Managing Complex Geometries through Rhinoceros: Voronoi cells

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Abstract

This research describes the possibility of handling complex geometries through Rhinoceros, three-dimensional modeling software, by means of some parameterization blocks. More specifically, by processing programming codes in an internal Rhinoceros environment (Grasshopper), it is possible to parameterize geometries, allowing accurate and dynamic modeling. The geometry analyzed in the following discussion is an unconventional closed-cell structure. It is based on the application of Voronoi diagrams in the three-dimensional field, thus creating closed cells in a foam-like manner. The objective pursued is to further study the properties of such structures and possible applications, ranging from the automotive to the biomedical field. The possibility of modeling complex structures with easiness, combined with the possibility of realizing them through modern additive manufacturing techniques, leads to the development of new, unconventional materials with unique mechanical properties and fully customizable behavior.

Keywords
Parameterization blocks, Complex geometries, Voronoi cells, Rhinoceros, Grasshopper.

1. Introduction
1.1 Rhinoceros and Grasshopper
Rhinoceros is a commercial software for 3D modeling so called freeform surfaces. The modeling process is done through NURBS (Non-Uniform Rational B-Splines), an opposite approach to polygon meshes applied by other competing software (Gou & Zhu, 2022; Grillanda et al., 2022; Herrema et al., 2017). It is known for its diversity, multidisciplinary functionality, low learning curve, relatively low cost and the ability to export files in different formats, making it also a true conversion tool within a workflow. The software has numerous plug-ins, which allow many different functionalities and increase the potential of the basic software. In general components made with Rhinoceros are used in structural and energy optimizations, but also in the naval industry thanks to the possibility to realize complex shapes, which do not follow classical geometric primitives (Keskas et al., 2022; Willis, 2016; Wu et al., 2022). The Rhinoceros plug-in used is Grasshopper: it is a block programming environment (high level) that makes Rhinoceros parametric. Grasshopper allows complex 3D shapes to be generated by defining a node diagram (algorithm) that can describe the mathematical and geometric relationships present within a model. Three-dimensional models developed in this context are dynamic systems that can be modified in real time by varying the parameters defined during the construction of block diagrams, with immediate advantages in controlling geometries (Hsu et al., 2015).

1.2 Voronoi structure
The Voronoi diagram is named after Georges Voronoi and is also called Voronoi tessellation, or Dirichlet tessellation (from Peter Gustav Lejeune Dirichlet)(Dirichlet, 1889; Voronoi, 1908). The mathematical definition of a Voronoi diagram is basically a subdivision of a plane into zones close to a given set of objects. In the simplest case, considering a plane and thus only two dimensions, these objects are a finite number of points in the plane and are called seeds. For each point, a corresponding region, called the Voronoi cell, is determined; this consists of all points in the plane closer to that specific seed than to any other. The process is carried out for each seed. Voronoi diagrams have both practical and theoretical applications in many fields, especially in science and technology (Cheng et al., 2022; Frizziero et al., 2021; Pan et al., 2020).

The application of Voronoi subdivision in 3D is an extension of the two-dimensional model. The seeds are placed in a volume and the cells become a set of points that form closed volumes that minimize the distance between one seed and the others (Ledoux, 2007). If seeds are placed in the volume in a regular and equidistant manner a volume with regular cells of the same volume will be obtained, vice versa the cells will have different volume.

1.3 Introduction to FDM (Fused Deposition Modelling) and close cell printing

Fused Deposition Modeling (FDM) is a 3D printing technology that relies on the deposition through lines and layers of thermoplastic polymers. A variety of polymers can be processed in FDM 3D printing, from conventional PLA to engineering polymers such as PVB and PEEK (Ferretti, Santi, Leon-Cardenas, Freddi, et al., 2021; Ren et al., 2022). With appropriate arrangements, flexible elastomers such as TPU and TPE can also be printed (Ferretti, Leon-Cardenas, et al., 2021). The special feature of FDM printing is the possibility of making closed-cell latex structures (Ben Ali et al., 2019). This is possible because there is no need to remove powder, as is the case with Selective Laser Sintering (SLS) printing, or the risk of encapsulating resin in the cells as is the case with Stereolithography (SLA) printing (Capel et al., 2013). The geometry of the cells that can be made is very constrained by one of the limitations of FDM printing, which is the need for the support beyond a certain angle of inclination. This value also varies as the polymer itself being printed varies (Jiang et al., 2018).

