Cost Minimization: A Comparison Study on Various Types of Transportation Methods

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Abstract

This is an era of globalization. At present the market growth in our country is remarkable.But sometimes we see some fickleness in market because taste and choice of consumersare changing nowadays. And market is very much dependent to consumers' taste and choice. Taking steps according to consumers' is very necessary in business market. One of the most important and successful applications of quantitative analysis to solve business problem has been in the physical distribution of products, commonly referred astransportation problems. The purpose is to minimize the cost of shipping goods from onelocation to another so that the needs each arrival area are met and every shipping locationoperates within its capacity. We could set up a transportation problem and solve it using the simplex method as with any Linear programming problem. But the special structure of the transportation problemallows us to solve it with a faster, more economical algorithm than simplex. This work is on comparative study of determining transportation cost by different methodsand choosing those methods which offer least cost and maximize the profit.

Keywords
Transportation scenarios, Linear Programming Problem, Transportation Cost, MATLAB and Initial Basic Feasible Solution.

1. Introduction
Transportation cost depends on several factors, such as distance, quantity, and method of transportation (Saharan et al. 2020; Gupta et al. 2021). Fjellstrom determined the transportation cost of ore and waste material to the crusher and backfilling rooms in the underground Renstrom mine using the software package "AutoMod." Two models were used to generate an optimized solution for a transportation problem that minimized the cost of transporting iron ore from two ore mines to three steel plants (Hussain et al. 2022). The authors compared their results to determine the most practical model for a real-world situation and how significant the difference in cost would be (Gayialis et al. 2022; Sarkar et al. 2021). The transportation problem is a special kind of Linear Programming Problem (LPP) in which goods are transported from a set of sources to a set of destinations subject to the supply and demand of the sources and destination respectively such that the total cost of transportation is minimized (Chen et al. 2022). It is also sometimes called as Hitchcock problem. The objective of transportation problem is to minimize the cost of distributing a product from a number of sources or origins to a number of destinations. The problem of minimizing transportation cost has been studied since long and is well known (Blanco et al. 2019; Zhong et al. 2022; Zhou et al. 2021). In this work we have added a new algorithm that provides a. better IBFS, for both the balanced and unbalanced TP, than those algorithms just (J Nahar et al. 2018).

Types of Transportation problems:
• **Balanced:** When both supplies and demands are equal then the problem is said to be a balanced transportation problem.
• **Unbalanced:** When the supply and demand are not equal then it is said to be an unbalanced transportation problem. In this type of problem, either a dummy row or a dummy column is added according to the requirement to make it a balanced problem. Then it can be solved similar to the balanced problem.

Methods for solving transportation problem
The transportation problem as a major problem in linear programming problems is important. To solve the transportation problem we need to find a feasible solution (J.Reeb and S. Leavengood 2002). The feasible solution of the transportation problem can be obtained by using:

1. North-West Corner method, 2. Row Minima method, 3. Column Minima method, 4. Matrix Minimum method, 5. Vogel's Approximation method. After obtaining feasible solutions, we use the existing methods to achieve the optimal:
1. Modi method, 2. Method of stepping stones

Mathematical formulation of transportation problem: The transportation problem deals with the transportation of any product from m origins, $O_1 \cdots, O_m$ to n destinations,$D_1 \cdots, D_n$ with the aim of minimizing the total distribution cost,where:

- The origin hasO_i a supply of a_i units,$i = 1, \cdots, m$

- The destinationD_j has a demand for b_j units to be delivered from the origins, $j = 1, \cdots, n$

- •c_{ij} is the cost per unit distributed from the origin O_i to the destination D_j, $i = 1, \cdots, m$, $j = 1, \cdots, n$

In mathematical terms, the above problem can be expressed as finding a set of $x_{ij}'s$, $= 1, \cdots, m$, $j = 1, \cdots, n$, to meet supply and demand requirements at a minimum distributioncost. The corresponding linear model is:

$$\min z = \sum_{i=}^m \sum_{j=1}^n c_{ij}x_{ij}$$

subject to

$$x_{ij} \leq , i = 1, \cdots, m$$
$$\sum_{j=1}^n x_{ij} \geq , j = 1, \cdots, n$$
$$\sum_{i=1}^m$$

$$x_{ij} \geq 0, i = 1, \cdots, m, j = 1, \cdots, n$$

Thus, the problem is to determine x_{ij}, the number of units to be transported from to D$_j$, so that supplies will be consumed and demands satisfied at an overall minimum cost. The first m constraints correspond to the supply limits, and they express that the supply of commodity units available at each origin must not be exceeded (HUNJET et al. 2003; Henry Otoo et al. 2019). The next n constraints ensure that the commodity unit requirements at destinations will be satisfied.The decision variables are defined positive, since they represent the number of commodity units transported.

