Modeling and Simulation of Four-Wheel Steering System for Shorter Wheelbase Vehicle

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Abstract

In-vehicle handling one of the most important systems is the steering system. Typically, all vehicles have a steering system that allows the vehicle to be turned to either side, that is, left or right. In this paper, an idea of implemented considering a steering system for the rear wheel in addition to the front wheel, resulting in a four-wheel steering system. The turning radius required for four-wheel steering is less than that necessary for front-wheel steering after evaluating the effect of the four-wheel steering system. Some cars use four-wheel steering to improve steering response, increase vehicle stability while driving at high speeds, or reduce turning radius while maneuvering at low speeds. However, the improvement in turning radius is discussed using Ackermann steering geometry (ASG), from which the wheel critical turning angles are calculated for minimum turning radius. The steering system is implemented using a rack and pinion mechanism, and the design of the pinion and rack are obtained from ASG. In the study, the vehicle behavior for 4-wheel steering is done using a physical prototype and MATLAB Simulink model. Also, the steering of the front and rear wheel is performed using a servo motor which is controlled by the microcontroller. The vehicle's turning radius is reduced by 33% due to the four-wheel steering system. Improvements have also been made in high-speed handling while changing lane with the four-wheel steering system, in which excessive lateral displacement has been reduced by 77% and reduce the time required to change the lane. Also, the study of these forces for the four-wheel steering system is discussed for low-speed maneuvering.

Keyword
Steering mechanism, Ackermann steering, front wheel steering, four-wheel steering, MATLAB Simulink and dynamics.

1. Introduction

The steering system is the most important component of any vehicle. It allows the driver to have complete control over the vehicle’s maneuvering. The steering function is to steer the vehicle with the manual input to the steering wheel. The majority of vehicles, including cars, trucks, tractors, and other vehicles, have front-wheel steering. Except for special purpose vehicles, front-wheel steering input has been utilized to control the direction of vehicles since the invention of the automobile. Because of its importance in managing vehicle dynamics, yaw stability management of the vehicle is one of the intriguing challenges to focus on in this scenario. It is generally known that a simple bicycle model which can be used to examine the vehicle's maneuvering and handling stability dynamics. In four-wheel steering systems, a mechatronics system steers the rear wheels. While turning using four-wheel steering system wheels rotate by angle $\Delta \theta_1, \Delta \theta_2$ and $\Delta \theta_3, \Delta \theta_4$ for the front-wheel and rear-wheel respectively. This study shows vehicle steering at low and high speeds, with the low-speed study demonstrate the vehicle’s turning radius can be reduce, which is useful for parking and U-turns.

1.1. Objective

In this study, a four-wheel steering system with a shorter wheelbase is introduced as it was not reported before, particularly to the hatchback category of vehicle. In this paper,
1. To study the steering mechanisms for a four-wheel steering system.
2. Using a four-wheel steering mechanism study the vehicle behavior for a shorter wheelbase.
3. Reduce the turning radius and improve the high-speed maneuvering of the shorter wheelbase vehicle.
4. To study the dynamic behavior of four-wheel steering vehicle.

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2. Literature review

The four-wheel steering system has been the subject of extensive research. The primary purpose of rear-wheel steering is to improve a vehicle’s maneuverability, particularly at lower speeds of up to 50 km/h. This solution has recently been seen mostly in larger automobile manufacturers for longer wheelbases vehicle (Katoh, et. al. 1990). The dynamics of a multi-function four-wheel steering system were studied using four methods. Method of constant yaw rate, technique of equal side slip angle, method of zero side slip, and method of negative side slip or managing side slip angle were the four methods. A four-wheel steering system with multiple functions could improve a vehicle's directional stability, sharp turning performance, and parking performance (I. Nyoman et. al. 2000). Also, it is reported that a four-wheel steering system can shorten the time it takes for lateral acceleration to respond to steering movement, perhaps improving the driver's control; and a vehicle with the four-wheel steering system may be better able to avoid collisions (Shoichi et. al. 1985, Takiguchi et. al. 1986).