1.4 Objectives of the study

This paper describes the modeling of a Voronoi-type closed-cell structure intended for the fabrication of a 3D printed polymer foam. This is the first step before the complete mechanical characterization of this cell structure, following an approach similar to that of Ge et al., 2018.

The objective is the realization of a "macroscopic cell", that means a cubic volume filled with a 3D Voronoi structure. This structure has a known density value and changes as the input parameters change. To make the cell investigable by modern finite element software, it needs to be symmetrical on opposite faces, so that a structured mesh (symmetry of nodes on opposite faces) can be made. This is a basic requirement for the creation of a so called RVE (Representative Volume Element), one of the most common methodologies to study foam like materials (Ferretti, Santi, Leon-Cardenas, Fusari, et al., 2021). Clearly, not all 3D Voronoi structures respect the printability limits required by the FDM manufacturing process. A lot of work has been done to achieve a structure that minimizes the creation of too excessive overhangs and also reduce to a minimum area of "bridging", while keeping a constant wall thickness of the various cells. The "macroscopic cell" can then be multiplied to obtain a larger cell, thanks to the symmetric geometry. The realization of the final component with 3D Voronoi cell structure can be achieved by Boolean cutting of solids, between the final cell and the part (Novak, 2019).

In Figure 1 is schematized the Workflow followed for this study. In the following, the methodologies, arrangements and aspects that characterize such a structure are analyzed, as well as the problems that arise from the implemented methodology.

Figure 1. Workflow

2. Methods

2.1 Domain Definition

Initially, it is necessary to define the domain within which the non-symmetrical model is created with the 3D Voronoi structure inside it. A parametric cube was used as a border region for the purpose.

The first step is to define the plane on which the parametric box rests and its size. In this case the XY plane is used with origin at the point (0,0,0) and the coordinates of the box are 5x5x5 (mm) (Figure 2). In Grasshopper environment is very simple to manage input parameters: numbers can always be used with input and almost all command blocks can be used as both input and output.
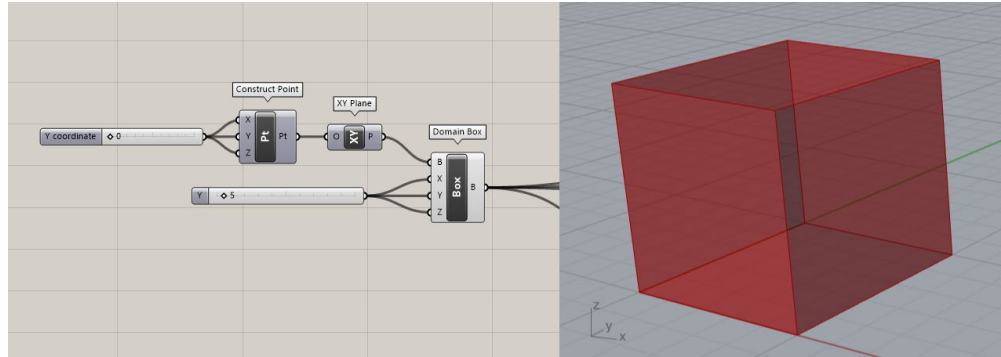
Figure 2. Domain Definition