The transportation problem in standard form is shown below:

$$\min z = \sum_{i=}^m \sum_{j=1}^n c_{ij}x_{ij}$$

subject to,

$$x_{ij} = , i = 1, \cdots, m$$
$$\sum_{j=1}^n x_{ij} = , j = 1, \cdots, n$$
$$\sum_{i=1}^m$$

$$x_{ij} \geq 0, \quad i = 1, \cdots, m, j = 1, \cdots, n$$

The matrix format for the transportation problem

The relevant data for any transportation problem can be summarized in a matrix format using a tableau called the transportation costs (A. Seethalakshmy, N. Srinivasan 2019) tableau (see Figure 1). The tableaudisplays the origins with their supply, the destinations with their demand and the transportation per-unit costs.

	D_1	D_2	$\cdots$	D_n	Supply
O_1	c_{11}	c_{12}	$\cdots$	c_{1n}	a_1
O_2	c_{21}	c_{22}	$\cdots$	c_{2n}	a_2
⋮	⋮	⋮	⋱	⋮	⋮

O_m	c_{m1}	c_{m2}	...	c_{mn}	a_m
Demand	b_1	b_2	...	b_n	

Figure 1. Transportation cost tableau

1.1 Objectives
This paper's objectives are to find the least cost using various transportation methods and conclude that Vogel's approximation method is one of the best methods for finding an initial basic feasible solution. The Modi method is recommended for optimality testing because it takes less time than the Stepping Stone method. Moreover, to justify our result from Vogel's approximation method, we also use MATLAB, which also gives the same result.

2. Literature Review
The transportation problem is a fundamental network-structured linear programming problem that arises in several contexts (Ladji, et al. 2021). We propose a two-step method for solving fuzzy transportation problems where fuzzy triangular numbers represent all parameters. The results show that the proposed way is more straightforward and computationally more efficient than existing methods in the literature (Kané et al. 2021). Cost functions are built up for each operation by each mode/means/combination. The total transportation cost is minimized concerning the choices of routes, modes and means (Jourquin et al. 1996). Transportation plays a crucial part in a vehicle and determines the efficiency of moving products; the study is limited to retail distribution networks in the business-to-customer (B2C) channel (Parkhi et al. 2014). The model includes the carbon emission costs at all steps and the variable production rate for bioenergy production (Sarkar et al. 2021).

We have already discussed the objectives of some articles. The north-west corner, row minima, column minima, Vogel's approximation, and matrix minima are the five methodologies that this study compares and contrasts in depth to provide distinct solutions to transportation problems. When compared to other methods, we find that only Vogel's approximation has the lowest cost. We also obtain the same result by using MATLAB. Additionally, we use the Modi method and Stepping Stone method to assess the optimality of our desired initial basic feasible solution. In addition, this study advises using the Modi approach rather than the Stepping Stone method. This is due to the stepping stone method's requirement that we draw numerous loops to assess if our chosen initial value is optimal. Drawing loops for every cell in the Stepping Stone method is lengthy.

3. Methods
3.1 Solving Transportation Problem with Various Methods
Problem: A company has factories at F1, F2 and F3 which supply to warehouses at W1,W2, and W3. Weekly factory capacities are 200, 160 and 90 units, respectively. Weeklywarehouse requirements are 180, 120 and 150 units, respectively. Unit shipping costs (in taka) are as follows:

Table 1. Demand and Supply

	W1	W2	W3	Supply
F1	16	20	12	200
F2	14	8	18	160
F3	26	24	16	90
Demand	180	120	150	450

We will find out the optimal distribution for this company to minimize total shipping costusing various types of transportation that we have discussed above.

3.1.1 By North-West Corner method
From Table 1, total number of supply constraints: 3 & total number of demand constraints: 3. The rim values for F1 = 200 and W1 = 180 are compared. From Table 2, the smaller of the two i.e. min(200, 180) = **180** is assigned to F1W1.

This meets the complete demand of W1 and leaves 200 - 180 = 20 units with F1. From Table 3, the rim values for F1 = 20 and W2 = 120 are compared. The smaller of the two i.e. min (20, 120) = **20** is assigned to F1W2. This exhausts the capacity of F1 and leaves 120 - 20 = 100 units with W2. From Table 4, the rim values for F2 = 160 and W2 = 100 are compared. The smaller of the two i.e. min (160, 100) = **100** is assigned to F2W2. This meets the complete demand of W2 and leaves 160 - 100 = 60 units with F2 in Table 4.

