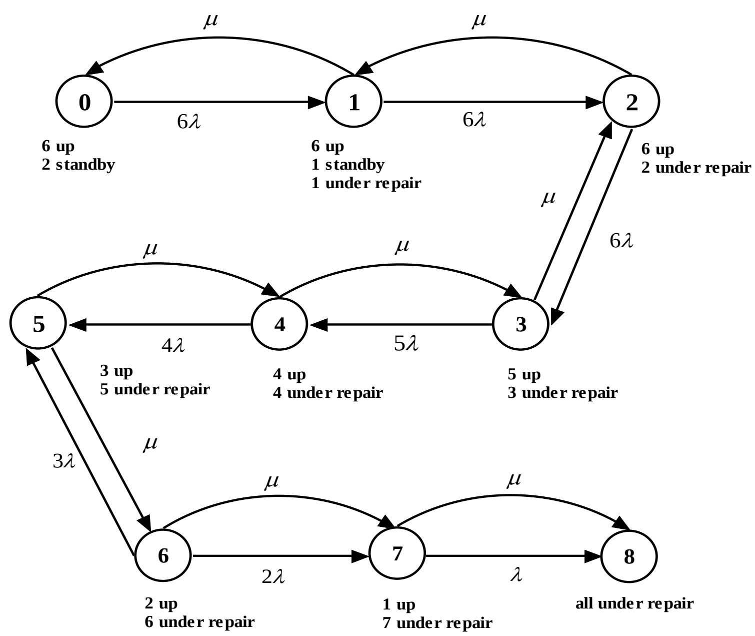


Figure 4: Markov Transition Diagram for 6-Out-Of-8: Cold Standby System.

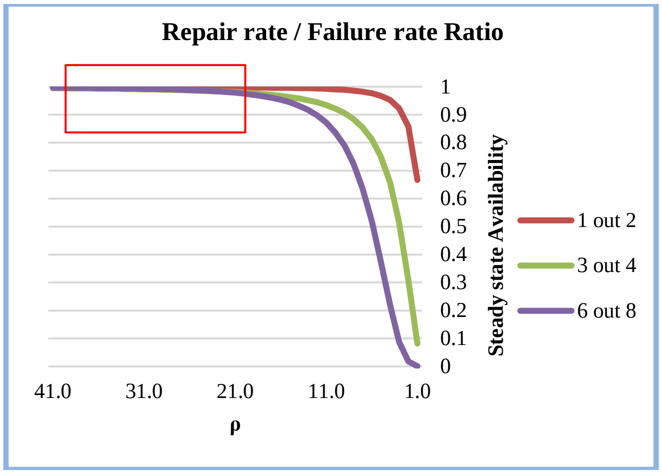
Figure 5 shows the expected steady-state availability of the three systems with respect to repair-failure ratio. As expected, the 1-out-of-2 system performed better than the other two systems due to its high level of redundancy. The other two systems have the same degree of redundancy; however, the 3-out-of-4 should have a better cost due to the economy of scale of its components. With the low repair to failure ratio, the 3-out-of-4 system performs much better. The incremental improvement of the 6-out-of-8 system with the ratio increase is higher than the 3-out-of-4 system. Consequently, for higher repair-failure ratio, the 6-out-of-8 performs better than standard 3-out-of-4. It can be concluded that designers of critical systems with extremely high availability may scarify the economy of scale and use multiple lower capacity components to improve the system availability. On the other hand, the designers of less risky systems are recommended to stick to the components economy of scale to have lower system cost and higher system availability.

7. Conclusion

The analysis of the two models revealed that at relatively low availability target, using larger economic components results in higher availability. On the other hand, targeting an extremely high availability requires to scarify the components' economy of scale.

8. Future Work

The above conclusion should be verified for more complex system setup as well as for heterogynous systems. It is also research worthy to address the effect of the redundancy level, the component economy of scale and reliability, and the repair capacity on the cost of achieving a certain availability level.



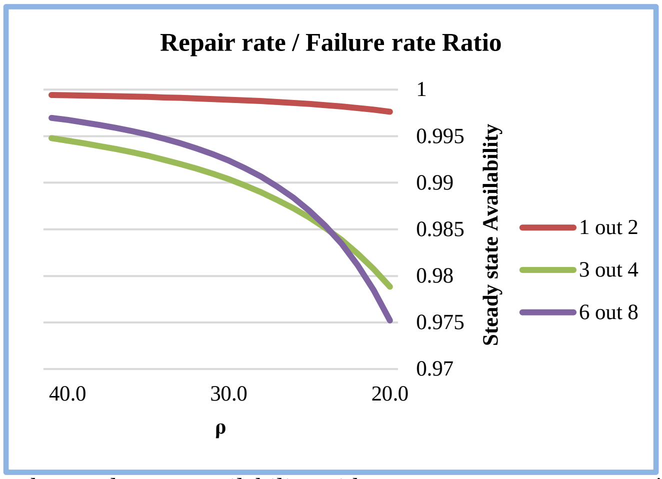


Figure 5**:** The steady state availability with respect to components setting up and ρ

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Biography

Mohamed Grida is an assistant professor of Industrial Engineering, in Faculty of Engineering, Zagazig University Zagazig, Egypt.
e-mail: mogrida@zu.edu.eg
Mohamed Grida is Vice President for Europe and Middle East (2B Technology), an assistant professor in Industrial Engineering at Zagazig University. He previously studied as Visiting Researcher at HKUST univeresty, Clear Water Bay, Kowloon, Hong Kong. He earned his master degree in industrial engineer at American University in Cairo. He earned his doctoral degree in industrial engineer at Zagazig Univeresty.

Abdel Nasser H. Zaied, is prof. of Information Systems, Dean, Faculty of Computers and Informatics, Zagazig University, Egypt.
e-mail: nasserhr@zu.edu.eg
He previously worked as an associate professor of Industrial Engineering, Zagazig University Egypt, an assistant professor of Technology Management, Arabian Gulf University, Bahrain; and as visiting professor at Oakland University, USA. He supervised 17 PhD. thesis and 50 MSc. thesis, and examined 10 PhD. thesis and 51 MSc thesis

Ghada Kholief is an engineer in Egyptian Radio and Television union Cairo, Egypt
e-mail: eng.ghadakholief@yahoo.com
Ghada kholief is an operation and maintenance engineer who is responsible for NTN media channels group systems operation. She earned her bachelors degree in electronics and communication at the Arab Academy for science, technology and maritime transport. She earned her master degree in industrial and engineering management at the Arab Academy for science, technology and maritime transport. She is preparing for doctoral degree in industrial engineer at Zagazig University.