A Model for Production Possibility Frontier of an Enterprise using Multi-Objective Optimization Approach

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Abstract

The paper aims to propose a model for production possibilities frontier for commodities of an enterprise. The motivation of the paper is to contribute to one of the Sustainable Development Goals set by the United Nations, which is to reduce carbon footprint towards the net zero emissions by 2050. A Multi Objective Optimization is used in the study, to help managers decide what mix of products to produce in order to minimize the production costs and carbon dioxide emissions with their current resources. Data in the model comes from a case study company to have numerical parameters and test the feasibility and efficiency of the proposed model. The input parameters and all constraints mentioned in the study was transformed into mathematical model and was coded in Linear programming software. The results show a Pareto Optimal mix of products or production possibility frontier in which the optimum solution points are converged within the boundary of their resources.

Keywords
Pareto Optimization, Production Possibility Frontier (PPF), Multi-Objective Optimization, Carbon Footprint, Net Zero Emissions

1. Introduction
The concept of sustainable development was formally introduced in UN report Our Common Future (Development, 1987). Likewise, in year 2000, the United Nations Development Programme (UNDP) provides a development paradigm about sustainable development. In 2015, the Paris Agreement by 196 member parties committed to transformed the trajectory of sustainability to limit global warming to below 2^0C, above pre –industrial levels (UN, 2021). In order to meet these goals, the carbon dioxide emissions should be reduced by 45% by 2030 from 2010 levels and reach net zero emissions by 2050 (UN, 2021).

There is a strong concern for implementing sustainable development in all business activities and human activities. Sustainable development is defined as "development that meets the needs of the present without compromising the ability of future generations to meet their own needs" (Drexhage & Murphy, 2010). Resource scarcity and environmental pollution have increasingly a major concern globally. Strict regulations and the demand for innovations have driven organizations to implement sustainable practices (Yacob P. et al 2019). Manufacturing is a critical contributor to a country's economic growth, consequently, the integration of sustainable production systems should be implemented in the complete life cycle of the products (Belhadi et al, 2020).

Carbon footprint measures the products or services environmental impact on global warming. It is the total gas emissions caused by companies, product or person. Manufacturing companies had been a major source of global carbon emission (Olantunji et al, 2019). Sustainable development is link to manufacturing and consumption of both goods and services to reduce carbon footprint. Organizations are forced to integrate reduction of carbon footprint as part of their operational strategies. Some of the motivating factors are regulation (Lee, 2012) and evolving environmental criteria as basis for being competitive (Baumgartner & R., 2018). The future product demands are linked to the manufacturing and consumption of goods and services, which lead to sustainable development and reduction of carbon footprint. This paper supports business goals of having a sustainable production system which considers the achievement of two objectives, the production costs and the production carbon footprint. Furthermore, this project helps contributes to the achievement of Sustainable Development Goals (SDG) number 13, which stated as "Take urgent action to combat climate change and its impacts".

A production possibility frontier is presented which indicated the production possibilities of two or more commodities or products which van be produced together that helps managers and leaders decide what mix of commodities are most beneficial. The production possibility frontier (PPF) assumes that technology is constant, resources are used efficiently.

1.1 Objectives

The paper aims to develop a mathematical model for the optimum mix of products to produce that considers two objective functions which are to minimize the production cost and at the same time, to minimize the carbon footprint of the production system.

2. Literature Review

In United Nations Environment Programme, which they develop the 2030 Agenda for Sustainable Development, in which the goal is to reduce the use of natural resources as production materials and reduce the emission of wastes and pollutions over the life cycle of the products (UNEP,2015). As more and more customers are purchasing sustainable products, companies need to adopt to new sustainable practices.

The increasing deterioration of our environment forced stakeholders to include sustainability in their business goals, an adoption of corporate standard for corporate environmental management was release in 2005(ISO 14001, 2005).

The United Nations goal is to limit the use of greenhouse gas emission among highly industrialized countries. Moreover, one of the key elements for achieving sustainable development is the transition towards Sustainable Consumption and Production (UNEP, 2015).

Manufacturing plays a critical role in our economy, with this, the integration of sustainable production practices increasingly being considered by a great number of manufacturing firms. Sustainable production should put together technology, design and production practices in the system (Belhadi, et al 2020).