Table 2. Step 1

	W1	W2	W3	Supply
F1	16 **(180)**	20	12	20
F2	14	8	18	160
F3	26	24	16	90
Demand	0	120	150	

Table 3. Step 2

	W1	W2	W3	Supply
F1	16 **(180)**	20 **(20)**	12	0
F2	14	8	18	160
F3	26	24	16	90
Demand	0	100	150	

Table 4. Step 3

	W1	W2	W3	Supply
F1	16 **(180)**	20 **(20)**	12	0
F2	14	8 **(100)**	18	60
F3	26	24	16	90
Demand	0	0	150	

Table 5. Step 4

	W1	W2	W3	Supply
F1	16 **(180)**	20 **(20)**	12	0
F2	14	8 **(100)**	18 **(60)**	0
F3	26	24	16	90
Demand	0	0	90	

From Table 5, the rim values for F2 = 60 and W3 = 150 are compared. The smaller of the two i.e. min (60, 150) = **60** is assigned to F2W3. This exhausts the capacity of F2 and leaves 150 - 60 = 90 units with W3. The rim values for F3 = 90 and W3 = 90 are compared. The smaller of the two i.e. min (90, 90) = **90** is assigned to F3W3 in Table-6. Table-7 gives Initial feasible solution.

Table 6. Step 5

	W1	W2	W3	Supply
F1	16 **(180)**	20 **(20)**	12	0
F2	14	8 **(100)**	18 **(60)**	0
F3	26	24	16 **(90)**	0
Demand	0	0	0	

Table 7. Step 6

	W1	W2	W3	Supply
F1	16 **(180)**	20 **(20)**	12	200
F2	14	8 **(100)**	18 **(60)**	160
F3	26	24	16 **(90)**	90
Demand	180	120	150	

From Table 7, the minimum total transportation cost = 16 x 180 + 20 x 20 + 8 x 100 + 18 x 60 + 16 x 90 = 6600. Here, the number of allocated cells = 5 is equal to m + n - 1 = 3 + 3 - 1 = 5Therefore, this solution is non-degenerate.

3.1.2 By Matrix Minimum method
From Table 1, Total number of supply constraints: 3 Total number of demand constraints: 3

Table 8. Step 7

	W1	W2	W3	Supply
F1	16	20	12	200
F2	14	8 **(120)**	18	40
F3	26	24	16	90
Demand	180	0	150	

Table 9. Step 8

	W1	W2	W3	Supply
F1	16	20	12 **(150)**	50
F2	14	8 **(120)**	18	40
F3	26	24	16	90
Demand	180	0	0	

Table 10. Step 9

	W1	W2	W3	Supply
F1	16	20	12 **(150)**	50
F2	14 **(40)**	8 **(120)**	18	0
F3	26	24	16	90
Demand	140	0	0	

Table 11. Step 10

	W1	W2	W3	Supply
F1	16 **(50)**	20	12 **(150)**	0
F2	14 **(40)**	8 **(120)**	18	0
F3	26	24	16	90
Demand	90	0	0	

The smallest transportation cost is 8 in cell F2W2. From Table 8,the allocation to this cell is min(160, 120) = **120**. This satisfies the entire demand of W2 and leaves 160 - 120=40 units with F2. From Table 9, the smallest transportation cost is 12 in cell F1W3The allocation to this cell is min (200, 150) = **150**. This satisfies the entire demand of W3 and leaves 200 - 150=50 units with F1. From table-10, the smallest transportation cost is 14 in cell F2W1.The allocation to this cell is min (40, 180) = **40**. This exhausts the capacity of F2 and leaves 180 - 40 = 140 units with W1. From table-12, the smallest transportation cost is 16 in cell F1W1.The allocation to this cell is min (50, 140) = **50**. This exhausts the capacity of F1 and leaves 140 - 50=90 units with W1. From table-11, the smallest transportation cost is 26 in cell F3W1the allocation to this cell is min (90, 90) = **90**. Table-7 gives Initial feasible solution.

Table-12: Step 11

	W1	W2	W3	Supply
F1	16(50)	20	12(150)	0
F2	14(40)	8(120)	18	0
F3	26(90)	24	16	0
Demand	0	0	0	

Table-13: Step 12

	W1	W2	W3	Supply
F1	16(50)	20	12(150)	200
F2	14(40)	8(120)	18	160
F3	26(90)	24	16	90
Demand	180	120	150	

The minimum total transportation cost $= 16 \times 50 + 12 \times 150 + 14 \times 40 + 8 \times 120 + 26 \times 90 = 6460$. Here, the number of allocated cells = 5 is equal to $m + n - 1 = 3 + 3 - 1 = 5$ Therefore, this solution is non-degenerate.

