Product Quality Analysis RS CD 10 SY-C9 Using Multivariate Statistical Process Control

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Abstract

There are nine variables affecting the production process of RS 10 SY-C9 yarn production. So we need analysis using Multivariate Statistical Process Control by forming T^2 Hotelling control chart. The results show that there are 6 points beyond the control limits with the main causes of TPI, Neps, and CSP. The calculation of process capability index of 1.456007 shows that the process is able to produce a product in accordance with the quality characteristics of the company.

Keywords:
Multivariate Analysis, Statistical Process Control, T^2 Hotelling **,** Capability Index

1. Introduction

Multivariate process control is one of the most important parts of statistical process control. Keep in mind, in industry, there are many situations where monitoring or control should be carried out simultaneously, of two or more interrelated quality process characteristics are required (Busababodhin and Amphanthong, 2016). Monitoring these quality characteristics independently can be very inappropriate and misleading. A process monitoring problem in which several interrelated variables are collectively known as Multi-variate Statistical Process Control. A very useful tool in multivariate statistical process control is the quality control chart (Frisen, 2011; Williams, 2004). This tool can also be used in the control of yarn production process at PT. Grand Tixtile Industry.

One of the yarn produced by PT Grand Textile Industry is RS 10 SY-C9 RS yarn. This type of yarn is one of the excellent products because of its high production amount, so there needs to be strict supervision in guaranteeing the quality. To monitor the quality of yarn RS CD 10 SY-C9 Grand Textile Industry company using univariate control diagram. This method is not considered appropriate because the existing data structure is multivariate, where there are nine characteristics measured by the company. This univariate method impact happening return (return) yarn, and repair of yarn (rewind). It was found that in June alone there were 72,814 cones returned, with production costs IDR 90,000.00 for each cones, then in June 2016 alone the company spent IDR 6,553,260,000.00 to produce the return yarn. This is certainly detrimental to the company, then more precise monitoring techniques are needed.

The T^2 Hotelling control chart, is a control chart that can be used in a joint control process, with quality characteristics that are examined more than one. The T^2 Hotelling control chart is used when both or more characteristics are technically dependent or suspected to be related (Hidayat et al., 2016; Jiang and Song, 2017). In this paper, the data is analyzed with variables that are more than one, and controlled together so it is deemed necessary to use T^2 Hotelling as a yarn quality control tool manufactured by PT. Grand Textile Industry. T^2 Hotelling multivariate control chart is used for the purpose of measuring two or more quality characteristics that have significant correlation.

2. Literature Review

In this section we discussed multivariate data, Statistical Process Control (SPC), the T^2 Hotelling control chart, and the T^2 Hotelling control charts free distribution.

2.1 Multivariate Data

According to Johnson (2002), univariate analysis is an analysis performed to analyze one variable from observation. Univariate analysis serves to summarize the measurement data collection in such a way that the data set is transformed into useful information. Multivariate analysis is an analysis related to the number of variables more than two are analyzed simultaneously on each observation with the interrelationship between variables.

2.2 Statistical Process Control (SPC)

SPC is a technique capable of controlling quality to monitor, analyze, predict, control, and improve the production process. In the SPC there are several tools used to analyze the stability of the process. There are seven main tools used in SPC: check sheet, histogram, Pareto chart, fishbone diagram (cause and-effect diagram), and stratification, scatter diagram and control chart chart) (Montgomery, 2009).

2.3 The T^2 Hotelling Control Chart

The $\boldsymbol{T^2}$ Hotelling control chart is a control chart to measure multivariate data or some variables introduced by Harold Hotelling. This control chart can be used for individual data and subgroup data.
According to Montgomery (2009), $\boldsymbol{T^2}$ Hotelling control chart is a control chart for individual data on multivariate data in the form of variable data both used in large shift size that is with the value of shift (δT) greater than three and suitable to be used in observation with large amount or more than 30 observations. In addition, the $\boldsymbol{T^2}$ Hotelling control chart analysis can be performed when the data held meet the normal multivariate assumptions (Rogalewicz, 2012; Hair et al., 2011).

This research used multifariate $\boldsymbol{T^2}$ Hotelling control chart with multivariate normal distribution. This is done based on the exploration of the data that has been obtained that the data possessed normally distributed visually, reinforced by the information that the company expects the product according to specification with small variations.

