Designing and Computing the Generalized Process Capability Indices under Neutrosophic Set

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Abstract

Process Capability indices (PCIs) such as C_p and C_{pk} are commonly used measures to evaluate a manufacturing processes performance. The validity of these measures is based on that quality characteristics data under evaluation is normality distributed and the manufacturing process in under statistical control. The C_p measure considers the process data is centering the specification band. However, it lacks to evaluate a process off-centering the given specification band. The C_{pk} measure which is 2nd generation considers the off-centering issue of any process being evaluated for its performance. Addressing the indeterminate and incomplete information about the different specification limits (SLs), the process capability indices C_p and C_{pk} is designed under the neutrosophic fuzzy environment. The neutrosophic fuzzy SLs contains three different aspects of membership degrees such as truth, indeterminacy, and falsity degrees which is more flexible to the fuzzy based approaches. Furthermore, the proposed neutrosophic PCIs C_p and C_{pk} is implemented on the rubber parts manufacturing company. The results show the more reliable values of PCIs while dealing with the indeterminacy degree present in the SLs. The wide range of getting different SLs is also benchmark findings of the proposed neutrosophic PCIs study. The conclusions and future work scope is also furnished in an efficient manner.

Keywords
Neutrosophic fuzzy set, Indeterminacy membership function, Neutrosophic numbers, Process Monitoring, Neutrosophic Process Capability Indices.

1. Introduction
Process capability indices (PCIs) played vital role to improve the operational efficiency of manufacturing products and processes thus resulting in significant reduction of process losses that occur due to non-compliance to customer specifications. Although process capability indices such as C_p and C_{pk} are being extensively applied in industry to assess process performances but there is a lack of understanding among quality practitioners that these capability measures are essentially based on statistical theory of normality. If the basic assumptions of statistical theory are violated the capability assessments can mislead to wrong conclusions.

Process capability analysis together with statistical process control and design of experiments are statistical methods that have been used for decades with the main purpose being to reduce the variability in industrial processes and products. The need to understand and control processes is getting more and more relevant due to the increasing complexity in technical systems in industry. Moreover, the use of statistical methods in industry is also increasing by the introduction of quality management concepts such as the Six Sigma programme, where statistical methods, including process capability indices, are important parts.

Process capability analysis deals with how to assess the capability of a manufacturing process, where information about the process is used to improve the capability. With process capability analysis one can determine how well the process will perform relative to product requirements or specifications. However, before assessing the capability of a process it is important that the process is stable and repeatable. That is, only natural (common) causes of variation should be present. It should be noted that a process capability analysis could be performed even if the process is unstable. However, such an analysis will give an indication of the capability at that very moment only and hence the results are of limited use.

When the process is found stable, different techniques can be used within the concept of process capability analysis in order to analyze the capability For instance, a histogram along with sample statistics such as average and standard deviation gives some information about the process performance and the shape of the histogram gives an indication about the distribution of the studied quality characteristic. Another simple technique is to determine the shape, center and spread of the distribution by using a normal probability plot.

1.1 Objective of the study

The processes capability analysis utilizes the specification limits between which the process are considered as under the control. Some certain statistical technique based measures are introduced for calculating the index called as process capability index. There are many approaches for analyzing the process capability but few are well-known such as C_p and C_{pk} . These indices measure the potential capability of the current process and suggest the monitoring strategies based on certain specification limits. It is often seen that the SLs may not be always precise and deterministic in nature, however it can be assigned with some vague values or parameters. Hence, the SLs with vagueness and ambiguousness are dealt with the fuzzy concept. The fuzzy set theory provides the opportunity to deal with the vague information and convert it into the precise information.

Sometimes, the situation may arise that contains the degree of indeterminacy or neutrality while assigning the lower and upper SLs. For those situations, the fuzzy approach will not work because the fuzzy set only considers the membership function of the element into the set. Hence, degree of indeterminacy or neutrality can be handled by using the neutrosophic set (NS). The NS theory contains the three different membership functions such as truth, indeterminacy and falsity degrees of the element into the set. Independency of truth-membership and indeterminacy membership functions from each other and expressing the fact that an individual does not have full control of the issue with the indeterminacy-membership function has an important place in modeling uncertainty problems. The NSs are more suitable to model these uncertainties for engineering and scientific applications. Thus, using the NSs concept is quite advantageous while dealing with uncertainty and can obtain more realistic results about the PCA. Furthermore, the proposed PCIs are implemented on the product manufacturing company.