3.1.3 By Vogel's Approximation method
From table-1, total number of supply constraints: 3 total number of demand constraints:

Table 14. Step 13

	W1	W2	W3	Supply	Row penalty
F1	16	20	12	200	4=16-12
F2	14	8	18	160	6=14-8
F3	26	24	16	90	8=24-16
Demand	180	120	150		
Colum penalty	2=16-14	12=20-8	4=16-12		

Table 15. Step 14

	W1	W2	W3	Supply	Row penalty
F1	16	20	12	200	4=16-12
F2	14	8 (120)	18	40	4=18-14
F3	26	24	16	90	10=26-16
Demand	180	0	150		
Column penalty	2=16-14	--	4=16-12		

Table 16:.Step 15

	W1	W2	W3	Supply	Row penalty
F1	16	20	12	200	4=16-12
F2	14	8 (120)	18	40	4=18-14
F3	26	24	16 (90)	0	--
Demand	180	0	60		
Column penalty	2=16-14	--	6=18-12		

From Table 14, the maximum penalty, 12, occurs in column $W2$.The minimum c_{ij} in this column is $c_{22}=8$. The maximum allocation in this cell is min (160, 120) = **120**. It satisfies demand of $W2$ and adjusts the supply of $F2$ from 160 to 40 (160 - 120=40). From Table 15, the maximum penalty, 10, occurs in row F3.The minimum c_{ij} in this row is $c_{33}=16$. The maximum allocation in this cell is min (90, 150) = **90**. It satisfies supply of F3 and adjusts the demand of W3 from 150 to 60 (150 – 90) =60. From Table 16, the maximum penalty, 6, occurs in column W3.The minimum c_{ij} in this column is $c_{13}=12$. The maximum allocation in this cell is min (200, 60) = **60**. It satisfies demand of W3 and adjusts the supply of $F1$ from 200 to 140 (200 – 60) = 140.

Table 17. Step 16

	W1	W2	W3	Supply	Row penalty
F1	16	20	12 (60)	140	16
F2	14	8 (120)	18	40	14
F3	26	24	16 (90)	0	--
Demand	180	0	0		
Column penalty	2=16-14	--	--		

Table 18. Step 17

	W1	W2	W3	Supply	Row penalty
F1	16 (140)	20	12 (60)	0	--
F2	14	8 (120)	18	40	14
F3	26	24	16 (90)	0	--
Demand	40	0	0		
Column penalty	14	--	--		

Table 19. Step 18

	W1	W2	W3	Supply	Row penalty
F1	16 (140)	20	12 (60)	200	4 |4 |4 |16 |-- |
F2	14 (40)	8 (120)	18	160	6 |4 |4 |14 |14 |
F3	26	24	16 (90)	90	8 |10 |-- |-- |- - |
Demand	180	120	150		

From Table 17, the maximum penalty, 16, occurs in row F1.The minimum c_{ij} in this row is $c_{11}=16$. The maximum allocation in this cell is min (140, 180) = **140**. It satisfies supply of F1 and adjusts the demand of W1 from 180 to 40 (180 – 140) = 40. From Table 18, the maximum penalty, 14, occurs in row F2.The minimum c_{ij} in this row is $c_{21}=14$. The maximum allocation in this cell is min (40, 40) = **40**.It satisfies supply of F2 and demand of W1. From Table 19, the minimum total transportation cost $= 16 \times 140 + 12 \times 60 + 14 \times 40 + 8 \times 120 + 16 \times 90 = 5920$. Here, the number of allocated cells = 5 is equal to $m + n - 1 = 3 + 3 - 1 = 5$.Therefore, this solution is non-degenerate.

3.1.4 By Column Minima method
Total number of supply constraints: 3 total number of demand constraints: 3

Table 20. Step 19   Table 21. Step 20   Table 22. Step 21

	W1	W2	W3	Supply
F1	16	20	12	200
F2	14 **(160)**	8	18	0
F3	26	24	16	90
Demand	20	120	150	

	W1	W2	W3	Supply
F1	16 **(20)**	20	12	180
F2	14 **(160)**	8	18	0
F3	26	24	16	90
Demand	0	120	150	

	W1	W2	W3	Supply
F1	16 **(20)**	20 **(120)**	12	60
F2	14 **(160)**	8	18	0
F3	26	24	16	90
Demand	0	0	150	

From Table. 20, in the 1st column, the smallest transportation cost is 14 in cell F2W1.The allocation to this cell is min (160, 180) = 160. This exhausts the capacity of F2 and leaves 180 - 160 = 20 units with W1.From Table 21, in 1st column, the smallest transportation cost is 16 in cell F1W1The allocation to this cell is min (200, 20) = **20**.This satisfies the entire demand of W1 and leaves 200 - 20 = 180 units with F1.From Table 22, in the 2nd column, the smallest transportation cost is 20 in cell F1W2.The allocation to this cell is min (180, 120) = 120. This satisfies the entire demand of W2 and leaves 180 - 120 = 60 units with F1.