2.2 3D Voronoi Cells Definition

To define 3D Voronoi Cells it is necessary to populate the domain with random points through the command "Populate 3D". The command "Voronoi 3D" generate Cells of the 3D Voronoi diagram from the collection of points. It is necessary to specify that the domain must be specified in both commands. The Figure 3 shows how to manage the number and arrangement of cells by changing the number of points to add and the random seed for insertion in the "Populate 3D" input.
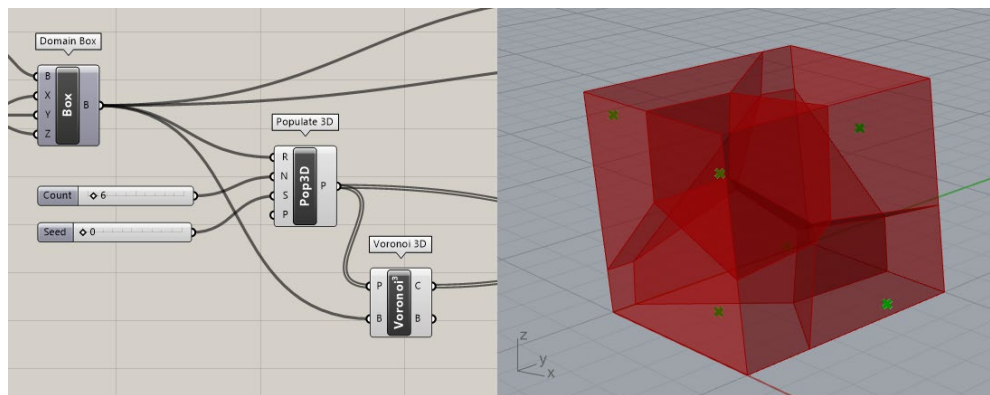

Figure 3. 3D Voronoi cells definition

2.3 3D Voronoi Non-Symmetrical Structure Definition

To obtain 3D Voronoi Non-Symmetrical Structure (Figure 4) it is necessary to scale both the domain and the Voronoi cells by the same factor: the first respect to its center and the latter with respect to its points of origin. Finally, a simple solids difference operation must be performed. The center of the box is easily found through the command "BoxProperties", which returns all the properties of the domain. This step is necessary because if only the cells are scaled in this environment is possible to perform the solid difference. Otherwise, some surfaces of Voronoi cells would overlap and the solid difference wouldn't work.
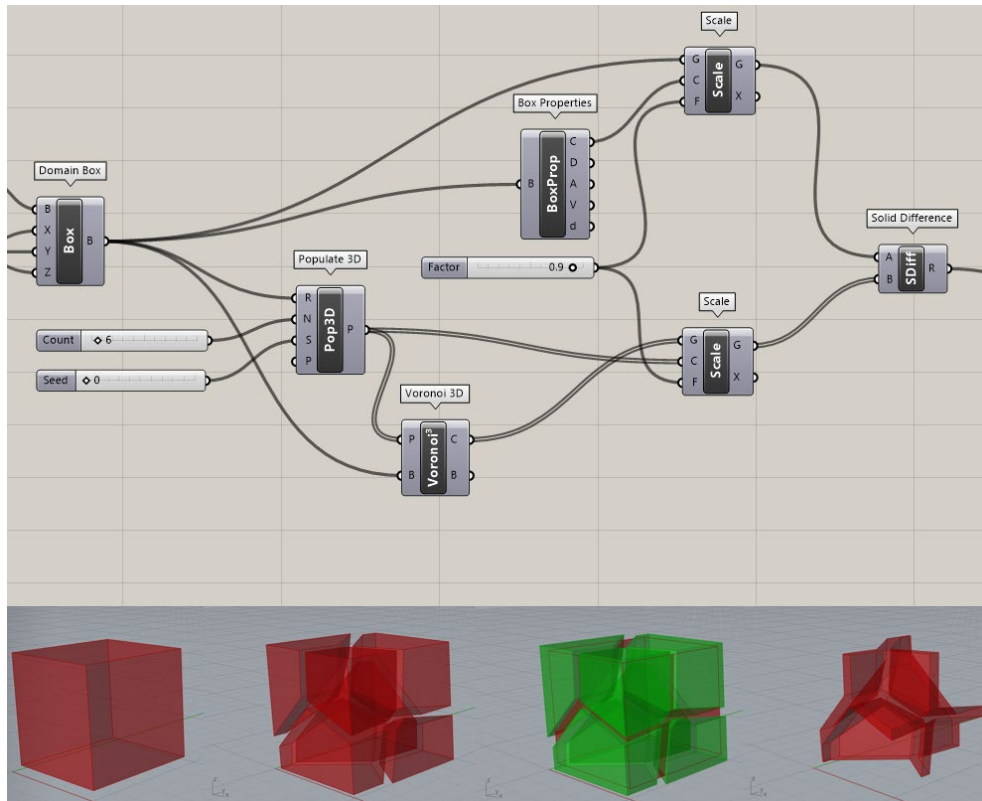
Figure 4. Non-Symmetrical structure definition

