Developing a quadratic programming model for time-cost trading off in construction projects under probabilistic constraint

Abbas Mahmoudabadi
Department Of Industrial Engineering
MehrAstan University, Guilan, Iran
mahmoudabadi@mehrastan.ac.ir

Fateme Pakzad
Industrial Engineering Department
Mehrastan University
Guilan, Iran
faeze32393@gmail.com

Abstract
Time and cost are the main important attributes in construction projects in which they have trading off relations with together. Therefore, decision makers are dealing with the problem to organize the project completion time based on its overall cost. In the present research work, a non-linear programming model has been developed for considering the relationship between time and cost over the construction projects while the overall cost should be probably restricted on the specified base line. Trapezoidal fuzzy numbers have been used to define the relationship between time and cost in each activity, so a quadratic programming has been developed for considering time and cost trading off as well as probabilistic constraint on project overall cost. The proposed model has been applied in the case study of a construction project named Weigh in Motion system (WIM) and results revealed that probabilistic constraints and non-linear relations between time and cost can be formulated using a quadratic programming approach.

Keywords: Quadratic Programming, Probabilistic constraints, Project Management, time-cost trade off, weigh in Motion System

1. Introduction
Infrastructure projects and infrastructure development projects are known as necessary for economic growth and development in the country and significant investments are accounted for them. Most project managers are trying to plan the approved budget and in accordance with the specified time to be finished. In 1961, Kelly Price-time balancing problem were discussed for the first time by taking a linear relationship between time and expense activities (Shankar, 2011). Researches in this area have led to the use of different methods to solve the problem of balancing cost and time. Methods and optimization algorithms are divided into two categories of accurate approximation algorithms and algorithms. Approximation algorithms include innovative methods and accurate algorithms of mathematical methods. The success of the innovative ways in order to answer questions on projects depends on the type of problem and achieves the optimal solution will not be guaranteed. In general, the rules required to develop and analyze these methods have been validated with experimental results. Innovative methods can be provided by Fundal, Prager, Moslehi methods (Taha, 2008) with increasing the size and complexity of the issues, while the meta-heuristic method is very common to do the above concern. One of these methods is known as genetic algorithm [Kim et. al. 2003 & Azaron et al., 2005) and the others are optimization of bird populations, Leap Frog and optimization of ACS noted. If mathematical methods are able to solve the problem, the answer is to determine the absolute optimum. Mathematical methods include the method used for linear programming, nonlinear programming, integer programming and so on (Feng et al., 1997) while the direct or indirect algorithms can be divided into two categories.
One of the examples of direct method is known as gradient algorithm which is used to direct searching the maximum (minimum) amount of the issue by following the most rapid rate of increase (decrease) in the objective function. In the indirect method, using optimization algorithms for solving optimization problems with the public is limited. Examples of these conditions include quadratic programming, separable programming and stochastic programming (Taha, 2008), while stochastic programming is assumed that the data (parameters) unknown random variables [Ke & Liu, 2005) that have a certain probability distribution. While, fuzzy numbers in various research projects and can be used to show uncertainty (Prad, 1979), this information is used to convert the plan into a definite equivalent (Wollmer, 1985) as well as one of the basic tasks in the estimated-duration and cost activities is the use of fuzzy theory.
Heris and Hejazi (2014) developed a model to balance cost and quality and time in their article of a random scenario-based modeling multi-objective. Scenario planning, strategic planning is a method used in some organizations, to long-term plans is flexible.

Hu and He in his article equilibrium model to provide quality cost and time involved in construction projects with resource allocation approach. Total Quality model projects of two researchers each activity is calculated as a weighted average quality. On the other hand the quality of each activity as weighted average quality of raw materials, equipment, labor and management is calculated. New techniques will not be possible and the scale of the problem is increasingly increases. For this purpose, the author of the three-storey building your model that only includes 13 activities to have solved with genetic algorithm (Hu & He, 2014).