Table 23. Step 22	Table 24. Step 23	Table. 25. Step 24

	W1	W2	W3	Supply
F1	16 **(20)**	20 **(120)**	12 **(60)**	0
F2	14 **(160)**	8	18	0
F3	26	24	16	90
Demand	0	0	90	

	W1	W2	W3	Supply
F1	16 **(20)**	20 **(120)**	12 **(60)**	0
F2	14 **(160)**	8	18	0
F3	26	24	16 **(90)**	0
Demand	0	0	0	

	W1	W2	W3	Supply
F1	16 **(20)**	20 **(120)**	12 **(60)**	200
F2	14 **(160)**	8	18	160
F3	26	24	16 **(90)**	90
Demand	180	120	150	

From Table 23, in the 3rd column, the smallest transportation cost is 12 in cell F1W3.The allocation to this cell is min (60, 150) = 60. This exhausts the capacity of F1 and leaves 150 - 60 = 90 units with W3. From Table 24, in the 3rd column, the smallest transportation cost is 16 in cell F3W3.form Table 25,the allocation to this cell is min(90, 90) = **90**. The minimum total transportation cost = 16 x 20 + 20 x 120 + 12 x 60 + 14 x 160 + 16 x 90 = 7120. Here, the number of allocated cells = 5 is equal to m + n - 1 = 3 + 3 - 1 = 5.Therefore, this solution is non-degenerate.

3.1.5 By Row Minima method
From table-1, total number of supply constraints: 3and total number of demand constraints: 3

Table 26. Step 25	Table 27. Step 26	Table 28. Step 27

	W1	W2	W3	Supply
F1	16	20	12 **(150)**	50
F2	14	8	18	160
F3	26	24	16	90
Demand	180	120	0	

	W1	W2	W3	Supply
F1	16 **(50)**	20	12 **(150)**	0
F2	14	8	18	160
F3	26	24	16	90
Demand	130	120	0	

	W1	W2	W3	Supply
F1	16 **(50)**	20	12 **(150)**	0
F2	14	8 **(120)**	18	40
F3	26	24	16	90
Demand	130	0	0	

From Table 26, in the 1st row, the smallest transportation cost is 12 in cell F1W3.The allocation to this cell is min (200, 150) = **150**.This satisfies the entire demand of W3 and leaves 200 - 150 = 50 units with F1. From Table 27, in the 1st row, the smallest transportation cost is 16 in cell F1W1.The allocation to this cell is min (50, 180) = **50**.This exhausts the capacity of F1 and leaves 180 - 50=130 units with W1. From Table 28, in the 2nd row, the smallest transportation cost is 8 in cell F2W2.The allocation to this cell is min (160, 120) = **120**.
This satisfies the entire demand of W2 and leaves 160 - 120 = 40 units with F2.

Table 29. Step 28

	W1	W2	W3	Supply
F1	16 **(50)**	20	12 **(150)**	0
F2	14 **(40)**	8 **(120)**	18	0
F3	26	24	16	90
Demand	90	0	0	

Table 30. Step 29

	W1	W2	W3	Supply
F1	16 **(50)**	20	12 **(150)**	0
F2	14 **(40)**	8 **(120)**	18	0
F3	26 **(90)**	24	16	0
Demand	0	0	0	

Table 31. Step 30

	W1	W2	W3	Supply
F1	16 **(50)**	20	12 **(150)**	200
F2	14 **(40)**	8 **(120)**	18	160
F3	26 **(90)**	24	16	90
Demand	180	120	150	

From Table 29, in the 2nd row, the smallest transportation cost is 14 in cell F2W1.The allocation to this cell is min (40, 130) = **40**. This exhausts the capacity of F2 and leaves 130 - 40 = 90 units with W1. From Table 30, in the 3rd row, the smallest transportation cost is 26 in cell F3W1.The allocation to this cell is min (90, 90) = **90**. From Table 31, the minimum total transportation cost =16 x 50 + 12 x 150 + 14 x 40 + 8 x 120 + 26 x 90 = 6460. Here, the number of allocated cells = 5 is equal to m + n - 1 = 3 + 3 - 1 = 5.Therefore, this solution is non-degenerate.

