Analytical Solution of a Prestressing Strand Subjected to Axial Strain

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Abstract

Steel wire ropes or strands are used as lifting element in various branches of industry such as elevators, cranes, bridges. Those are mainly subject to external axial load while performing their principal duties. Axial strain occurs in accordance with carried axial load. In this study, a certain strain value is applied to the one end of a prestressing strand to extract analytical solution proposed firstly by Love and expanded by Costello later. Load carried by each wire, total load carried by strand, center wire and outer wire stresses, twisting moments have been determined for a prestressing strand having 11.11 mm diameter by using analytical solution proposed by Costello.

Keywords
Prestressing strand, Analytical solution, Wire.

1. Introduction
Wire ropes are flexible structures and consist of many wires to distribute the total load to the each wire in order to increases operational safety. It is very important to know the safely transport ranges and maximum load lifting capacities of steel wire ropes or strands. In order to determine rope lifting capacity and stresses occurred on ropes, various experimental, analytical and numerical studies have been conducted. The general theory of bending and twisting of thin rods is discussed by Love (1944). Costello (Costello, 1990 and Onur, 2016) expanded the theory of the thin rod derived by Love and analytically formulated the static behavior of the wire, strand and entire rope, taking into account the effects of curvature and torsion change of a thin wire, respectively. In this study, a prestressing strand having 11.11 mm diameter has been investigated to determine load carried by each wire, total load carried by strand, center wire and outer wire stresses, twisting moments by using analytical solution proposed by Costello when strain value of 0.003 is applied.

2. Static Response of a Prestressing Strand
Prestressing strands consist of a center wire having radius of R_1 located in the middle of strand and six helical wires having radius of R_2 braided around center wire as shown in Figure 1.

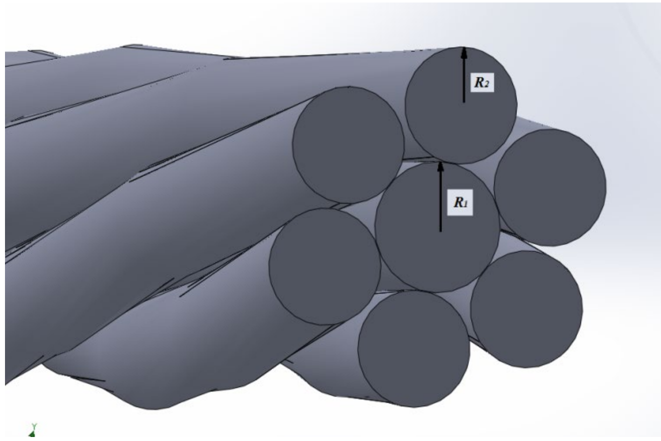

Figure 1. Investigated prestressing strand [3]

Six differential equilibrium equations for the thin wire loaded as shown in Figure 2 have been presented below as equation set (1) by using the direction cosines and summing the forces and moment along element of length *ds* (Costello, 1990).
N and N' are the components of the shearing force in the x and y directions, T is the axial tension, G and G' are the components of the bending moments in the x and y directions, H is twisting moment, X, Y and Z are the components of the external line load per unit length in the x, y and z directions, K,K' and Θ are the components of the external moment per unit length in the x and y directions, κ and κ' are the components of the curvature in the x and y directions respectively and τ is the twist per unit length of the wire (Costello, 1990).

$$
\begin{aligned}
&\frac{dN}{ds} - N'\tau + T\kappa' + X = 0\\
&\frac{dN'}{ds} - T\kappa + N\tau + Y = 0\\
&\frac{dT}{ds} - N\kappa' + N'\kappa + Z = 0\\
&\frac{dG}{ds} - G'\tau + H\kappa' - N' + K = 0\\
&\frac{dG'}{ds} - H\kappa + G\tau + N + K' = 0\\
&\frac{dH}{ds} - G\kappa' + G'\kappa + \Theta = 0
\end{aligned}
\tag{1}
$$