Hua and Junjie (2014) in his article of Modeling project time–cost trade-off in fuzzy random environment For complex environment with more than one type of uncertainty, this paper presents three types of time–cost trade-off models, in which the project environment is described via introducing the fuzzy random theory. The expected value and the chance measure of fuzzy random variable are introduced for modeling the problem under different decision-making criteria. for solving the time–cost trade-off problem in mixed uncertain environment with randomness and fuzziness, the fuzzy random cost minimization model, the fuzzy random expected cost minimization model and the fuzzy random chance maximization model were built. To solve the models, the hybrid intelligent algorithm integrating the fuzzy random simulations and GA was designed. The effectiveness of the proposed algorithm was illustrated by three numerical experiments (Hua & Junjie, 2014)

Hua Ke, Weimin Ma and Xiaowei Chen in his article of Modeling stochastic project time–cost trade-offs with time-dependent activity durations, in some projects, activity durations show their complexity with time-dependence as well as randomness. a stochastic time–cost trade-off problem with time-dependent activity durations was formulated with objective of minimizing the project cost with completion time limits. For solving the problem, three decision-making criteria were introduced, based on which the expected cost minimization model, the a-cost minimization model and the \ probability maximization model were established to satisfy different practical managing requirements. To solve the models, an intelligent algorithm integrating the stochastic simulations and GA was built. The effectiveness of the proposed GA-based intelligent algorithm was illustrated by numerical experiments (Ke et al., 2012).

Karen et al., in their article of Multistage quadratic stochastic programming, Quadratic stochastic programming (QSP) in which each sub-problem is a convex piecewise quadratic program with stochastic data, is a natural extension of stochastic linear programming. This allows the use of quadratic or piecewise quadratic objective functions which are essential for controlling risk in financial and project planning. Two-stage QSP is a special case of extended linear-quadratic programming (ELQP).The recourse functions in QSP are piecewise quadratic convex and Lipschitz continuous. Moreover, they have Lipschitz gradients if each QP sub-problem is strictly convex and differentiable. Using these properties, a generalized Newton algorithm exhibiting global and super-linear convergence has been proposed recently for the two stage case. We extend the generalized Newton algorithm to multistage QSP and show that it is globally and finitely convergent under suitable conditions. We present numerical results on randomly generated data and modified publicly available stochastic linear programming test sets. Efficiency schemes on different scenario tree structures are discussed. The large-scale deterministic equivalent of the multistage QSP is also generated and their accuracy compared (Karen et al., 2001).

Zheng and Ng (2005) presented a new approach for time–cost optimization model by integrating fuzzy set theory and non-replaceable front with genetic algorithms, where fuzzy set theory was introduced to model the managers' prediction on activity times and costs as well as the associated risk levels.

In the current issue of the balance between the cost and duration of projects nonlinear programming technique used to optimize the duration of the project. The limit for completion of the project cost is determined a probable range. The relationship between activity and costs for quadratic equation is considered. Axle weighing system moving in Isfahan Nain mode is used for the study.

2. Developing Mathematical Model

The mathematical model is the first step to better plan better signs, parameters and decision variables and constraints and objective functions implies introduced them. The duration of each activity trapezoidal fuzzy numbers (a, b, c, and d) considered and in any case cost also includes an arrangement with C (a), C (b), C (c), C (d) expressed. By increasing the duration of each activity, cost increases (bad-sighted). on the other hand if the activity is too low, operating costs also increased (optimistic case). Contribution can be a curve of the second degree is. on the other hand any quadratic equation contains coefficients that includes power factor of the second variable power factor of variable and fixed value. Time curve for activity i is formulated as equation (1).

Where in:

$$C(\mathrm{T}_i) = \beta_{2i}(\mathrm{T}_i)^2 + \beta_{1i}(T_i)^1 + \beta_{0i} \tag{1}$$

Where:

β_{2i} : The second time power factor for activity i in operating cost.

β_{1i} : The first time power factor for activity i in operating cost.

β_{0i} : Constant coefficient of cost for activity i.

Decision variable: The analysis model earliest start time and duration of each project activity and the project completion time as the decision variable in the model. The objective function is to minimize the duration of the entire project, which is obtained by minimizing the time required to complete the project, so the project completion time as a decision variable in the model. Starting time is also considered to be variable in the decision because the project completion time, maximum n is the earliest start time.

