Behavioral Modelling of Corundum under the Effect of Porosity Rate

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Abstract

Corundum, which is a form of Alumina, is used to manufacture various components and mechanical, electrical and electronics parts. Known for its relatively high hardness, its other characteristics depend on the rate of porosity that varies depending on the manufacturing process. One of the characteristics to consider is the value of the tensile stress in a standard test to failure as a function of the force applied and the rate of porosity. In this study, to analyse the simultaneous effect of two parameters, we used the statistical method of design of experiments. The study and analysis make it possible to establish a response of the effect of the two parameters in isolation from each other, then their effects simultaneously. Consequently, this will allow us to establish a mathematical model by which we can explain the tribological behaviour of porous corundum, with curves and graphs, and optimize the final result.

Keywords
Behavioural modeling, Corundum, Design of experiments, Porosity, Compression test.

1. Introduction
Alumina, Al_2O_3, is a well studied and universally used ceramic material possessing such attractive properties as the excellent wear and oxidation resistance, thermal and electrical insulation, good mechanical strength at elevated temperatures, excellent thermal shock resistance etc. Their wider use, however, is limited by its relatively low strength and fracture toughness. Engineering grade polycrystalline Corundum products are usually made by sintering Corundum powder at high temperature (>1300 °C). It has been demonstrated by the fact that the ceramic always contain cracks and pores largely, reduces their tenacity. The manufacturing process limits the component and section size that can be produced in reasonably full density, but it is also a major source of the initial defects that through fracture toughness will limit the strength of Corundum components in service. As consequence, strength of Corundum is not a strict material property but dependent on stressed volume [1]. The intention of the present work is to study the influence of porosity rate on the mechanical behaviour of the material Corundum under nominal compression. It is also an occasion to present the modelling of behaviour by design of experiments method; which will allow us to predict a change of stress, through a mathematical model of studied material under compressive loading, and to predict consequently the optimal parameters of work.

2. Experimental procedure
Alumina powder (95 wt.% α-Al_2O_3 and 5 wt.% additives) was used for the fabrication of porous Corundum, by the sintering method under high pressure and different temperatures in the range of 1350-1550°C, with which we had obtained blocks of Corundum at different rate of porosity of 38, 32.7, 7.7, and 3.3 %. The surface measurement of pores was done from micrographs obtained by using "Scion Image" software. The average of the percentages is calculated after several surface measurements of different sections from the same sample. The dimensions of test samples are summarized in Table 1. The study of the mechanical behaviour of Corundum was evaluated by exposing the samples to the compression test by applying a uniaxial load at room temperature, using the universal testing machine (Fig. 1).

Table 1. A summary of the test sample dimensions.

Samples	Porosity rate (%)	Diameter (mm)	Initial length Lo (mm)	Initial section So (mm)
1	3.3	20	100	$L_0 = 5{,}65 \times \sqrt{S_0}$
2	7.7			
3	32.7			
4	38			

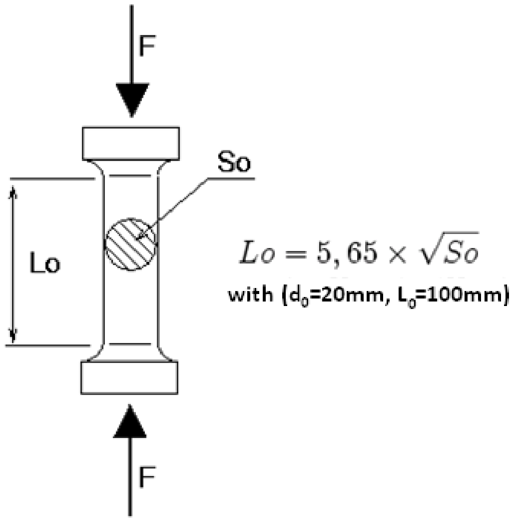

Fig. 1: Uniaxial compression test

The test measurements were conducted as follows: For every given porosity rate of the sample, the stress is measured at different values of strain during its variation, this last that depends, of course, of the change in length, then all the results and parameters were associated to the modelling process by the method quoted before, in order to understand its mechanical behaviour (stress-strain) as well as the mechanisms which explain that.

3. EQUATION AND MODELLING

In order to investigate the effect of porosity rate on the behaviour of Corundum under continued increase loading, the modelling by the design of experiments method seems adequate. It makes possible to predict other responses of stress values in the experimental field, basing of course on test results already performed, in other words, without any required to additional experiments. This method brings a solution that makes possible to minimize the number of experiments to carry out, and thus, saving time and money without sacrificing the precision of the results [2]. In the present study, we use the no-conventional design of experiments which makes possible to use data at our disposal without following a plan defined in advance [3].