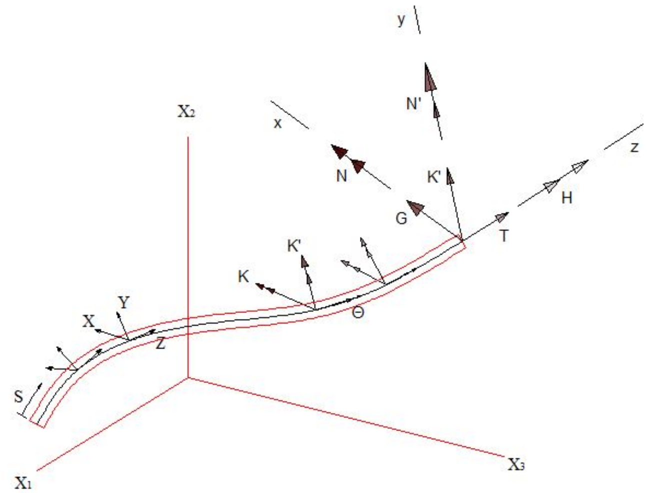

Figure 2. Loads exerted upon a thin wire [2]

It is assumed that outer wire is not exposed to external bending moments (K_2=K'_2=Θ_2=0), that the outer wire is not exposed to external force in the y and z axes in unit length (Y_2=Z_2=0) and that the axial wire tension is constant along the wire length (T_2). Six differential equations presented above become as equation set (2) considering assumptions (Costello, 1990).

$$
\begin{aligned}
&-N'_2\overline{\tau_2}+T_2\overline{\kappa'_2}+X_2=0\\
&Y_2=0\\
&Z_2=0\\
&-G'_2\overline{\tau_2}+H_2\overline{\kappa'_2}-N'_2=0\\
&N_2=0\\
&\Theta_2=0
\end{aligned}
\tag{2}
$$

The subscript 2 refers to outer wires and barred symbols refer to the deformed state of wire. Costello's theory is valid when the outer wires do not come into contact with each other.
Equations (3) and (4) can be derived for the strain of the straight center wire (ξ_1) and rotational strain (β_2) of an outer wire respectively (Costello, 1990).

$$\xi_1=\xi_2+\frac{\Delta\alpha_2}{\tan\alpha_2}=\varepsilon \tag{3}$$

$$\beta_2=r_2\tau_s=\frac{\xi_2}{\tan\alpha_2}-\Delta\alpha_2+\nu\frac{(R_1\xi_1+R_2\xi_2)}{r_2\tan\alpha_2} \tag{4}$$

Now following equations can be written for an outer wire (Costello, 1990).

$$\frac{G'_2}{ER_2^{\,3}}=\frac{\pi}{4}\left(-\frac{2\sin\alpha_2\cos\alpha_2}{r_2/R_2}\Delta\alpha_2+\nu\frac{(R_1\xi_1+R_2\xi_2)}{r_2}\frac{\cos^2\alpha_2}{r_2/R_2}\right) \tag{5}$$

$$\frac{H_2}{ER_2^{\,3}}=\frac{\pi}{4(1+\nu)}\left(\frac{(1-2\sin^2\alpha_2)}{r_2/R_2}\Delta\alpha_2+\nu\frac{(R_1\xi_1+R_2\xi_2)}{r_2}\frac{\sin\alpha_2\cos\alpha_2}{r_2/R_2}\right) \tag{6}$$

$$\frac{N_2'}{ER_2^{\ 2}} = \frac{H_2}{ER_2^{\ 3}} \frac{\cos^2\alpha_2}{r_2/R_2} - \frac{G_2'}{ER_2^{\ 3}} \frac{\sin\alpha_2\cos\alpha_2}{r_2/R_2} \tag{7}$$

$$\frac{T_2}{ER_2^{\ 2}} = \pi\xi_2 \tag{8}$$

$$\frac{X_2}{ER_2} = \frac{N_2'}{ER_2^{\ 2}} \frac{\sin\alpha_2\cos\alpha_2}{r_2/R_2} - \frac{T_2}{ER_2^{\ 2}} \frac{\cos^2\alpha_2}{r_2/R_2} \tag{9}$$

Equation (10) is obtained by taking the projections of the forces acting on the outer wires in the strand axis.

$$\frac{F_2}{ER_2^{\ 2}} = m_2\left(\frac{T_2}{ER_2^{\ 2}}\sin\alpha_2 + \frac{N_2'}{ER_2^{\ 2}}\cos\alpha_2\right) \tag{10}$$

F_2 is the axial load exerted upon all outer wires, m_2 is the number of outer wires. The total torsional moment M_2 affecting the outer wires is calculated using equation (11) (Costello, 1990).

$$\frac{M_2}{ER_2^{\ 3}} = m_2\left(\frac{H_2}{ER_2^{\ 3}}\sin\alpha_2 + \frac{G_2'}{ER_2^{\ 3}}\cos\alpha_2 + \frac{T_2}{ER_2^{\ 2}}\frac{r_2}{R_2}\cos\alpha_2 - \frac{N_2'}{ER_2^{\ 3}}\frac{r_2}{R_2}\sin\alpha_2\right) \tag{11}$$