X_i= The earliest start time of n the activity

T_i= Duration of n the activity

M= the maximum amount of time to start of n the activity (completing time of project)

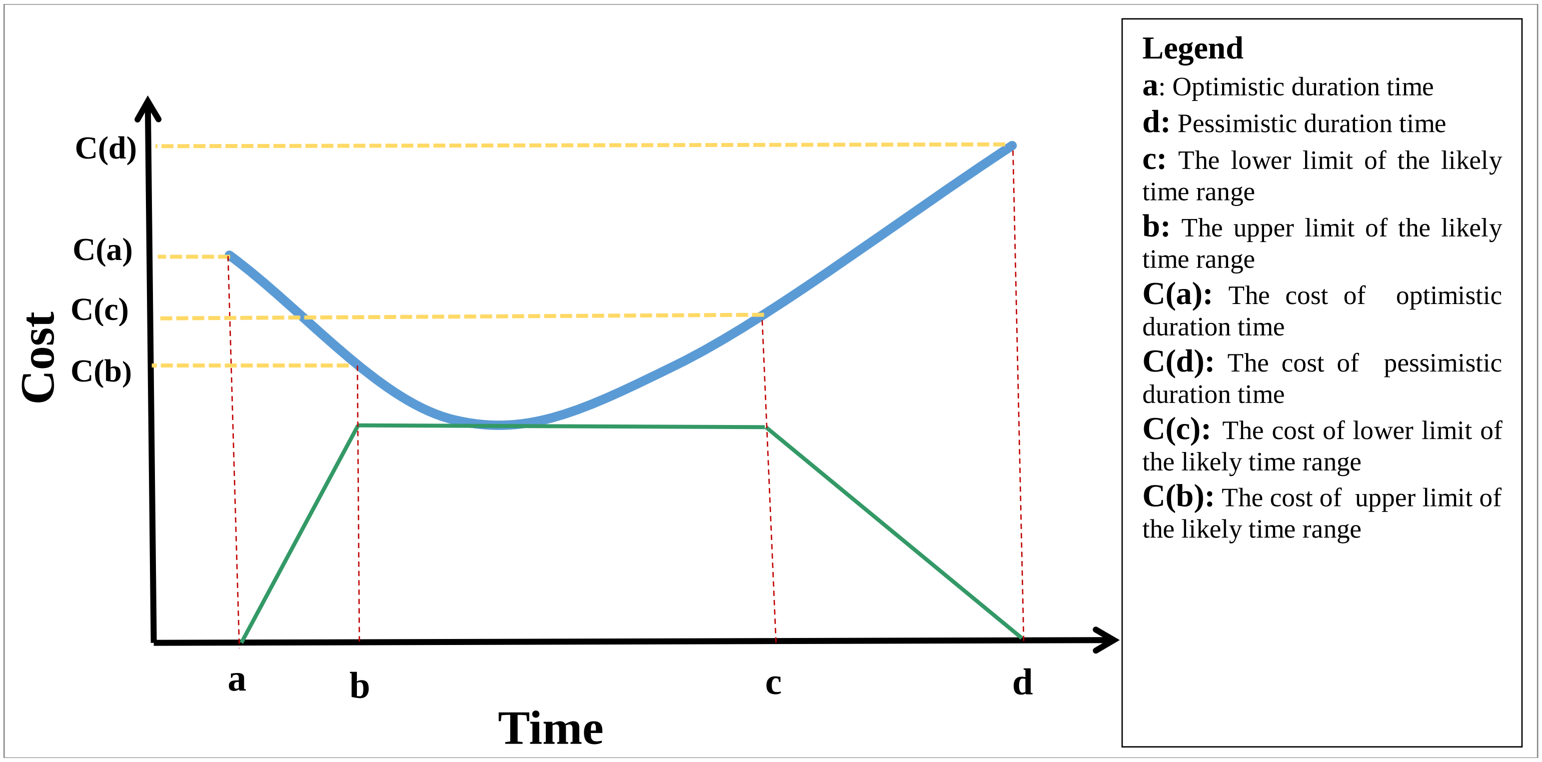


Figure 1: The relationship between the cost and duration of activity

Parameters: Coefficients of quadratic curves associated with each activity are considered as one of the parameters. Time optimistic, pessimistic time, the range of possible activities, fuzzy mean and variance of operating costs as well as the parameters of the model are:

$lowT_i$= Possible to carry out activities optimistic that the time ith and talked with a trapezoidal fuzzy numbers will be displayed.