The demand for (Electric Vehicles) EVs has increased in recent years. The environmentally friendly EV has become a primary focus of study in the automobile industry due to the development of modern technologies for safety and high mobility (Chen and Li 2014). EVs are regarded to be more manageable than internal combustion engine vehicles (ICEV) due to the introduction of new technology and control inputs (Hori 2004). The introduction of the independently actuated (IA) drive in electric vehicles (EVs) opened up a slew of new research opportunities for improving maneuverability and active safety (Zhang et.al. 2015)-(Wang et. al. 2016). In-wheel motors can provide faster and more accurate torque distribution as well as more effective actuation for vehicle motion control (Wang et. al. 2011, Xiong et.al. 2009) (Zhang et. al. 2014a). Numerous studies have shown that EV mobility, stability, and safety may all be significantly improved by fully utilizing the increased control capabilities. The four-wheel independent steering (4WIS) and four-wheel independent drive (4WID) EV, which uses four steering motors and four drive motors to regulate the steering angle and drive torque of each wheel individually, is the most ideal for an AGV (Zhang et. al. 2014b). When steering a high-speed tracked vehicle, it is critical to understand the interrelationship between terrain factors and vehicle characteristics. The handling behavior during non-stationary motion is studied when operating at high and low speeds using a five-degree-of-freedom (DOF) steering model of a tracked vehicle (Janarthanan et. al. 2011). A transfer case and torque vectoring system are used for the improvement of vehicle handling, where the transfer case defines to control the torque distribution between the front and rear axle. A sliding mode control method was used to apply the controller for lateral dynamics (Kim et.al. 2014).

3. Handling System

A steering system is designed using various mechanisms such as Ackermann, Davis, and others, but for this study, the Ackermann steering mechanism is used. It consists of four linkages, and the Ackermann steering geometry is a geometric arrangement of linkages in the steering of a vehicle. Ackermann steering geometry is a geometric arrangement of linkages in an automobile or other vehicle’s steering designed to handle the problem of wheels on the inside and outside of a turn tracing out circles of differing radii. By pushing the steering pivot points inward such that they lie on a line drawn between the steering kingpins and the center of the rear axle, a simple approximation to ideal Ackermann steering geometry can be achieved. A stiff bar called the tie rod connects the steering pivot points, which can also be part of the steering system, such as a rack and pinion. At whatever angle of steering, the center point of all the circles traced by all wheels will lie at a common point with perfect Ackermann mechanism. As shown in Figure 1 and radius of the circle is termed as the turning radius of the vehicle. When this condition satisfy it is termed as Ackermann perfect steering condition and it is given by equation (1) (Figure 1):

\[
\cot \Delta \theta_2 - \cot \Delta \theta_1 = \frac{w}{L}
\]  

(1)
3.1. Four-wheel Ackermann Steering Geometry
Extending the concept of Ackermann steering geometry used in the front wheel steering to four-wheel steering as shown in Figure 2, solving the geometry the equation for the perfect steering condition is,

\[
\frac{(\cot \Delta \theta_4 - \cot \Delta \theta_3) \times (\cot \Delta \theta_2 - \cot \Delta \theta_1)}{\left(\cot \Delta \theta_2 - \cot \Delta \theta_1\right) + \left(\cot \Delta \theta_4 - \cot \Delta \theta_3\right)} = \frac{w}{L} \tag{1}
\]

Eq.1 and Eq.2 is perfect steering condition in theory. Symmetrical linkages are used in this system for turning to the right and left, resulting in similar turning on both sides. In Figure 1 and Figure 2 it is noticed that in front-wheel steering, the common point of intersection is located on the rear wheel axle axis, far from the vehicle body, whereas in four-wheel steering, it is relocated higher and closer to the vehicle body (Figure 2).

Figure 2. Ackermann Steering geometry for four-wheel steering.

4. Steering Mechanism Parameters
4.1. Turning angle of the wheel
The steering mechanism used by Ackermann is a simplified version of the steering mechanism. This is known as a 4R linkage. If the vehicle follows a straight path and the point at CG follows a straight line, the radius of curvature is infinity because the axis of rotation is parallel to the x-axis. As per Eq.2, the axis of rotation of both wheels must now be correlated for perfect steering (Table 1).

<table>
<thead>
<tr>
<th>Model Parameter</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wheelbase of vehicle (L)</td>
<td>250mm</td>
</tr>
<tr>
<td>Wheel track of the vehicle (w)</td>
<td>150mm</td>
</tr>
</tbody>
</table>

Table 1. Vehicle Parameter of Prototype
Three pairs of values can have three accuracy point syntheses to satisfy a 4-R linkage. However, symmetrical linkages should be the same because this allows for an identical turn in both the clockwise and counter-clockwise directions. A two-point synthesis is performed in this case.

\[ \alpha = \cot^{-1} \frac{w}{2l} \]  