2. Literature Review

The capability assessment of the processes has been widely discussed in the literature. A large numbers of research studies are available for the different techniques and methodologies for the performance analysis of the process in various scenarios. Specially, the literature is broadly accompanied with the normal and non-normal processes or both. Some more steps or procedures are required for the non-normal data-set while assessing the performance of the prevailing processes before arriving at any conclusions. On the other hand, when the data-set are normally followed then few easier analyses are needed to be performed. Thus, there are ample number of methodologies are available in the literature for dealing with the normal and non-normal data-set. Some of the relevant work can be discussed for better improvement in the research study. Parchami and Mashinchi (2007) investigated a methodology based on Buckley's estimation approach which has taken the shape of triangular fuzzy numbers and developed a family of confidence intervals to estimate process capability indices. Kahramanand Kaya (2009) presented a novel fuzzy approach for PCIs controlling the levels of pH, dissolved oxygen (DO) and temperature (T) in dam's water for irrigation and applied to the normal data-set. Kaya and Kahraman, (2010) also investigated the new PCIs called robust PCIs (RPCIs) for a piston manufacturing company. Ahmadini and Ahmad (2021) also discussed the preference relation which can be used for PCIs. The developed RPCIs used the concept of fuzzy set theory for increasing the PCIs' flexibility and sensitivity by defining specification limits and standard deviation. Parchami et al. (2014) discussed the generalized version of PCIs to measure the capability of a fuzzy-valued process in producing products on the basis of a fuzzy quality. Furthermore, the spontaneous computing formulas for the generalized PCIs are computed for normal and symmetric triangular fuzzy observations, where the fuzzy quality is defined by linear and exponential fuzzy SLs. et al. (2019) suggested the a critical value from statistical hypothesis testing and investigated process capability indices chart, which both lowers the chance of quality level misjudgment caused by sampling error and provides reference for the processes improvement in poor quality levels. Also, the validity and applicability of the PCIs are demonstrated by applying to the bottom bracket of bicycles. Hesamian et al. (2021) used the intuitionistic fuzzy concept and the relevant information to develop a process control criterion while assuming the population is normal with intuitionistic fuzzy mean and exact variance.

In the recent years, many more research work is presented by considering the extension of the previous research. Yalcin and Kaya (2022) derived a new extension of intuitionistic fuzzy sets, which is called Penthagorean fuzzy sets and utilized for the process capability analysis in a production processes of an Autonomous Underwater Vehicle. Oztaysi, et al. (2018) investigated the performance measurement for determining the performance indicators and their weights. The proposed performance measurement indicators using interval valued intuitionistic fuzzy AHP are implemented in a law office which deal with follow-up of unpaid bills. Aslam and Albassam (2019) proposed a variable sampling plan for the PCI using neutrosophic statistics. The neutrosophic plan parameters have been determined using the neutrosophic optimization and a comparison between plans based on neutrosophic statistics and classical statistics has been explored. Almarashi and Aslam (2021) also presented a repetitive sampling control chart for the gamma distribution under the neutrosophic environment. The performance of the designed chart has been noted using the average run length measurements. Kumar et al. (2022) discussed the generalized process capability index in Bayesian scenario by considering the quality characteristic follows normal distribution. The PCIs are modified with the these asymptotic confidence interval, generalized confidence interval, three bootstrap confidence intervals and highest posterior density credible intervals. Saha et al. (2021) developed the PCIs based on the proportion of conformance and which is applicable to normally as well as non-normally and continuous as well as discrete distributed processes. For the estimation of PCIs, the five classical methods of estimation have been used when the process follows exponentiated exponential distribution. Aslam et al. (2019) also presented the way of designing of a sampling plan using the process loss consideration for the multiple dependent state sampling under the neutrosophic statistics. Thus, in continuation of the above discussed work, we have presented the PCIs under the flexible neutrosophic environment and made an attempt to explore the further real-life application.

3. Process Capability Indices C_p and C_{pk}

The performances of the process can be monitored using the specific statistical tools. After analyzing system processes, the corresponding assessment are done to further improve the process capability. Among the available process assessment techniques, there are very popular statistical approach which is based on the process location and dispersion. The measure which takes into account the potential performances of the process is the target and SLs. Most of the statistical measure uses the measure of dispersion and central tendency for providing the numerical values to the processes. There are some well-known PCIs (such as C_p, C_{pk}, C_{pm} and C_{pmk}) which are discussed extensively in the literature. The PCIs also takes the specification limits, such as upper specification limit (USL) and lower specification limits (LSL) while measuring the processes capability. Therefore, here we have presented the general structure of the PCIs $C_p(x,y)$ which can represent the C_p and C_{pk} for the particular value of the x and y, respectively.