$medT1_i$= The lower limit of the possible range of time doing that in the trapezoidal fuzzy numbers are shown with the letter b.

$medT2_i$= The upper limit of the possible range of time doing that in the trapezoidal fuzzy numbers are shown with the letter c.

$uppT_i$= The longest times possible to complete the work i th my cynical time in the trapezoidal fuzzy numbers are defined with the letter d.

$MeanC_i$= Average cost of doing ith, fuzzy numbers mean that optimistic, pessimistic and likely range is obtained.

$VarianceC_i$= Fuzzy variance cost, the variance of fuzzy numbers optimistic, pessimistic and likely range is obtained.

$V_1(J,I)$= Xi variable coefficients in the model are discussed in the limit J with V_1. The coefficients zero, (1) and (1) in the fall. Factor V1 to work i am so that for the earliest onset of activity i am against (+1), for all the activities that previously needed Activity i are, (1) and for the rest of the activities (the prerequisite i) are zero.

$V_2(J,I)$= The coefficient model variable T_i the restrictions J with V_2 shown. These factors include the zero and +1 is to work i the coefficient V_2 is the case for all activities that are a prerequisite for activity i, the value of (+1) and for activities that are prerequisites Activity i do not, the value will be zero.

First objective function: The objective function is to minimize the duration of the project. This is achieved by minimizing the maximum amount X_i (the earliest start time of i) is obtained. Therefore, the maximum value of the M X_i considered that the objective can be achieved by minimizing the amount of M… and by (2) is as follows:

$$M = Max(X_i) \tag{2}$$

So the objective function for (3) is defined as:

$$MinZ = (M) \tag{3}$$

Constraints: In order to achieve the goal of minimizing the time constraints that include:

In the network structure of the project from legitimate activities as the activities project will be used to maximize its earliest start time X_i the maximum total duration of the project. The limit for (4) is shown:

$$X(i) \leq M \tag{4}$$

Any activity on the network has prerequisites that are effective in determining the limits. This limitation is required by the activity coefficients and V_1, V_2 in section (2-2) have stated, are defined as follows:

As soon as i should start as soon as the sum of (j) (i activity is a prerequisite) and activity (j) is. V_1 Factor for the earliest start time of i-fold (+1), a prerequisite for activity (j) to (1) is. V_2 Coefficient equals to the time of activity i +1.

The limit as (5) is shown:

$$V_1(X_i - X_j) \geq V_2(T_j) \tag{5}$$

As a prerequisite for starting any activity, this limit according to (6) is assumed.

$$X_1 = 0 \tag{6}$$

The duration of each activity possible in the time domain has a low (optimistic time) and a top (the cynical time) is. That should not be too high than too low. The limit is for 7 into the model.

$$low(T_i) \leq \text{T}_i \leq upp(T_i) \tag{7}$$

The mathematical model of planning will become a quadratic programming model.

$$C(\text{T}_i) = \beta_{2i}(\text{T}_i)^2 + \beta_{1i}(T_i)^1 + \beta_{0i} \tag{8}$$

Since the tender stage is very important. Price Check out the entire project should be in a range likely to determine the amount does not exceed a certain, reassuring. So in this study, the possibility that the amount of total $C(\text{T}_i)$ (total project cost) less than the total average cost of activities is fuzzy Z_α likely to be checked. The project cost is likely to be determined Z_α a plausible range.