For four-wheel steering, a 4-R linkage mechanism is used for front and rear wheel \( l_1 \) is vehicle axle, \( l_2 \) steering drive linkage, \( l_2 \) and \( l_4 \) are symmetrical links termed as a tie rod. The angle between the tie rod and vehicle axle is calculated using Eq.3. From the vehicle parameter as shown in the Table, \( \alpha \) is approximately 70°. Now for evaluating linkage lengths radius of curvature is necessary for which the mechanism is designed and it is calculated using Eq.4 which is obtained as 350mm by setting the limits of curvature to \( x = \pm0.002857 mm^{-1} \)

\[ \therefore R_G = \frac{x}{x} \]  

For the value of radius calculated using Eq.4 values of wheels rotation angle \( \Delta \theta_1 \), \( \Delta \theta_2 \), and \( \Delta \theta_3 \), \( \Delta \theta_4 \) for the front-wheel and rear-wheel respectively are obtained as shown in the Table.(2) from Freudestein’s method, which also helps to calculate the link length ratios.

### 5. Modeling and Simulation

MATLAB Simulink (MATLAB 2021a) is utilized in the methodology to understand the behavior of vehicles based on computations. The yaw motion is calculated using a two-axle vehicle body model. For low-speed and high-speed maneuvers, a comparison of four-wheel steering and front-wheel steering systems is performed. The CAD model of the vehicle prototype was also constructed to understand its practical behavior, as illustrated in Figure 5. Linkages length were calculated in the prototype using the wheel angle calculation described above.

#### Table 2. Change in Angle with Straight Configuration (± indicates the Left or Right side turning)

<table>
<thead>
<tr>
<th>Change in Angle</th>
<th>Values(Deg.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>front-wheel right side ( \Delta \theta_1 )</td>
<td>± 22.6</td>
</tr>
<tr>
<td>front-wheel left side ( \Delta \theta_2 )</td>
<td>± 16</td>
</tr>
<tr>
<td>rear-wheel right side ( \Delta \theta_3 )</td>
<td>± 19.5</td>
</tr>
<tr>
<td>rear-wheel left side ( \Delta \theta_4 )</td>
<td>± 13.6</td>
</tr>
</tbody>
</table>

#### 5.1 MATLAB Simulink Model

##### 5.1.1. Low-speed turning performance

The steering geometry of Ackermann is as shown in Figure 2, with the front and rear wheels rotating in opposite directions, referred to as an out-phase configuration. The axes of rotation of the front and rear wheels cross at point \( O \) in the out-phase configuration, resulting in a decreased turning radius. The out-of-phase configuration vehicle can only run at low speeds, which required a range of 0-35 km/hr. In this model, as the vehicle’s speed grows, the lateral forces on the wheels increase, reducing the perpendicular forces on the inner wheels. As the vehicle approaches critical speed, the likelihood of the wheels rolling increases. After several iterations, a speed range is established. As shown in Figure 3 the vehicle turning performance at low speed for Four-wheel steering and front-wheel steering MATLAB Simulink models are compared. In which 3-DoF Vehicle Model block is used to set the vehicle parameters mentioned...
in the Table. (1). Constant Speed input and Steering input to the model are Constant block and Ackermann Steering block in which range of steering angle are provided as shown in Table (2). Similar blocks re used in the High-speed turning performance (Table 3 and Figure 3).

Figure 3. Vehicle turning performance at low-speed for Four-wheel steering and front-wheel steering MATLAB Simulink models are compared.

<table>
<thead>
<tr>
<th>Model Parameter</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>(m)</td>
<td>2000 kg</td>
</tr>
<tr>
<td>(L)</td>
<td>2.5 m</td>
</tr>
<tr>
<td>(w)</td>
<td>1.5 m</td>
</tr>
<tr>
<td>(h)</td>
<td>0.165m</td>
</tr>
<tr>
<td>(c_1)</td>
<td>1.15 m</td>
</tr>
<tr>
<td>(c_2)</td>
<td>1.35 m</td>
</tr>
<tr>
<td>(E)</td>
<td>0.2 m</td>
</tr>
<tr>
<td>(I_w)</td>
<td>0.587 kg.m(^3)</td>
</tr>
<tr>
<td>(I_e)</td>
<td>2.6 kg.m(^2)</td>
</tr>
</tbody>
</table>

### 5.1.2. High-speed turning performance

The common point \(O\) of the axis of rotation is changed by Ackermann’s steering geometry. An in-phase configuration occurs when both the front and rear wheels revolve in the same direction at high speeds. A common point approaches infinity due to in-phase arrangement. This arrangement aids lane change since the lateral shift is faster and more dynamically stable, that is normal forces on the wheels remain constant and lateral forces are minor. High Speed is considered from 54km/hr and above, and as shown in Figure 4 the vehicle turning performance at high speed for Four-wheel steering and front-wheel steering MATLAB Simulink models are compared (Figure 4).
6. Dynamic effect of four-wheel steering