2.4 The T^2 Hotelling control charts Free Distribution

The analysis of the $\boldsymbol{T^2}$ Hotelling control chart must satisfy the assumption that the data is normally multivariate distribution. In fact, however, the data do not always satisfy the assumption of multivariate normal distribution. This leads to $\boldsymbol{T^2}$ Hotelling control chart with the free distribution procedure as described by Mason & Young (2002).
For the analysis on the $\boldsymbol{T^2}$ Hotelling control chart with the free distribution procedure is the same as the $\boldsymbol{T^2}$ Hotelling control chart with the differences found in the upper control limit determination (BKA). The upper control limit value (BKA) is derived from the use of Chebyshev theorem which does not pay attention to the distribution of data.

3. Material and Methodology

In this study the quality of RS 10 SY-C9 yarn thread was measured by the NE, single strength, tenacity, CSP, elongation, TPI, U%, NEPS, and hairiness variables. Based on data obtained known that multivariate normal distribution data so that in this research used T^2 Hotelling control chart.

3.1 Material

The data used in this research is secondary data that is data from RS 10 SY-C9 RS yarn which is the superior product of PT Grand Textile Industry. Acquired 164 observations of production data of RS 10 SY-C9 yarn production in December 2015 until July 2016. The research variables used in this case are nine characteristics that determine the quality of the yarn. The nine variables are listed in Table 1.

Table 1.Characteristics of Yarns

Variables	Variable Names	Unit	Ccontrol Limits
X_1	NE (length of yarn per unit weight)	hk/lbs	9.82 – 10.18
X_2	U% (yarn unevenness)	%	14 - 20
X_3	Neps (unbreakable weight)	piece	≤ 60
X_4	Single Strength (strength per piece of yarn)	Kg	≥ 750
X_5	Elongation	%	≥ 6.8
X_6	Tenacity (yarn strength)	CN/Tex	≥ 12.5
X_7	TPI (the number of loops per inch)	Kg	14.06 – 15.54
X_8	Hairiness (feather yarn)	%	≤ 7.7
X_9	CSP (Cone Strength Product)	Lid	≥ 2400

3.2 The T^2 Hotelling Control Chart

After testing the normal multivariate assumptions, then the control diagram is formed. Because normal multivariate assumptions are met then the control chart formed is a $\boldsymbol{T^2}$ Hotelling control chart. This diagram can be formed in accordance with the steps below,

1. Calculate the average of each observed characteristic.
2. Calculating the variance-covariance matrix of the observed characteristics.
3. Calculate the statistical value of T2 Hotelling using the following equation,

$$T_i^{\,2} = (x_i - \bar{x})' S^{-1} (x_i - \bar{x}),\ \ i = 1,2, \dots n \tag{3.1}$$

For each observation *i* = 1,2,3, ..., n for the value of n> 100 compare the value of T2 with the control limit specified using the following equation,

$$UCL = \frac{p(n-1)}{n-p} F_{(\alpha,p,n-p)} \tag{3.2}$$

and $LCL = 0$

Where p is the number of characteristics or variables measured and n is the number of observations. Create a control chart by creating a plot between values T_i^2 with observations *i* = 1, 2, 3, *n*.

3.3 Process Capability Analysis of Multivariate Data

The multivariate capability index calculation was performed using MCM process capability index method. MCM process capability index method was first introduced by Taam, Subbaiah, and Liddy in 1993. Calculation of process capability index with this method is considered more sensitive and easier to apply. Another advantage of this method can also indicate variability, centeredness, and both (Scagliarini & Vermiglio: 2007).

According to Scagliarini & Vermiglio (2007) calculation of process capability index for multivariate normal data, formulated as follows:

$$\widehat{M}C_{pm} = \frac{\hat{C}_p}{\hat{D}} \tag{3.3}$$

where $\widehat{M}C_{pm}$: value of process capability estimate

$\hat{C}_p$: an approximation value of capability proportional to the univariate C_p value, is obtained by the formula

$$\hat{C}_p = \frac{\dfrac{2\prod_{i=1}^{p} a_i \pi^{p/2}}{p \times \Gamma(p/2)}}{|S|^{1/2} (\pi K)^{P/2} \left[\Gamma\left(\frac{p}{2}+1\right)\right]^{-1}} \tag{3.4}$$

with $\Gamma(p/2)$: value of gamma distribution for variable p

K : quartiles of 99.73% of chi-square, degrees of freedom p

$\hat{D}$: the estimated value of the distance between the process mean and the target is obtained by the formula

$$\hat{D} = [1 + \frac{n}{n+1} (\mu - \mu_0)' S^{-1} (\mu - \mu_0)]^{1/2} \tag{3.5}$$

