

# Ultradiscrete exponential Newell-Whitham model : A cellular-automaton model for traffic flow

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## **Abstract**

The exponential Newell-Whitham (eNW) model, defined by a differential equation with time-delay, is one of the most important models for traffic flow. In this presentation, we propose the ultradiscrete eNW model, which is a cellular-automaton model, by applying the ultradiscrete method to the eNW model. We first point out that there is a close relationship between the eNW model and the Lotka-Volterra (LV) equation, which is a soliton equation. The time-discrete analogue of the LV equation which keeps its integrability is well-known in the field of integrable systems. Considering these facts, we then give a time-discrete analogue of the eNW model. Furthermore, we present its exact solution using the bilinear method as well as Kanai and Tutiya did for the original eNW model. Also, since the ultradiscrete method can result in reducing soliton equations to cellular automata which inherit the solitonic nature such as an infinite number of conservation laws and soliton solutions, we apply it to the discrete-time eNW model and its exact solution to obtain the ultradiscrete eNW model and its exact solution. Finally, we evaluate the validity of this model as a traffic flow model by conducting simulation.

## **Keywords**

traffic flow, delay differential equation, cellular automaton, integrable systems, ultradiscretization, soliton equations

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## **Biographies**

**Kouyo Suzuki** is a graduate student of Hosei University. When he was doing BA degree, he majored in information engineering. Currently his research interests are about traffic flow using integrable systems.

**Shin Isojima** is an associate professor of Hosei University. His area of specialty is the nonlinear integrable systems. He especially studies integrable cellular automata by means of the ultradiscretization method and examines how to extend the method to general systems. He hopes that the ultradiscretization method and cellular automata become helpful tools to analyze various phenomena and he tries to contribute it.