As the response y_i depends on two factors, both having several levels, so it is judicious to express the estimated response in matrix form as the following notation:

$$[Y_{calculated}] = [X].[a_i] \tag{1}$$

3.1 MODEL COEFFICIENTS

Modelling is done by a polynomial of a second degree equation (1), which derives from the development of function in Taylor series, in which, each effect of parameter is represented by variable x_i attached to a coefficient a_i. Other coefficients a_{ij} are related to the interaction of effects x_i and x_j [21]. The general form of the polynomial model is written in the following form:

$$y_i = a_0 + \sum_{i=1}^{k} a_i x_i + \sum_{i=1}^{k} a_{ii} x_i^2 + \sum_{i=1}^{k-1} \sum_{j=i+1}^{k} a_{ij}\, x_i x_j \tag{2}$$

The developed form of polynomial model is written as below.

$$y_i = a_0 + a_1 x_{i,1} + a_2 x_{i,2} + a_{12} x_{i,1} x_{i,2} + a_{11} x_{i,1}^2 + a_{22} x_{i,2}^2 + e_i \tag{3}$$

where $x_{i,1}$ is porosity rate factor; $x_{i,2}$ is strain factor; y_i is response factor and e_i is gap between measured and predicted values obtained from the model of stress variation for any cycle i under applied load test.

The treatment of design of experiments consists on estimating, by the method of least squares, the P coefficients of a mathematical model while their number is lower than that of experiments performed [4]. By using "Matlab" for treatment of equation (2), we obtain the coefficients values by the following equation.

$$a = ({}^{t}XX)^{-1}.{}^{t}X.y \tag{4}$$

where a is coefficient matrix; $({}^{t}XX)^{-1}$ is inverse of matrix$({}^{t}XX)$; ${}^{t}X$ is transpose of matrix X and y is experimental measured response.

3.2 MATHEMATICAL MODELLING

After estimating coefficients of model (porosity rate; strain), we can now establish the mathematical model that linked response to factors. The model takes the following form.

$$y_{stress} = 345{,}796 - 242{,}272x_1 + 301{,}988x_2 - 222{,}025x_1x_2 - 4{,}72018x_1^2 - 26{,}884x_2^2 \tag{5}$$

where y_{stress} is a predicted value of stress; x_1 is porosity rate; and x_2 is strain value.
Through the estimating values of model coefficients and as a preliminary analysis, we can already indicate that the acting of porosity rate is important.

3.3 MODEL QUALITY

The equation (5) drifting from a statistical method is only an approximation of reality, from where appear the differences between the experimental and the prediction values. The estimate of coefficients of the second-degree polynomial model is based on test results, which are particular values with a random variable, for each treatment of design of experiments. So we have to make a judgment, by the quality of model coefficients, on the obtained results. We distinguish two types of qualities which are as follows.

3.3.1 MODEL VALIDITY

It makes possible to know the degree of rapprochement of the predicted values compared to the measured values. This coefficient translates the contribution of the model in the restitution of the measured response variation. It can be calculated by the relation in equation (6).

$$R^2 = 1 - \frac{SSE}{SST} \tag{6}$$

where SSE is sum of squares of errors and SST is sum of squares total [5,6].

In our case, the model validity coefficient is 0.988, it is estimated very good since it checks the following relation.

$$0 \leq R^2 \leq 1 \tag{7}$$

3.3.2 MODEL REPRODUCIBILITY

It makes possible to judge the ability of the polynomial model to predict the response without carrying out other tests than those already done. Its theoretical value can vary between minus infinity (-∞) and (+1), but the model is considered reproducible [6], if the value of Q2 is close to 1.

$$Q^2 = 1 - \frac{PRESS}{SCM} \tag{8}$$

$$\infty - \leq Q^2 \leq 1 \tag{9}$$

where PRESS is prediction residual error sum of squares and TSS is total sum of squares.

The model reproducibility obtained by applying the equation (8), of which Q2 is 0.975, is even good since it checks the equation (9).

The results mentioned before show that the quality of mathematical model obtained is satisfactory. It means that the new responses values, obtained from model, will be close to experimental values.

4. GRAPHICAL ANALYSIS AND DISCUSSION OF THE RESULTS

In this part, we will expose a series of curves traced with the obtained results, basing, of course, on the theory of design of experiments. These curves interpret the mechanical behaviour of Corundum, under applied load.

Fig. 2 shows a graphical description of the real values of stress as a function of strain at different porosity rate. We note that the evolution of stress increases in all of them. It is clear that the increase in strain causes an increase in the response. It should also be noted that the degree of inclination of the curves is different; it means that the rate of porosity has an effect on the steepness of variation. We can see on the graph for the slope obtained with a porosity rate of 3.3 %, the steepness is the greatest one, more than those obtained with 7.7, 32.7 and 38 % respectively.

Generally, for lower rates of porosity, the small increase in strain leads to a large increase in response. However, for higher rates, the increase of the strain leads to a small increase in response.