As mentioned in the previous chapter, different structures can be obtained by changing the number of cells and the arrangement. It is also possible to change the scale factor and increase the thickness of part of the structure. In this way, different densities are obtained for the structure. It is important to emphasize that to achieve a good result, these parameters should be chosen wisely. If the complexity of the geometry is increased, it is not possible to manage the algorithm easily. It is not possible to correctly perform the solid difference and obtain a single boundary representation through the command "SolidUnion". This problem is analyzed in the next chapter. Figure 5 shows different structures:
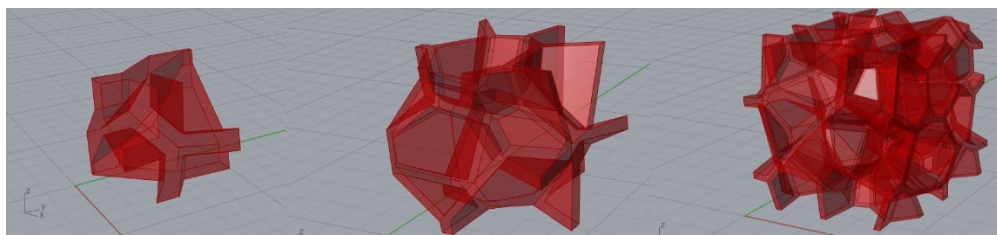