In the case of a four-wheeled vehicle, none of the wheels must rise off the ground during a turn. The requirement is met as long as the ground’s physical reaction to one of the wheels is favorable (or upwards). A four-wheeled vehicle having a mass \( m \) assuming that the weight is equally divided among the four wheels. The reaction of the ground on each wheel,

\[
R_w = \frac{mg}{4} \text{ (upwards)} \quad (5)
\]

On a rotating body, a reactive gyroscopic torque or couple also has effects that are comparable to centripetal and centrifugal forces. The gyroscopic effect refers to the result of the reactive gyroscopic pair. Thus, when a car turns, or when the axes of spin are subjected to some angular motion, this effect is felt by cars with spinning parts like wheels or engine rotors. The reaction gyroscopic couple on turning is provided by equal and opposite forces on the outer and inner wheels of the vehicle. Thus the force on each wheel outer or inner is similar to weight in the upward or downward direction respectively. The effect of a centrifugal couple acts on the vehicle as it moves on the curved path in an outward direction at the center of mass of the vehicle. This force would tend to overturn the vehicle outward and the overturning couple is,

\[
C_c = \frac{m\nu^2}{R} h \quad (6)
\]

Now, the vertical reaction on each wheel,

\[
= \frac{W}{4} + \frac{C_g}{2w} + \frac{C_c}{2w} \quad (7)
\]

Similarly for the lateral forces on the front and rear wheels,

\[
F_{yf} = \frac{mv^2\zeta_1}{R_L} \quad \text{(Front wheels)} \quad (8)
\]

\[
F_{yr} = \frac{mv^2\zeta_2}{R_L} \quad \text{(Rear wheels)} \quad (9)
\]

7. CAD Model of the Prototype

The CAD model of the prototype is a scaled-down design of the actual vehicle with a ratio of 1:10. Freudenstein’s method is used to design the prototype vehicle steering mechanism. The vehicle parameters are given in the Table 1, these are used to calculate the steering torque. In the prototype, as shown in Figure 5 the servo motors are used to drive the rack and pinion mechanism, this rack, and pinion mechanism are for both front and rear-wheel steering. The calculation of the torque is necessary to select the proper specification servo motor so that it could drive the rack and pinion. Using given parameters from the Table 1 steering force is calculated.

\[
\text{Force on wheel} = \mu_f \times g \times \text{Corner Mass} \quad (10)
\]

Now, Input torque from the ground is due to the frictional force and perpendicular distance from the contact patch to the kingpin axis which is theoretically the scrub radius which is the distance in the front view between the kingpin axis and the center of the contact patch of the wheel. For the Prototype, the torque due to frictional force on the front wheel is obtained as 317.75Nmm. This torque is equal to the lateral push from the tie rod. Now, the torque due to lateral push from tie rod is a force on tie road, and the perpendicular distance between the outer tie rod point ends to kingpin axis. From this total force required on a rack to displace is 11.15N and torque on pinion of radius 18mm is 200Nmm (Figure 5).
8. Result and Discussion
The methodologies used to study the vehicle maneuvering behavior were both simulation and physical prototyping. The optimal parameters are used in simulation to investigate the vehicle turning performance at low and high speeds. A physical prototype was also designed and developed to test the performance of the four-wheel steering system and the front-wheel steering system. The physical prototype focuses primarily on low-speed turning performance in order to examine the vehicle's turning radius.

8.1. Simulation result for turning
Simulations of the four-wheel steering system was carried out in MATLAB on a virtual car body block using the parameters shown in the Table. (1). The turning angle computations were fed into the simulation’s steering block.

- In the case of low-speed turning performance, the steering is turned to its extreme angles as shown in Table. (2) to observe the behavior.
- The position of the vehicle in XY-plan tracing the circular path as shown in Figure 6 and Figure 7, where Figure 6 is the result of the four-wheel steering system and Figure 7 is the result of the front-wheel steering system.
- In comparison to front-wheel steering, the four-wheel steering system traces circles with a smaller radius.
- The steering input for high-speed maneuvering differs from that for low-speed maneuvering because the wheels rotate in-phase at high speeds.
- Lane change frequently necessitates high-speed maneuvering. The yaw position of the vehicle is indicated in Figure 8, with the lateral displacement of the vehicle being less in the case of four-wheel steering than in the case of front-wheel steering (Figure 6).
• Figure 7 and 8, illustrates that a vehicle with four-wheel steering requires less space to change lanes, whereas a vehicle with front-wheel steering requires more space to change lanes
• In Figure 9, the yaw rate of the input signal, four-wheel steering, and front-wheel steering are displayed for high speed.