In the Philippines several law and decree were enacted to address environmental concerns. In 1999, Clean Air Act provides for a comprehensive air pollution control policy and a national programme to prevent, manage, control, and reverse air pollution through both regulatory and market-based instruments. For better implementation of less pollution in production, investigation should be conducted (Rehman & Shrivastava, 2018). Moreover, studies on Greenhouse gas impact generated in production that affects health and ecosystem still limited (Chen et al , 2022).

3. Methods

The proposed solution techniques is a systematic process that minimizes production cost and also at the same time minimizes production carbon footprint which will result to a sustainable production systems. A multi objective optimization model is propose. This sustainable production system model helps business organizations improved their performance on production cost and production footprint.

The first objective is to minimize the cost within the production. The production costs are assumed to be fixed production cost and variable cost. The second objective is the environmental concern which is minimizing the carbon footprint within the production. The carbon footprint considered is production carbon footprint. Since there are two (2) different objectives with different units of magnitude, a multi-objective optimization technique was used. The weighted-sum approach was used. Below is my proposed mathematical model for sustainable production systems.

MATHEMATICAL MODEL FORMULATION

A. Indices and Notations

j represents production facility

i represents product

m represents carbon footprint

B. Input Parameters

Aij = Fixed production cost for product i at production facility j

Bij = Variable cost for product i at production facility j
Fij= Carbon Footprint of production facility j per unit of product i
Gik = Production Capacity of production facility j to produce product i
Oi = Total demand of product i
C. Decision Variables
Xij = Quantity of product i to produce in production facility i
Yij= 1 if product i is produced at production facility j; = 0 otherwise
D. Objective Functions
The objective is to minimize the economic cost and carbon footprint the firm.
D.1 Economic Objective
The economic objective is to minimize the cost within the production. The production costs are assumed to be fixed production cost and variable cost. That is, Minimize Total Cost (Z1) = Fixed Cost + Variable Cost

Min Z1=$\sum n$ (Aij x Yij) + $\sum I$iI=1
(Bij x Xij)
D.2 Environmental Objective
The environmental objective is to optimize by minimizing the carbon footprint within the production. The carbon footprint considered is production carbon footprint. That is,
Minimize Carbon Footprint (Z2) =Production Carbon Footprint
E. Multi-objective Optimization
Since there are two (2) different objectives with different units of magnitude, a multi- objective optimization technique was used. The weighted-sum approach was used. The general form of this technique is,
Min w1 Z1 + w2 Z2; The equation for the multi objective function becomes a single objective function. The weights are set for economic performance (Z1) and the environmental performance (Z2).
In order to remove the units of the two objective functions, thus become single objective function. We will use Minimization Q (deviations) = w1*(actual value-target value)/target value + w2*(actual value-target value)/ target value.
F. Constraints
Xij <=Gik x Yij, ∀j
Xij>=Oi
Xij>=0 and integer
Yij= 0, 1
The first constraint (1) is about the number of units of product k to be produce should not exceed the capacity of each production facilities j. The second constraint (2) is about the number of units of product k to produce should be greater than or equal to the total product demand. Constraints (3) is integer variables while constraint (4) is a binary variable.

4. Data Collection

The case study company has three major products that can produce in any of their three production facilities.
The company wants to minimize the production costs and the carbon footprint. The production costs and carbon footprint parameters were gathered from the company. To test its effectiveness, the proposed mathematical model is used to this company. The input parameters for the proposed model were shown on Tables 1 to 5.

Table 1. Fixed Cost of Product if produced in production facility (Php)

	Production Facility 1	Production Facility 2	Production Facility 3
Product 1	350	405	375
Product 2	430	360	400
Product 3	380	360	350

Table 2. Variable Costs per Unit of the Product if produced in production facility (Php)

	Production Facility 1	Production Facility 2	Production Facility 3
Product 1	17	18	17

Product 2	21	20	21
Product 3	18	19	17

Table 3. Carbon Footprint per unit of product (kg) in each production facility

	Production Facility 1	Production Facility 2	Production Facility 3
Product 1	4.1	4.3	4.2
Product 2	4.6	4.5	4.4
Product 3	3.9	4	3.7

Table 4. Capacity in each production facility

	Production Facility 1	Production Facility 2	Production Facility 3
Product 1	825	775	800
Product 2	550	650	650
Product 3	650	580	550

Table 5. Product Demand(units)

	Production Facility 1
Product 1	1450
Product 2	1150
Product 3	1100

5. Results and Discussion

Applying the proposed mathematical model in linear programming software the results are as follows:

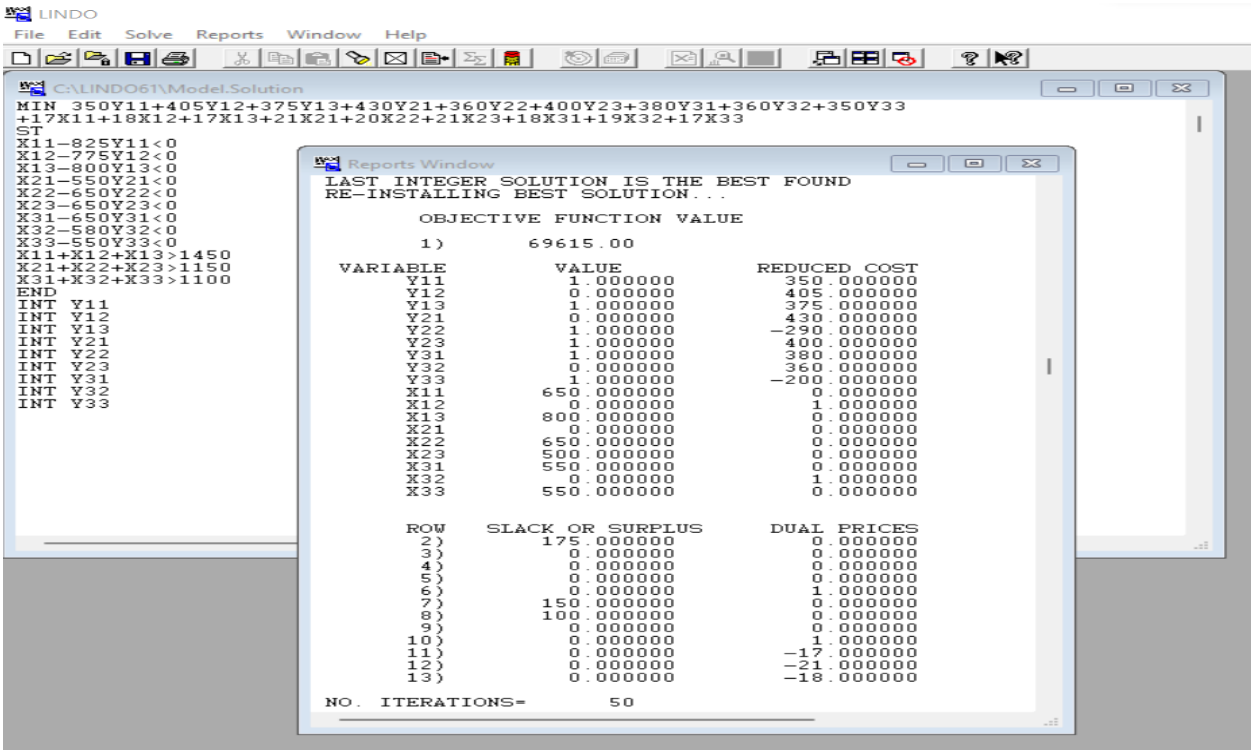

Figure 1. Optimal Solution for Economic Objective

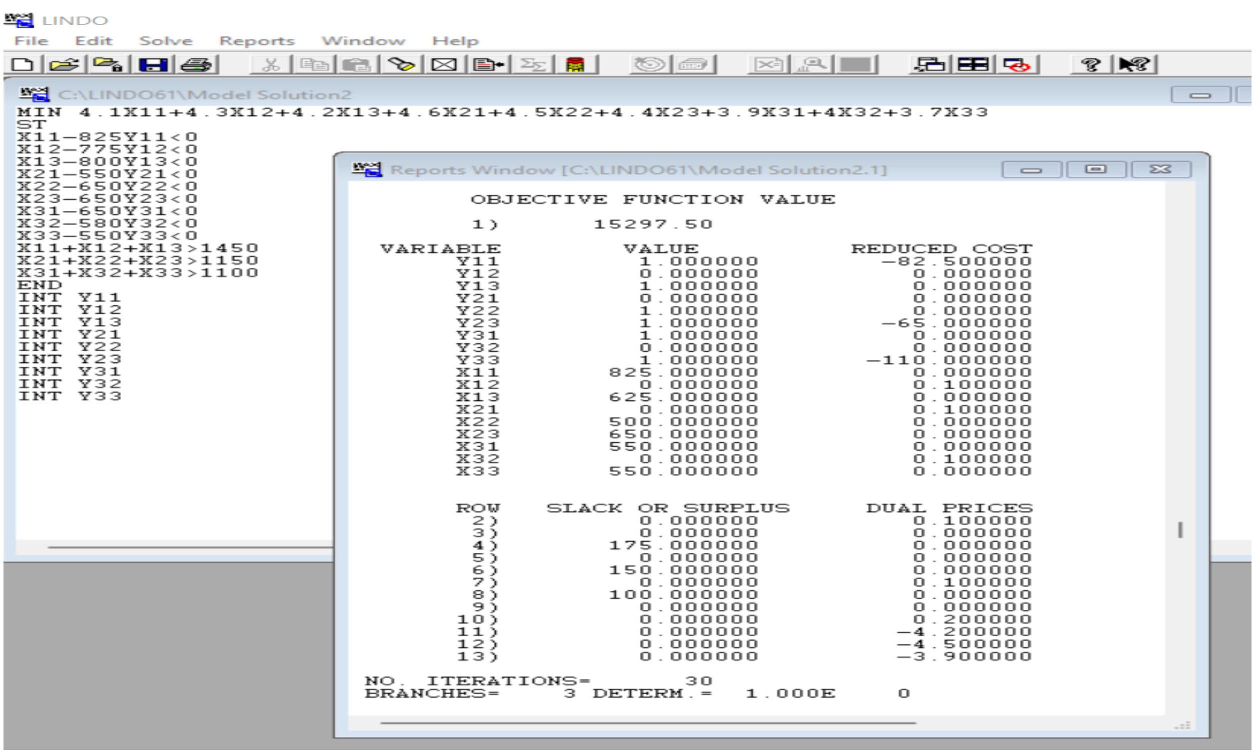

Figure 2 .Optimal Solution for Carbon Footprint Objective

Using the input parameters and all constraints mentioned, the model was coded in Linear programming software to compute for the decision variables and the objective functions. Figure 1 and Figure 2 are the results of the mathematical model formulation in software for linear programming model. The summary is also shown in below Table 6.

Table 6. Summary of optimal solutions

Decision Variables	Solution 1	Solution 2
X11	650	825
X12	0	0
X13	800	625
X21	0	0
X22	650	500
X23	500	650
X31	550	550
X32	0	0
X33	550	550
Min Z1(Cost)	69615	69765
Min Z2(C.Footprint)	15330	15297
Min Q(Deviation) (*w1*=.5; *w2*=.5)	0.11%	0.04%

The single function value of this multiobjective problem was computed below where multi objective function Min Q(deviations) equation was used. This objective function was set to minimize the percentage deviation from the target. Since solution 2 has a smaller deviation, the study chooses solution 2 which will serve as product mix planning schedule or the production possibility frontier. The solution 2 identifies the quantity of product i to be produced at production facility/plant j. In this case, 825 units of product 1 should be produced at plant 1 (i.e. X11 = 825). Product 1 should not be produced in plant 2 (X12 = 0). Plant 3 should produce 650 units of product 1(X13 = 625). And so on and so forth. With the given units of product i to be produced at plant j, the overall total production cost of the company considering the resulted production plan is 69, 765 Php. This is for the monthly basis. At the same time, the carbon footprint is 15, kgs. 297.. Hence, it was validated that solution 2 is the optimal solution with 0.04 deviations from the target value.

6. Conclusion

The study found an optimal solution for the production possibility frontier of the case study company. The single function value of this multiobjective problem was computed and solution 2 will served as product mix planning schedule. The proposed model resulted in a pareto optimal for the achievement of the 2 objectives which are to minimize production cost and ,at the same time minimize carbon footprint. The feasibility of the model and its effectiveness in achieving the goals of the company are demonstrated in the proposed model.

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Biography

Ma. Teodora E. Gutierrez is a full-time faculty member in the Industrial Engineering Program at the Technological Institute of the Philippines. She is an Asean Engineer (AE) and Professional Industrial Engineer (PIE). Currently enrolled for a degree in Doctor of Engineering in Engineering Management at Polytechnic University of the Philippines. She earned her degrees in Master of Science in Industrial Engineering at the University of the Philippines, Diliman in 2006 and Master in Business Administration at the Philippine School of Business Administration in 2001. Also, she finished her Bachelor of Science in Industrial Engineering at Polytechnic University of the Philippines (PUP) in 1997. Before joining TIP, she had 4 years' experience in various industrial companies which involved production planning, facility layout, quality improvement projects, and other Industrial Engineering works. She has published several scientific publications in the international refereed journals in operations research and engineering domain.