Figure 5. Different structures

2.4 3D Voronoi Symmetrical Structure Definition

To make the cell be symmetrical on opposite faces, it is necessary to refer to the symmetries of the cube. By appropriately mirroring the previous structure and joining the intermediate results, selecting the mirroring planes with order and accuracy, a macroscopic cell with opposite symmetrical faces is obtained (Figure 6). The volume of each B-Rep is easily measurable with the command "Volume". It solves volumed properties for closed B-Reps and mesh. It is also evident in this case that it is very simple to manage input parameters. For example, the point of origin of the planes is obtained by extrapolating a vertex of the previously defined scaled domain and applied for all of them.
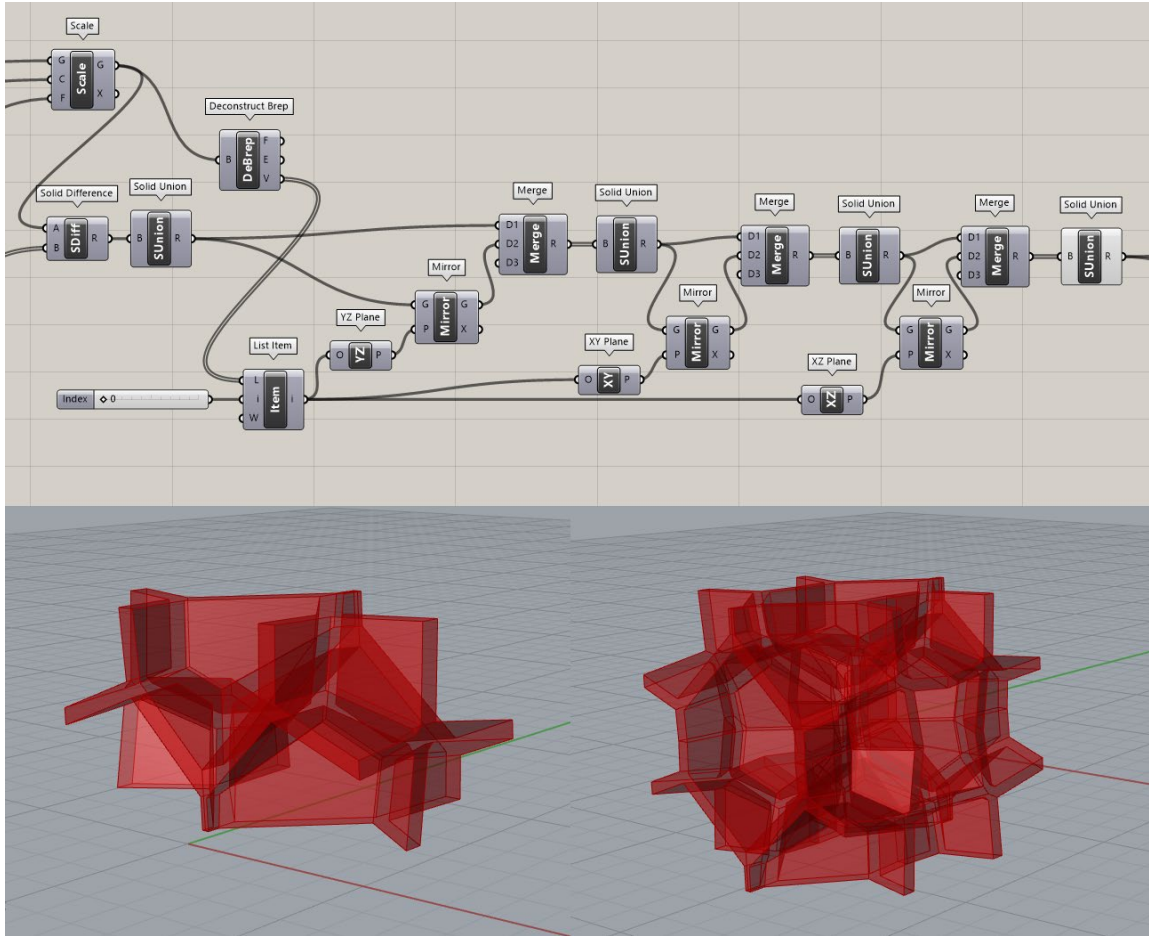
Figure 6. Symmetrical Microcell Definition

As it is important to carefully choose the input parameters, it is essential that the structure obtained is a single boundary representation through the command "SolidUnion". Above all, it is critical that once the structure is repeated, it allows the creation of a closed cell structure. It is necessary to check this at the end of this operation. In particular, after numerous attempts, it was found the best choice is to have at most 6 Voronoi cells and 0 seed numbers, in order to get a good result and to complete the next steps. The Figure 7 shows what happens if this indication is not respected.
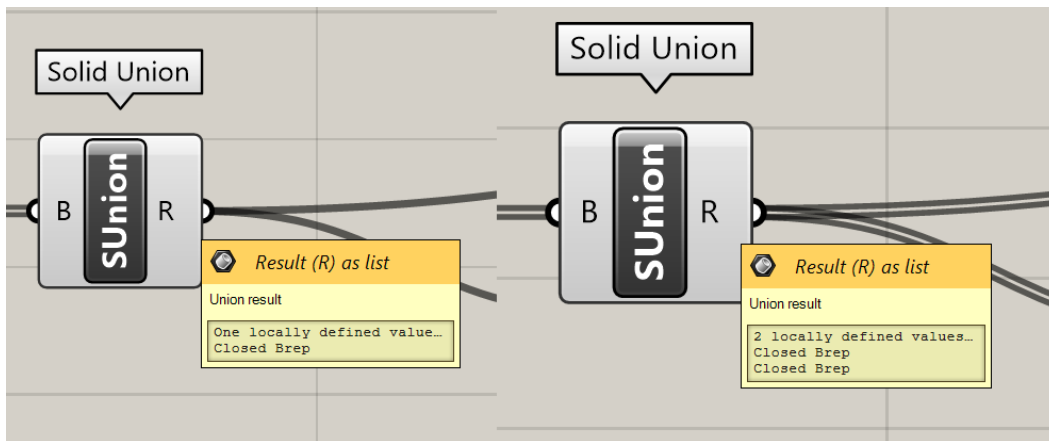