Figure 7. Vehicle turning performance at low-speed for front-wheel steering system

Figure 8. Comparison of yaw positions of vehicle for Steering at High-Speed turning performance (Magnitude in Deg.)

Vehicle behavior for both four-wheel and front-wheel is presented for the same input signal, where the input signal is the rate of change in steering wheel angle. These findings reveal that four-wheel steering at high speeds allows for faster lateral shifting than front-wheel steering. In a four-wheel steering system, the turning radius is shortened due to higher angular change.

Figure 9. Comparison of yaw rates of vehicle for Steering at High-Speed turning performance. (Magnitude in Deg./s)
8.2. Simulation result for dynamics
In the study of the four-wheel steering system, simulations have also been performed in MATLAB on a virtual vehicle body with parameters mentioned in the Table. (3). In the simulation, a virtual car body block was used. The input to this block is the kinematic steering block which is specifically for the steering mechanism. Where Ackermann steering mechanism is used to steer the vehicle. In the simulation two different types of steering angle inputs were provided, that is constant and sinusoidal. In both input cases, the vehicle dynamic behavior was different. Also in the case of different maneuvering modes, the normal and lateral forces changed.

![Figure 10. Input velocity profile](image)

Low-speed maneuvering is performed in an out-phase configuration. This configuration provides a minimum turning radius which turns the vehicle into a smaller space. The rear wheels can not rotate as far as front wheels but in this configuration, the front and rear wheels steers in proportion with each other. The normal force on each wheel while taking a curve path is as shown in Figure (11). This Figure 12 illustrates that upward normal reaction on front left and rear left increases whereas for the front right and rear right reduces with the increase in velocity.

![Figure 11. Normal forces for constant steering angle at Low-speed out-phase steering.](image)

In the case of sinusoidal steering input, the normal and lateral forces are minimum for speed calculated from the control parameter and mathematical method. In this case also, normal and lateral forces increase with increased velocity, which affect the vehicle stability. In Figure (13) and Figure (14) the normal forces and lateral forces are increasing with increase in velocity respectively.

![Figure 12. Lateral forces for constant steering angle at Low-speed out-phase steering](image)
8.3 Physical Prototype result
In the simulation of the vehicle prototype the result of the low speed and high speed are compared with actual physical prototype to study the vehicle turning performance. The CAD model prototype actual vehicle dimensions are scaled down to the physical prototype. It is expected that the behavior of the actual and scale down model should be the same, as the ratio for perfect steering conditions remains the same. As shown in figure (6) the simulation result shows the four-wheel steering system vehicle traces the curve of a radius of 350 mm at low speed but in the scaled model it is 400 mm. In Table (4) comparative analysis of the simulation and physical prototype is given.
Figure 15. Physical prototype test runs for low-speed turning performance

For the measurement of turning radius marker pen was attached to rear side of the vehicle as shown in Figure 15. As vehicle drives the traces of the marker appeared on the floor. Circle trace by vehicle at low speed is of radius 400mm. The difference in the radius of the simulation result and the physical prototype is due to design and manufacturing constraints.

9. Conclusion

It has been observed that system design for four-wheel steering employing Ackermann steering geometry mathematically satisfies the ideal steering condition. Also, the following points are concluded:

• A four-wheel steering vehicle, a smaller turning radius minimizes the time it takes to turn the vehicle.
• It has been noted that the circle trace by the four-wheel steering is of radius 350 mm and the ellipse trace by front-wheel steering is of radius 525mm.
• Steering on all four wheels at high speed, the vehicle’s lateral movement is faster and more stable than when steering on the front wheels.

A reduction in turning is noted as a function of the yaw position and yaw rate. This concludes that the four-wheel steering system helps to reduce the turning radius of the vehicle by approximately 33%. There are also improvements in high-speed maneuvering; while these results are likewise observed for the prototype; the physical prototype the turning radius of the vehicle is reduced by approximately 35%; given that the prototype is a scaled-down version of the actual car, it can be conclude that the calculations for the prototype are also applicable for the actual vehicle. Also, it has been observed that the stability of vehicle factors is different for low-speed and high-speed. Also, the normal force and lateral forces were affected while vehicle steering.

• The normal and lateral forces increase with increased velocity.
• The low-speed maneuvering normal and lateral forces affect the vehicle stability.
• The suitable turning for the four-wheel steering vehicle is 18km/hr.

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