3.1.6 Optimality test using Stepping Stone method

We use table-19 as allocation table. Iteration-1 of optimality test. Creating closed loop for unoccupied cells, we get,

Table 32. Step 31

Unoccupiedcell	Closed path	Net cost change
F1W2	F1W2→F1W1→F2W1→F2W2	20 - 16 + 14 - 8=10
F2W3	F2W3→F2W1→F1W1→F1W3	18 - 14 + 16 - 12=8
F3W1	F3W1→F3W3→F1W3→F1W1	26 - 16 + 12 - 16=6
F3W2	F3W2→F3W3→F1W3→F1W1→F2W1→F2W2	24 - 16 + 12 - 16 + 14-8=10

From Table 32, since all net cost change ≥ 0. So the final optimal solution has arrived.

Table 33. Step 32

	W1	W2	W3	Supply
F1	16 **(140)**	20	12 **(60)**	200
F2	14 **(40)**	8 **(120)**	18	160
F3	26	24	16 **(90)**	90
Demand	180	120	150	

From Table 33, The minimum total transportation cost =16 × 140 + 12 × 60 + 14 × 40 + 8 × 120 + 16 × 90= 5920.

3.1.7 Optimality test using Modi method

Allocation table is Table 17. Iteration-1 of optimality test. Find u_i and v_j for all occupied cells (i, j), where $c_{ij} = u_i + v_j$

1. Substituting, $u1 = 0$, we get
2. $c11 = u1 + v1 \Rightarrow v1 = c11 - u1 \Rightarrow v1 = 16 - 0 \Rightarrow v1 = 16$
3. $c21 = u2 + v1 \Rightarrow u2 = c21 - v1 \Rightarrow u2 = 14 - 16 \Rightarrow u2 = -2$
4. $c22 = u2 + v2 \Rightarrow v2 = c22 - u2 \Rightarrow v2 = 8 + 2 \Rightarrow v2 = 10$
5. $c13 = u1 + v3 \Rightarrow v3 = c13 - u1 \Rightarrow v3 = 12 - 0 \Rightarrow v3 = 12$
6. $c33 = u3 + v3 \Rightarrow u3 = c33 - v3 \Rightarrow u3 = 16 - 12 \Rightarrow u3 = 4$

Table 34. Step 33

	W1	W2	W3	Supply	uij

F1	16 **(140)**	20	12 **(60)**	200	u1=0
F2	14 **(40)**	8 **(120)**	18	160	u2=-2
F3	26	24	16 **(90)**	90	u3=4
Demand	180	120	150		
vij	v1=16	v2=10	v3=12		

From Table 34, Finding d_{ij} for all unoccupied cells (i, j), where

$$d_{ij} = c_{ij} - (u_i + v_j)$$

1. $d12 = c12 - (u1 + v2) = 20 - (0 + 10) = 10$
2. $d23 = c23 - (u2 + v3) = 18 - (-2 + 12) = 8$
3. $d31 = c31 - (u3 + v1) = 26 - (4 + 16) = 6$
4. $d32 = c32 - (u3 + v2) = 24 - (4 + 10) = 10$

Table 35. Step 34

	W1	W2	W3	Supply	uij
F1	16 **(140)**	20 **[10]**	12 **(60)**	200	u1=0
F2	14 **(40)**	8 **(120)**	18 **[8]**	160	u2=-2
F3	26 **[6]**	24 **[10]**	16 **(90)**	90	u3=4
Demand	180	120	150		
vij	v1=16	v2=10	v3=12		

Table 36. Step 35

	W1	W2	W3	Supply
F1	16 **(140)**	20	12 **(60)**	200
F2	14 **(40)**	8 **(120)**	18	160
F3	26	24	16 **(90)**	90
Demand	180	120	150	

From Table 35, since all $d_{ij} \geq 0$. So final optimal solution is has arrived. From Table 36, the minimum total transportation cost = $16 \times 140 + 12 \times 60 + 14 \times 40 + 8 \times 120 + 16 \times 90 = 5920$.