Figure 7. B-Rep problems

2.5 Array

The macroscopic cell obtained can be multiplied to make a larger cell. Its symmetry allows to guarantee the continuity of the structure. To do this it is necessary to confine it through the command "BoundingBox" before providing inputs for the array. The Figure 8 shows that this software fails to get a single B-rep from the array. This is a common problem in this environment. Also trying to create the union of the representations with further algorithms or by increasing the tolerance range of the environment the procedure fails. Therefore, once created, it is necessary to export the structure in formats consistent with those used by other software. Rhinoceros allows to export geometries in many and different formats, preserving their characteristics and dimensions, but losing the design tree.
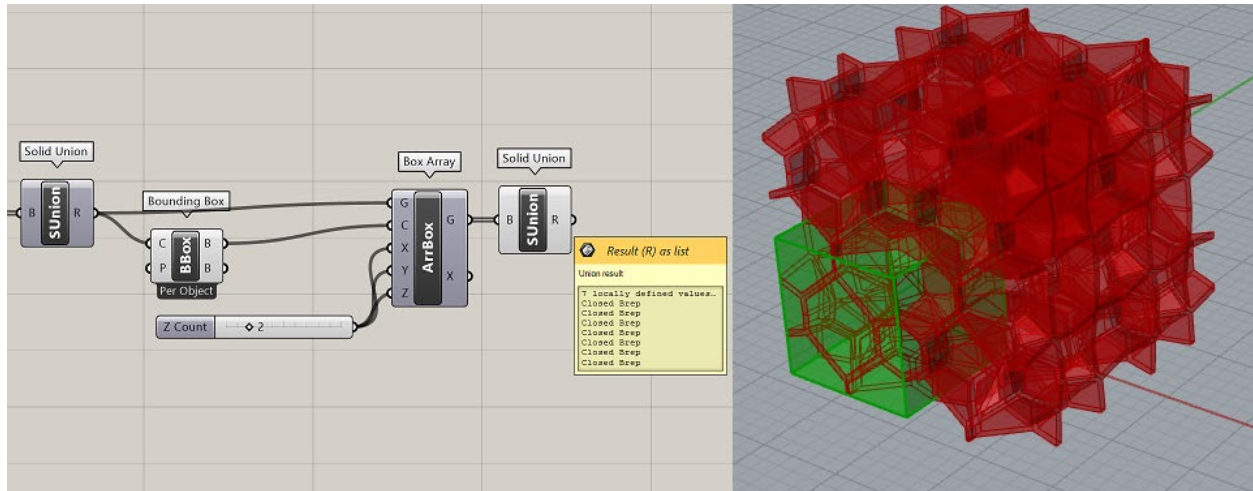

Figure 8. Box Array

3. Results and Discussion

After the macrocell was made on Rhinoceros, the result was exported in .STL format. The export was customized in order to better capture the geometry as best as possible. The "maximum angle" was set to 0.1° and the "maximum edge length" to 0.1 mm.

The STL was placed inside the Ultimaker CURA 5.2.1 slicer in order to generate the proper toolpath (G-code) needed for printing. The filament chosen was Ninjaflex 85A from Ninjatek, a TPU (Thermoplastic polyurethane) filament that exhibits good printability while requiring specific parameters for printing. Table 1 shows the main values of the mechanical properties of this filament, as stated by the manufacturer itself. It is possible to notice the high elongation-at-break values typical of elastomeric polymers, but also the moisture-absorption specifications that determine that this filament must be stored properly and with the appropriate precautions in order to avoid extrusion problems during printing.