$$P\left\{\sum MeanC_i \geq \sum C(\text{T}_i)\right\} \geq \alpha \tag{9}$$

Since the total average cost experiences can fuzzy random variable $E\left\{\sum MeanC_i\right\}$ and $Var\left\{\sum MeanC_i\right\}$ is normally distributed. So we have:

$$P\left\{\frac{\sum MeanC_i - E\left\{\sum MeanC_i\right\}}{\sqrt{Var\left\{\sum MeanC_i\right\}}} \geq \frac{\sum C(\text{T}_i) - E\left\{\sum MeanC_i\right\}}{\sqrt{Var\left\{\sum MeanC_i\right\}}}\right\} \geq \alpha_i \tag{10}$$

Where $\sum MeanC_i - E\{\sum MeanC_i\}/\sqrt{Var\{\sum MeanC_i\}}$ standard normal is with zero mean and variance one the relation (11) is obtained:

$$\frac{\sum C(\text{T}_i) - E\left\{\sum MeanC_i\right\}}{\sqrt{Var\left\{\sum MeanC_i\right\}}} \leq Z_\alpha \tag{11}$$

So random restriction has converted into definite limitations and relationship is used in the model [5].

$$\sum C(T_i) \leq E\left\{\sum MeanC_i\right\} + Z_\alpha\sqrt{Var\left\{\sum MeanC_i\right\}} \tag{12}$$

So the objective function considering the limitations mentioned, is optimal.

General model: After identifying the decision variables, the parameters, limitations and objective function in the previous section, the general model is as follows:

Min $Z = M$

Subject to:

$X_1 = 0$

$X(i) \leq M \qquad \forall i = 1, 2, ..., \text{n}$

$$lowT_i \leq T_i \leq uppT_i \qquad \forall i = 1, 2, ..., n$$

$$V_1(X_i - X_j) \geq V_2(T_j) \qquad i \ \forall i = 1, 2, ..., n, j \in$$

$$\sum C(T_i) \leq E\left\{\sum MeanC_i\right\} + Z_\alpha \sqrt{Var\left\{\sum MeanC_i\right\}} \qquad \forall i = 1, 2, ..., n$$

3. Case Study

In thus project we use Construction of the weighing system in Isfahan-Naeen as case study for checking recommended model that cost and time would be seemed as fuzzy numbers. As we said in before relation between time of activity and its cost would be related to one of them which is shown in first equation. For example, supply activities suitable for road pavement to install the system with the code D is shown, curve equation (13) can be expressed as:

$$C(T_d) = 1.5038(T_d)^2 - 29.445(T_d) + 641.44 \tag{13}$$

By solving the problem by software GAMS, the duration of each project activity and any of its earliest start time and the completion of the project (the decision) and the value of objective function (optimal duration of the project) and limit values of the including completion of the project costs, potential costs of the project (95%), the total cost of the project activities and the total variance fuzzy average cost of the project activities are identified to be set in accordance with the project plan.

The objective function, the optimal duration of the project can be achieved by taking limits. In (Table 1) amounts earliest start time of any activity, duration of each activity and total project completion time is shown:

Table 1. Project planning and implementation of activities

code	Activity description	Start time	Duration of project
A	Installing the necessary approval review	0	1
B	The whole system is installed on a network location	1	1
C	Location system installed in the bow part	2	1
D	Supply of road pavement perfect platform for system installation	3	1
E	The project is designed to install in place	4	1
F	Network communications and information systems	5	1
G	Operation of construction (foundation and rig)	7	2
H	Supply and installation of detection sensors weight, speed and vehicle class	8	1
I	Power supply	6	1
J	Communications infrastructure	7	1
K	Tender for the supply and installation of hardware and software for the control canter	6	1
L	Supply and installation of hardware and software for the control canter	11	1
M	Weighing supplying software systems, license plate reading, by grade and speed	7	1
N	Launching systems, installed software and hardware	8	2
O	calibration	10	1
P	Temporary delivery	11	1
Q	end	12	0
Time of ending project		12	

The issue of possible restrictions, the possible range of the cost of the project can be completed by taking 95% gains. In (Table 2) limit values that include, the cost of completing the project, the potential cost of the project (95%), the total cost of the project activities and the total variance fuzzy average cost of project activities is shown:

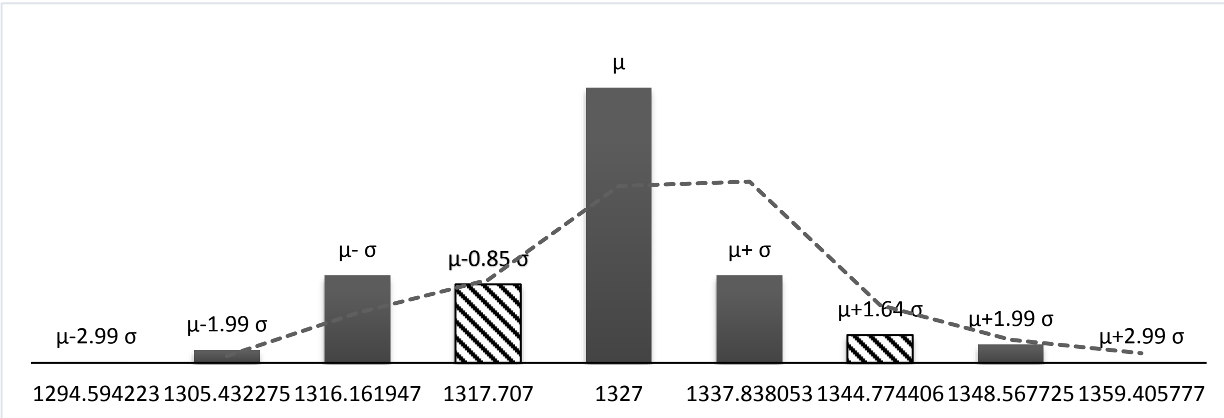
Table 2. Planning the cost of implementing activities

Limitations	Amount of limitations
Total average phase project cost (millions)	1327
The total project cost variance fuzzy (million)	117.463
The total project cost is fuzzy standard deviation (million)	10.808
Possible completion of the entire project cost (millions)	1344.829
Out of the total project cost (millions)	1317.707

Since the completion of the total project cost amounts possible variables with a normal distribution. The cost of completing the project is within ± 3σ of the average fuzzy costs to determine the probability of each of them to the area of the normal distribution curve to the costs to be calculated. That conforms to the figure 2.

4. Summary and Conclusion

Model using non-linear programming problem was solved with possible restrictions. And based on objective function, optimal project completion time, and then taking into account the normal probability distribution for the cost of the project, a probable range for the cost of the project was completed. GAMS software model to help the software that is powerful in solving programming problems were solved. The duration of projects can take up to 12 months and the cost of doing it in the range of 1317.707 to 1344.829 million USD is likely. The results indicate that a possible approach to control the cost of the project will be implemented through non-linear programming stages.



4. Figure 2. Values cost of completing the project in the normal distribution curve

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Biography

Abbas Mahmoudabadi, corresponding author (mahmoudabadi@mehrastan.ac.ir), is Ph.D. in Industrial Engineering and director of Master Program in Industrial Engineering at MehrAstan University, Guilan, Iran and deputy of Planning and Coordination in Transport and Fuel Management Centre, at Road Maintenance and Transport Organization, Tehran, Iran. He achieved his Ph.D. degree in January 2014 in the field of optimization in Hazmat transportation and received Thesis Dissertation Award from IEOM society in March 2015, Dubai, UAE. He has published near 60 journal or international conference papers and one book chapter published in the field of industrial engineering, transportation, traffic and road safety. He teaches transport and industrial engineering courses at universities and has around

25 years of executive experiences on traffic and road safety planning in developing countries. He has also strong cooperation with national and international agencies traffic safety and more with international agencies in the field of industrial engineering. Some national transportation projects have been implemented under his supervisory roles with the results of fatality reduction in intercity transportation.

Fatemeh Pakzad has Bachelor and Master of Science degrees in Industrial Engineering. She graduated from MehrAstan University, Guilan, Iran in June 2016. Her thesis dissertation is on studying the probabilistic cost-time trading-off in civil engineering projects and published her papers in this field.