4. Results and Discussion

The aim of any industry is to transport goods from sources to destination with minimumcost. Minimization of transportation cost is one of the main objectives to maximize the profit. In our paper, the initial basic feasible solutions we found using the methods Vogel'sApproximation, North-West Corner, Row Minima, Column Minima, Matrix Minima are5920, 6600, 6460, 7120, and 6460 respectively.
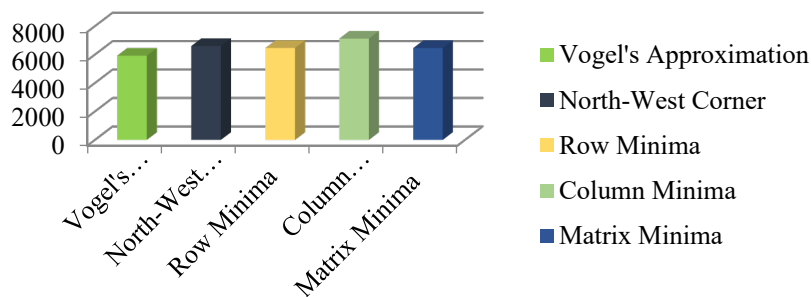

Figure 2. Initial basic feasible solution in different transportation method

Since our objective is to minimize the cost, from our above discussion we have already seen that, VAM (Vogel's Approximation Method) gives comparatively least cost of our given transportation problem. Other methods are inferior to VAM. Therefore, VAM givesbest initial basic feasible solution (which is non-degenerate) of our given transportation problem. After then, we checked the optimality of the result by using Modi method (u-v method)or Stepping Stone method. But one of the disadvantages of Stepping Stone method is that to write a loop for every empty cell which is exhausting and time consuming. That's why, for optimality test we use Modi method rather than the Stepping Stone method. Finally we obtain our desired optimal result. We would also want to testify that,if anyone wants to use the research work for further mathematical optimization and real life application, they can use it independently.

We also have solved the transportation problem in MATLAB using Vogel's Approximation is given the below.

4.1 Vogle's Approximation Method

Program:

```
%%% Vogle's Approximation method
format  short clear
all clc
%%% input data
Cost = [16 20 12; 14 8 18; 26 24 16];
A = [200 160 90]; % supply
B = [180 120 150];   % demand

if sum(A) == sum(B)
       fprintf("Given transportation problem is balanced\n")else
       fprintf("Given transportation problem is unbalanced\n")if sum(A)<sum(B)
              Cost(end+1,:)=zeros(1,size(A,2));
              A(end+1)=sum(B)-sum(A);  elseif
       sum(B)<sum(A)
              Cost(:,end+1)=zeros(1, size(A,2));
              B(end+1)=sum(A)-sum(B);
       end
end
ICost  =  Cost;  %  Save  the  Cost  copy
X=zeros(size(Cost)); [m,n]=size(Cost);
BFS = m+n-1; % total BFS
for i=1:m*n
       Col = sort(Cost, 1); % Ascending orderRow = sort(Cost, 2);
       pRow  =  Row(:,2)-Row(:,1);  %Penalty  Row  pCol  =
       Col(2,:)-Col(1,:); %Penalty ColR = max(pRow); % Find
       max penalty row C = max(pCol); % Find max penalty col
       Rmax  =  find(pRow==max(R,C));  %  max  penalty  value Cmax  =
       find(pCol==max(R,C)); %
       Cr  =  Cost(Rmax,:);  %  Penalty  rows  Cc  =
       Cost(:,Cmax); % penalty colsif max(pRow)  ~=
       max(pCol)
              if max(pRow)>max(pCol)
                     [rowind, colind]=find(min(min(Cr)) == Cost(Rmax,:));row1 = Rmax(rowind);
                     % findin row index
                     col1 = colind; % preserve column
              else
                     [rowind,colind]=find(min(min(Cc))  ==  Cost(:,Cmax));row1  =  rowind;  %
                     preserve row
              end    col1 = Cmax(colind); % find column index
```

```
            x11 = min(A(row1), B(col1));
            [val,ind] = max(x11); % find max allocationii = row1(ind); %
            identity Row position
            jj = col1(ind); % identity Col positionelse
            [rowind1,   colind1]=find(min(min(Cr))   ==   Cost(Rmax,:));   row1   =
            Rmax(rowind1); % finding row index
            col1 = colind1; % Preserve column
            C1 = Cost(row1,col1); % allocated cost
            [rowind2, colind2] = find(min(min(Cc)) == Cost(:, Cmax));row2   = rowind2; %
            preserve row
            col2 = Cmax(colind2); % find column indexC2 = Cost(row2,
            col2); % allocated cost
            if C1 < C2
                  x11 = min(A(row1), B(col1));
                  [val, ind] = max(x11); % find max allocationii = row1(ind); %
                  identify row position
                  jj = col1(ind); % identify col position
            else
                  x11 = min(A(row1), B(col2));
                  [val, ind] = max(x11); % find max allocationii = row2(ind); %
                  identify row position
            end   jj = col2(ind); % identify col position
end

      y11 = min(A(ii), B(jj)); % find the valueX(ii, jj) = y11; %
      assign allocation A(ii) = A(ii) - y11; % reduce row value B(jj)
      = B(jj) -y11; % reduce col value Cost(ii, jj) = Inf; % cell
      covered
   end


   %%% print the Initial BFS fprintf("Initial
   BFS = \n");IB = array2table(X); disp(IB);
   %%% check for degenerate & non-degenerate TotalBFS =
   length(nonzeros(X));
   if TotalBFS == BFS
         fprintf("Initial BFS is Non-Degenerate \n");else
         fprintf("Initial BFS is Degenerate \n");
   end

   %%  compute  the  initial  transportation  cost  InitialCost  =
   sum(sum(ICost.*X)); fprintf("Initial BFS Cost = %d\n", InitialCost);
Output:
   Given transportation problem is balancedInitial
   BFS =
      X1   X2   X3
      _    _    _

      140   0   60
      40   120   0
       0    0   90
```