Table 1. Ninjaflex 85A technical specification

General properties	Unit	Value	Test Method
Specific Gravity	g/cc	1.19	ASTM D792
Moisture Absorption - 24 hours	%	0.22	ASTM D570
Tensile Strength, Yield	MPa	4	ASTM D638
Tensile Strength, Ultimate	MPa	26	ASTM D638
Tensile Modulus	MPa	12	ASTM D638
Elongation at Yield	%	165	ASTM D638
Elongation at Break	%	660	ASTM D638

Hardness	ShoreA	85	ASTM D2240
Impact Strength (notched Izod, 23C)	kJ/m^2	4.2	ASTM D256
Heat Deflection Temperature (HDT) @ 10.75psi / 0.07 MPa	°C	60	ASTM D648
Heat Deflection Temperature (HDT) @ 66 psi / 0.45 MPa	°C	44	ASTM D648

Table 2 summarizes the printing parameters used, in accordance with those suggested by the manufacturer. In general, it was chosen to reduce the printing speed a lot, so as to avoid buckling effects due to filament flexibility, resulting in extrusion errors. The speed of the extrusion-free movements ("travel speed") was increased greatly, up to the printer's limit, this was combined with the "Z seam position" to the shortest possible path, to reduce the problems of oozing that are typical of flexible filaments. The "fan speed" is set to 100% to rapidly cool the extruded filament, maximizing overhang and bridging capability. The fan starts to run from the second layer onward to avoid warping phenomena on the first layer due to excessive cooling. The "flow" that determines the amount of extruded filament was slightly increased (105%) to compensate for the flexibility of the filament. Finally, the reduction of "retraction distance" was performed because of the elasticity of the filament itself, thus making very high values unnecessary. To compensate for filament leakage before the start of printing and to help with adhesion, a small brim was added to the outside of the part, as can be seen as a blue line in Figure 9. This set of values give us good reliability and minimize printing artifacts on the printed geometry.

Table 2. Slicing Printing Parameters Applied in Ultimaker Cura for Ninjaflex 8A5 TPU filament

Parameter	Unit	Value
Layer Height	mm	0.2
Line width	mm	0.4
Wall Line Count	-	3
Z seam position	-	Shortest
Top Layers	-	1
Bottom Layers	-	1
Infill Density	%	100
Printing Temperature	°C	215
Build Plate Temperature	°C	70
Flow	%	105
Print Speed	mm/s	35
Travel Speed	mm/s	200
Retraction Distance	mm	0.2
Fan Speed	%	100
Regular Fan Speed at Height	mm	0.4

Figure 9 shows the result once slicing is performed. It can be seen that the vertical walls print without the need for supports, this is thanks to the overhang's reduction optimization performed during the creation of the geometry inside Rhinoceros. Small bridging areas are still present, these are not a major problem, but can be further reduced in the future by acting directly on the initial geometry Printing is done using a delta-type 3D printer, the Anycubic Predator. This type of printer allows for fast printing and fast movement due to the extreme reduction in the mass of moving parts, compared to traditional Cartesian printers.
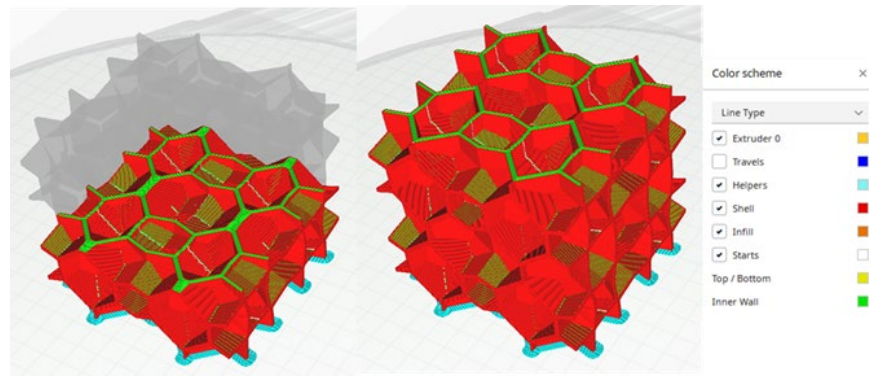
Figure 9. 3D Voronoi structure sliced inside Ultimaker Cura 5.2.1