4. Results and Discussion

The process is performed by calculating the average vector of the nine characteristics of quality:

$$\bar{X} = \begin{bmatrix} 10.00402 \\ 964.48228 \\ 16.05936 \\ 3079.50994 \\ 7.24335 \\ 14.42944 \\ 16.75049 \\ 46.96951 \\ 10.32012 \end{bmatrix}$$

and the covariance matrix estimates are as follows::

$$S = \begin{bmatrix} 0.01654 & -2.43606 & -0.01535 & \cdots & 0.00582 \\ -2.43606 & 2283.03246 & 32.88039 & \cdots & 0.92032 \\ -0.01535 & 32.88039 & 0.53363 & \cdots & 0.01929 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0.00557 & -1.20235 & -0.01304 & \cdots & -0.01938 \\ 0.57524 & 112.85817 & 2.67630 & \cdots & 0.59908 \\ 0,00582 & 0.92032 & 0.01929 & \cdots & 0.26999 \end{bmatrix}$$

Using equation (3.1) is obtained T^2 *Hotelling* for 164 data observation of RS CD 10 SY-C9 . The Upper Control Limits (UCL) and Lowr Control Limit are (LCL), UCL = 18,361161 and LCL = 0 so as to form the following control diagram:

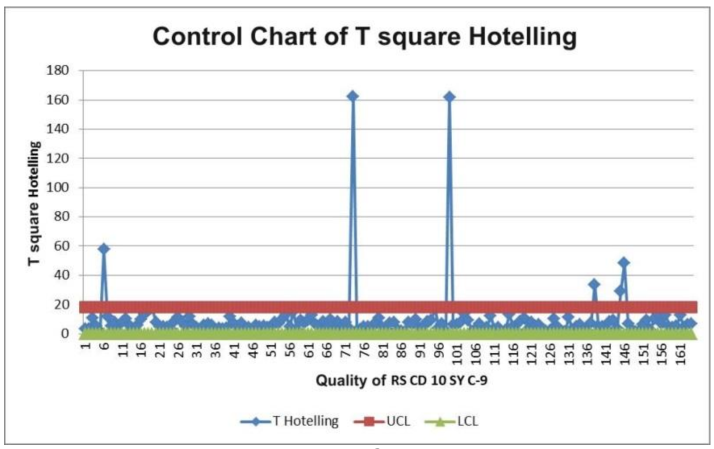

Figure 1 Control Chart of $\boldsymbol{T^2}$ Hotelling Yarns Quality

From the control chart, the points that are within the control limit of 158 points and the number of points beyond the control limit are 6 points, ie at points 6, 73, 99, 138, 145, and 146. Tracking is done at points is out of control to identify assignable causes. The most contributing variables make the point out of control be tracked using the decomposition control chart.

After all points are within the control limit, a capability analysis of the current process is performed. This analysis can answer whether the process is able to produce a product in accordance with the characteristics set by the company. From the normality test it is known that the data is normally distributed so that process capability analysis is done using equation (3.2). the calculation process in equation (3.2), is done using software R which produce $\widehat{M}C_{pm}$ = 1.456007. Because $\widehat{M}C_{pm} > 1.33$ then the process is considered capable of meeting the specifications set by the company.

5. Conclusions

Based on the analysis results, it is known that the quality of RS CD 10 SY-C9 in December 2015 to July 2016 is uncontrollable, as many as 6 points are out of control. The main causes of points beyond control are TPI, Neps, and CSP. The process capability index of 1.456007 shows that the process is capable of producing products that match the characteristics of the company's quality. PT Grand Textile Industry measures the quality of RS 10 SY-C9 yarn threads in a multivariate way so that the suitable method is Multivariate Statistical Process Control (MSPC) by forming a T^2 Hotelling control chart to monitor the quality of the yarn as it is multivariate structure, thereby reducing the return yarn.The results of the analysis show that the main cause of RS SY 10 C-C9 quality is beyond control limits are TPI, Neps, and CSP. Therefore, companies should pay more attention to the factors that influence the measurement of these three characteristics.

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