Initial   BFS   is   Non-DegenerateInitial BFS Cost
= 5920

5. Acknowledgement

6. Conclusion

The aim of any industry is to transport goods from sources to destination with minimumcost. Minimization of transportation cost is one of the main objectives to maximize the profit. In our paper, the initial basic feasible solutions we found using the methods Vogel'sApproximation, North-West Corner, Row Minima, Column Minima, Matrix Minima are5920, 6600, 6460, 7120, and 6460 respectively. We also have solved the transportation problem in **MATLAB** using Vogel's Approximation, North-West Corner, and Matrix Minima methods, which are given in the methods section. Since our objective is to minimize the cost, from our above discussion we have already seen that, VAM (Vogel's Approximation Method) gives comparatively least cost of our given transportation problem. Other methods are inferior to VAM. Therefore, VAM givesbest initial basic feasible solution (which is non-degenerate) of our given transportation problem. After then, we checked the optimality of the result by using Modi method (u-v method)or Stepping Stone method. But one of the disadvantages of Stepping Stone method is that to write a loop for every empty cell which is exhausting and time consuming. That's why, for optimality test we use Modi method rather than the Stepping Stone method. Finally we obtain our desired optimal result. We would also want to testify that,if anyone wants to use the research work for further mathematical optimization and real life application, they can use it independently.

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Biographies

This is **Md Habibur Rahman**. I graduated in Applied Mathematics from the University of Chittagong. Now I am working Teaching Assistant at Youth Society for Research & Action (YSRA). I am a member of IEEE and Organizing Secretary at IEEE Chittagong University (CU) Student Branch. My research interests are applied mathematics, data science, machine learning and artificial intelligence. I also working Research Assistant with last two years of experience working alongside the executive Machine Learning/AI field. I've attended several conferences and given successful research paper presentations online and in person. I've already had a conference paper published in IEEE. In addition, I am a gold award winner of Univ's 2nd International Competition for Young Researchers 2022, where I presented my research titled "Trends, Perspectives, and Prospects in Machine Learning." I inspired daily by my Supervisor and their two students. Recently I joined as a Research Assistant at Dr. Jamal Nazrul Islam Research Center for Mathematical and Physics, University of Chittagong. In my free time, I like to hike, crochet and play video games with my friends.

I am **Afsana Akter**. I graduated from the University of Chittagong with a BSc in Mathematics. My MS in Applied Mathematics from the University of Chittagong was recently finished. Under the guidance of Dr. H.S.Faruque Alam, professor in the department of mathematics at the University of Chittagong, I completed a project on operation research during my academic career. I have a strong enthusiasm for research. I will work in technology and am responsible for educating other employees on using progressive systems and applications, including accounting software, mass communication procedures, and organizational apps. I am a powerful force in the workplace and use my positive attitude and tireless energy to encourage others to work hard and succeed. I am inspired daily by my teacher and two senior brothers.

My name is **Md Miskat Hossain Siddique**. I'm now a research assistant at Jamal Nazrul Islam research center. I have successfully completed my graduation and post-graduation degree at university of Chittagong, department of Mathematics. I am deeply interested in operational research, data analysis, Machine learning, deep learning and similar field. I have done a successful project title "Steiner tree problem" based on operational research under the supervision of Prof Dr. M.M. Rizvi, the visiting research fellow, STEM UniSA. I have participated in various conferences and made successful poster presentation there both in online and onsite. Already I have a conference paper publication in IEEE. And I am a gold award winner of 2nd International Competition for Young Researchers 2022 organized by UniV where I have presented the research -titled "Trends, Perspectives and Prospects in Machine Learning".