The time required to print a cube of size 100x100x100 is 8 hours. This can be further reduced by optimizing the starting geometry and the nozzle path itself defined within the slicer. On larger prints or with a view to faster printing of this type of polymer, it would also be possible to optimize the starting geometry in order to be able to use larger nozzle sizes. These allow a considerable reduction in the printing time required at the cost of a typically lower surface finish. Also given the larger nozzle diameter, there would be less back pressure reducing buckling in the filament and being able to increase the printing speed itself as well. The printing temperature would then be raised to accommodate the higher volumetric flow rate, needing to heat a larger volume of filament inside the nozzle and extruding it in a shorter time.

4. Conclusion

Through the use of Rhinoceros and Grasshopper, it was possible to create a parametric workflow for the realization of closed-cells Voronoi like structure. The dimension and density of the cells can be adjusted in real time, without the need of a complete redrawing of the geometry. Thanks to FDM 3D printing process capability and the possibility of processing polymers such as TPU, it was possible to make a first foam like cube with this type of structure. The choice of printing parameters is vital to reduce the creation of artifacts on the printed geometry or under extrusion zones/extruder failure due to filament buckling. The combination of the presented geometry combined with the appropriate choice of settings gives us a stable result and good starting point for future development.

The next step is to improve both the obtained Voronoi geometry, improving its printability, and the realization of the geometry itself, optimizing the Grasshopper programming flow to allow Rhinoceros to process these structures even on larger array scales.

It may be of interest once the Grasshopper workflow has been optimized, to integrate the cell fabrication process with a density-based optimization process (continuously variable density) that changes the Voronoi cell density based on, for example, a stress or strain field. In this way, the density can be increased where needed, leading to foams optimized for the specific application or purpose.

Of course, accurate mechanical tests need to be conducted to evaluate the compressive stress-strain behavior of these types of structures, including evaluating the effect of density variation on mechanical properties. Having done that, it would be possible to make Ashby-Gibson curves in which to plot, for example, the reduced Young's modulus versus the reduced density, so as to get a better view of the behavior of these foams. Because of the anisotropy introduced during the layer-by-layer printing, it would be necessary to do compression tests not only along the Z direction of printing, but also in the XY plane.

This type of testing would require a very large number of samples, resulting in wasted material, time and high costs. The solution to this problem is the possibility to accurately predict the behavior at different density and different compression plane using FEA (Finite Element Analisys) simulation performed on specific software. The simplest method to simulate these porous materials is to create a nonlinear RVE and then validate it at some specific points on the Ashby-Gibson curve (defined reduced density values). This would greatly reduce the number of tests to be performed, but would ensure that the behavior of these foams can be predicted at any density value.

The realization of this FE (Finite Element) model is quite simple and many FE software have a dedicated integrated package. A further simplification occurs if the mesh is structured, that is, given a cube of material, there is symmetry of the nodes on the opposite faces. The simplest way to achieve this is to use a symmetrical starting geometry for the cube, like the one we made in this study.

Possible applications of this type of structure range from use as shock absorbers, for example in mountain bike guards or within the structure of helmets. In this case, the RVE model would be followed by a study through explicit FE simulations to validate the behavior of the Voronoi structure and material during impact and collapse of the cells themselves. Or intended for use as custom foams in the automotive industry to create custom seats or special motorcycle seats to improve comfort or create prototypes. All the way to the biomedical sector, where they could be used, for example, to produce orthopedic insoles customized to individual patient specifications.

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