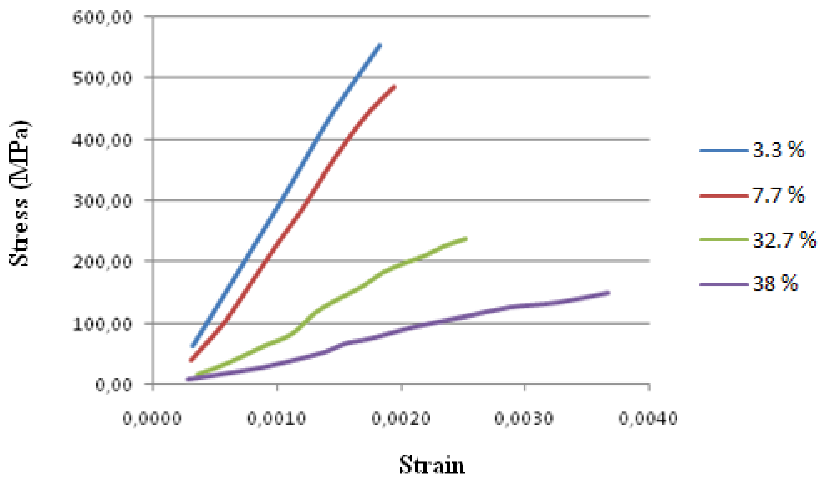

Fig. 2 Stress-strain curves of Corundum obtained at different rates of porosity

Among the advantages of design of experiments method is that the parameters interaction effect, is better investigated through the different graphical representations. Fig. 3 (a,b) shows typical 3D and 2D graphs of stress-strain variation under the effect of porosity rate; which are usually called Response-surface and Iso-response respectively; where this last is only a projection of the Response-surface on the down plane. In addition to the experimental values on the Fig. 2, the last graphical representations are plotted as a function of estimated values which are calculated by the mathematical model; so the graphs exhibit descriptive and predictive values.

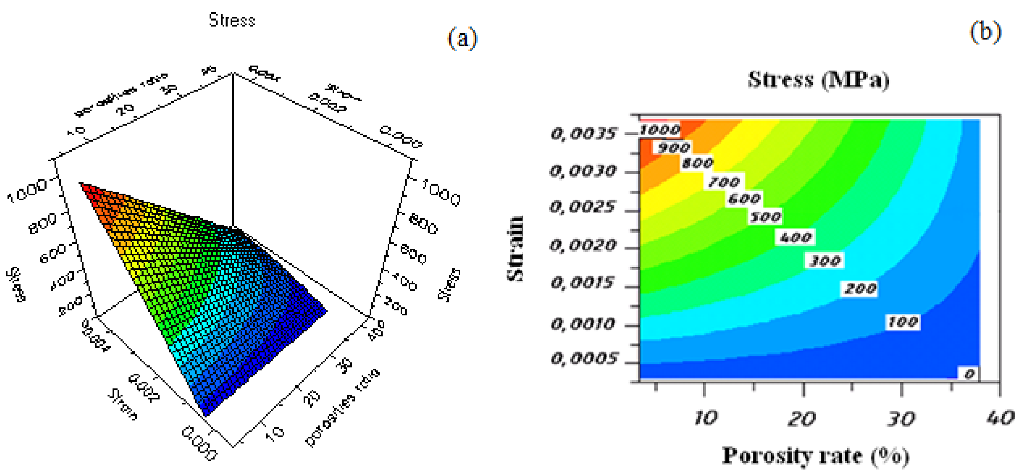

Fig. 3 Response-surface (a) and Iso-response (b) plot,
showing the effect of porosity rate on the behaviour of Corundum

In these graphs we clearly see the evolution of the behaviour stress-strain of Corundum according to the porosity rate. It is worth notifying, that the uniaxial load, is applied on the samples until rupture moment takes place. Overview of Fig. 3-a, we note that the response surfaces form arcs around a critical point of the stress and converge to it, on the top of the red zone.

Through the contours of Iso-response, Fig. 3-b, the parallel line to the porosity rate axis represents the stress axis and between axes, vertical lines (not showed in the graph) from each porosity rate, represent the evolution of stress as a function of strain increase, and it is clear that the number of zones traversed by these lines is different. It increases in the case of low porosity rate and decreases at a higher rate for same variation of strain. It means that the stress in Corundum of low porosity rate increases more rapidly than the one of a high porosity rate, with the increase of strain value.

In other hand, at this last condition, the material has a tendency to absorb more damage. The reason is that during the applied load, the grains on the top and bottom of the pores occupies the void. From which it increases slightly the plasticity of the material. but at the same time, the grains on the sides of the pores are moved further away from each other so that allow the propagation of cracks and as known for the brittle material when it is loaded to failure, it does so by the propagation of cracks. So it is worth to note that the pores, take part in this propagation. Consequently, that mechanism increases the fragility of the material.

5. Conclusion

Corundum, the studied material at different rate of porosity exhibits different mechanical behaviours (stress-strain). The rate of porosity has an effect on the steepness of stress-strain variation. The greatest one is the slope obtained with a porosity rate of 3.3%. For a lower rate of porosity, the small increase in strain leads to a large increase in stress in a Corundum. However, it leads to a small increase in response for an important porosity rate. The high porosity rate increases slightly the plasticity of the material, but also it increases the fragility of the material.

6. References

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