Axial load F_1 and torsional moment M_1 acting on the center wire are found by using equation (12) and equation (13), respectively (Costello, 1990).

$$\frac{F_1}{ER_1^{\ 2}} = \pi\xi_1 \tag{12}$$

$$\frac{M_1}{ER_1^{\ 3}} = \frac{\pi}{4(1+\nu)} R_1\tau_s \tag{13}$$

Total axial load F and total torsional moment M affecting the strand are found by using equation (14) and equation (15) respectively (Costello, 1990).

$$F = F_1 + F_2 \tag{14}$$

$$M = M_1 + M_2 \tag{15}$$

In the case of axial loading, the loads acting on the wires forming the straight strand can be found by using the equations given above. Wire stresses due to those loads can also be found.
Tensile stress on the center wire can be found by using equation (16) (Costello, 1990).

$$\sigma_1 = \frac{F_1}{\pi R_1^{\ 2}} \tag{16}$$

Tensile stress generated by axial wire load in the outer wire can be found by using equation (17) (Costello, 1990).

$$\sigma_2 = \frac{T_2}{\pi R_2^{\ 2}} \tag{17}$$

3. An Application of Theory to the Prestressing Strand

In this study, prestressing strand which has 11.11 mm diameter is used to apply Costello's theory to determine mechanical response of investigated prestressing strand. ASTM A416 standard (2012) describes important properties of prestressing strands. Geometric features and properties of investigated prestressing strand are as follows: The center wire radius (R_1) is 1.875 mm, outer wire radius (R_2) is 1.84 mm, lay length of strand is 165 mm, number of outer wires (m_2) is 6, Poisson's ratio is 0.3, modulus of elasticity is 196500 MPa, strand grade is 1860 MPa, helix radius (r_2) is found by adding R_1 and R_2. $\tan \alpha_2 = p_2/2\pi r_2$.

The results obtained by analytical calculations conducted in accordance with Costello equations are shown in Table 1.

Table 1. Loads, stresses, strains and twisting moments occurred on investigated strand

Results	**Unit**	**Costello Theory Results**
Strand strain, ε Center wire strain, ξ_1	m/m	0.003
Outer wire strain, ξ_2	m/m	0.0029239
Tensile stress on the center wire σ_1	MPa	589.502
Tensile stress on the outer wire, σ_2	MPa	574.542
Maximum axial load carried by center wire, F_1	N	6510.847
Maximum total axial load carried by outer wires, F_2	N	36305.44
Total axial load carried by strand, F	N	42816.287
Maximum twisting moment on center wire, M_1	Nmm	0
Maximum total twisting moment on outer wires, M_2	Nmm	18135.038
Maximum total twisting moment on strand, M	Nmm	18135.038

If results are examined outer wire strain is determined as 0.0029239 when strain value of 0.003 is applied to strand. Center wire strain equals to strand strain. Center wire carries 6510.847 N and each outer wire carries 6050.906 N. The entire prestressing strand carries 42816.287 N. It can be concluded that center wire carries 15.21% of total load and outer wires carry 84.79% of total load. Tensile stresses in the center wire and outer wire are determined as 589.502 MPa and 574.542 MPa respectively. There is no twisting on center wire since the center wire is straight. The maximum twisting moment calculated on the outer wires is 18135.038 Nmm.

4. Conclusions

In this study, a prestressing strand whose technical features are describes in ASTM A416 standard has been investigated. The maximum axial loads, maximum twisting moments and stresses to which the rope strand is exposed are calculated in accordance with Costello's theory and given in study. The entire prestressing strand carries 42816.287 N when prestressing strand having 11.11 mm diameter is strained by value of 0.003. Tensile stress on the center wire is little greater than the tensile stress on outer wire. Outer wire strain value is determined as 0.0029239. Maximum total twisting moment generated on strand is found as 18135.038 Nmm.

References

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Biographies

Yusuf Aytaç Onur is an Associate Professor at Zonguldak Bulent Ecevit University, Zonguldak, Turkey. He received his BSc degree at the University of Kocaeli, his MSc degree at Istanbul Technical University and his PhD at Istanbul Technical University, Turkey in 2003, 2006 and 2010 respectively. He has been conducting research into material handling and especially elevator systems.

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