Hence the superstructure for the PCIs ($C_p(x,y)$) can be given as follows (Eq. 1) (Chan et al. 1988):

$$C_p(x,y) = \frac{a - x\left|\mu - b\right|}{3\sqrt{\sigma^2 + y(\mu-\tau)^2}} \tag{1}$$

Where, μ and σ are the mean and standard deviation of the processes. The parameter $a=(USL-LSL)/2$ represents the half length of the specification interval, $b=(USL+LSL)/2$ is the mid-point of the between the specification limits. The target value is denoted by τ and for every $x, y \geq 0.$

For the different values of the x and y, one can obtain the process capability indices C_p and C_{pk}. Now it is trivial to obtain and verify that $C_p(x=0, y=0)=C_p$ and $C_p(x=1, y=0)=C_{pk}$. The indices C_p and C_{pk} can be furnished as follows Eqs. (2) and (3):

$$C_p = \frac{USL-LSL}{6\sigma} \tag{2}$$

$$C_{pk} = \min\left\{\frac{USL-\mu}{3\sigma}, \frac{\mu-LSL}{3\sigma}\right\} \tag{3}$$

The index C_p solely takes into account the variation in the process "σ" and do not perform the sensitivity analysis on the departure processes any more. The index C_{pk} deals with the process mean but are unable to differentiate between the on-target and the off-target processes of the process capability measures.

3.1. Neutrosophic set concept

Uncertainty is very common characteristics features which are found among the parameters' values. The existences of precise or well-known parameters are very rare in real-life. For instance, processing time to manufacture the unit product may not be well represented with the single values of time because there may be delay in processing time due to unforeseen breakdown of machines, some fault during the machine operations, un-attention of the user etc. In such cases, instead of assigning the processing time with single values it would be far better to assign within some interval or ranges which will ensure the processing time in a more realistic manner. The vagueness and ambiguousness present in the parameters' values are dealt with fuzzy concept. Firstly, Zadeh (1965) coined the term fuzzy set (FS) and used to address the vague and uncertain parameters. The FS contains the membership function of the element into the set and tries to maximize it as much as possible up-to 1 (maximum degree of belongingness). Afterward, Atanassov (1988) proposed the generalized version of FS and named it as intuitionistic fuzzy set (IFS). The IFS contains the membership and non-membership functions of the element into the set and tries to maximize the membership function as much as possible up-to 1 (maximum degree of belongingness) and minimize the non-membership function as much as possible up-to 0 (minimum degree of non-belongingness).

Most often, the membership and non-membership functions may not express the reality in a better way and such situation cannot be tackle with the FS and IFS. For example, if we take the opinion about the victory of team X in a cricket match, and supposing they have the possible chance of winning equaling 0.8, the chance team X has of losing would be 0.4 and the chance that the match would be a tie would be 0.5. All the possibilities are independent of each other and can take any value between 0 and 1.

First, Smarandache (1999) investigated the neutrosophic set (NS) which comprises three membership functions, namely, truth (degree of belongingness), indeterminacy (degree of belongingness up to some extent), and falsity (degree of non-belongingness) functions of the element into the neutrosophic set. The word *"neutrosophic"* is the hybrid mixture of two different words, namely, *"neutre"* taken from French, means "neutral", and *"sophia"*, derived from Greek, means "skill/wisdom", which literally gives the meaning *"knowledge of neutral thoughts"* (see Smarandache (1999)). Thus, the neutrosophic fuzzy decision set provides an ample opportunity to express the neutral thoughts of the decision maker(s) and consequently captures the behavior of indeterminacy in the decision-making problem.

The basic concept and related terminology of neutrosophic set (NS) theory can be furnished as follows:

Definition 1 Smarandache (1999): (Neutrosophic set) Let us consider a set $\widetilde{N}$ defined over the universal discourse X such that $x \in X$, then the set $\widetilde{N}$ is called as NS if

$$\widetilde{N} = \left\{x; \left[\, t_{\widetilde{N}}(x),\, i_{\widetilde{N}}(x),\, f_{\widetilde{N}}(x) \,\right] \| x \in X \right\},$$

where $t_{\tilde{N}}(x) : X \rightarrow [0,1]$ is called as the truth membership function and depict the degree of belongingness, $i_{\tilde{N}}(x) : X \rightarrow [0,1]$ is called as the indeterminacy/neutral membership function and depict the degree of unsurely, and $f_{\tilde{N}}(x) : X \rightarrow [0,1]$ is called as the falsity membership function and depict the degree of non- belongingness based on the decision makers perceptions and thought. The quantity $t_{\tilde{N}}(x),\ i_{\tilde{N}}(x),\ f_{\tilde{N}}(x)$ also follows the following condition:

$$^{-}0 \leq t_{\tilde{N}}(x) + i_{\tilde{N}}(x) + f_{\tilde{N}}(x) \leq 3^{+}$$

Definition 2 Smarandache (1999) (Single valued neutrosophic set) A single valued neutrosophic set (SVNS) $\tilde{N}_S$ based on the $\tilde{N}$ with the single valued independent variable x , can be defined as follows:

$$\tilde{N}_S = \left\{ x; \left\lfloor\, t_{\tilde{N}_S}(x),\ i_{\tilde{N}_S}(x),\ f_{\tilde{N}_S}(x) \right\rfloor\! \mid x \in X \right\},$$

where $t_{\tilde{N}_S}(x) : X \rightarrow [0,1]$, $i_{\tilde{N}_S}(x) : X \rightarrow [0,1]$, and $f_{\tilde{N}_S}(x) : X \rightarrow [0,1]$ represent the truth, indeterminacy and the falsity degrees under the SVNS, respectively with the following restrictions:

$$0 \leq t_{\tilde{N}_S}(x) + i_{\tilde{N}_S}(x) + f_{\tilde{N}_S}(x) \leq 1$$

Definition 3: Smarandache (1999) (Single valued trapezoidal neutrosophic number) A single valued trapezoidal neutrosophic number $\tilde{N}_T$ can be defined as $\tilde{N}_T = \left[\, (p^1, q^1, r^1\, s^1);\ \alpha \,\right], \left[\, (p^2, q^2, r^2\, s^{21});\ \beta \,\right], \left[\, (p^3, q^3, r^3\, s^3),\ \gamma \,\right]$ such that $\alpha,\ \beta,\ \gamma \in [0,1]$. All the associated membership function such as truth $\left(t_{\tilde{N}_T}\right) : R \rightarrow [0, \alpha]$, indeterminacy $\left(i_{\tilde{N}_T}\right) : R \rightarrow [0, \beta]$ and the falsity $\left(f_{\tilde{N}_T}\right) : R \rightarrow [0, \gamma]$ membership function can be furnished as follows:

$$t_{\tilde{N}_T}(x) = \begin{cases} t_{\tilde{N}_T l}(x), & p^1 \leq x \leq q^1 \\ \alpha, & q^1 \leq x \leq r^1 \\ t_{\tilde{N}_T u}(x) & r^1 \leq x \leq s^1 \\ 0 & Otherwise \end{cases}$$

$$i_{\tilde{N}_T}(x) = \begin{cases} i_{\tilde{N}_T l}(x), & p^2 \leq x \leq q^2 \\ \beta, & q^2 \leq x \leq r^2 \\ i_{\tilde{N}_T u}(x) & r^2 \leq x \leq s^2 \\ 1 & Otherwise \end{cases}$$

$$f_{\tilde{N}_T}(x) = \begin{cases} f_{\tilde{N}_T l}(x), & p^3 \leq x \leq q^3 \\ \gamma, & q^3 \leq x \leq r^3 \\ f_{\tilde{N}_T u}(x) & r^3 \leq x \leq s^3 \\ 1 & Otherwise \end{cases}$$

Using the more extensive concept of neutrosophic trapezoidal number, we will introduce the linear generalized neutrosophic trapezoidal numbers which are more flexible, informative and realistic in nature. The scientific logic and purpose behind the linear generalized neutrosophic trapezoidal numbers is to study and address the question that why the many researchers have taken the maximum values of the truth, indeterminacy and the falsity membership functions as 1. Logically, these values can be based on the decision maker point of view and degree of desirability. One can freeze any of these values between 0 to 1, but should be less than 1. Thus, we have made couple of assumption regarding the maximum values of the truth, indeterminacy and the falsity membership functions and presented in the Definition 4, 5 and 6 based on the different criteria.

Definition 4: Type-I Trapezoidal Single Valued Neutrosophic Number (T-I TSVNN): Assuming that the quantity of truth, indeterminacy and falsity are not dependent
In real-life scenario, for example, this situation can be realized in the process of selection of the research journal for publication. The "selection of the journal X among the databases of the journal" comes under this type of neutrosophic parameters. Some researchers will definitely suggest for the journal X which represent the truth membership function, some of them will be definitely against the journal X which represent the falsity membership function, and many of the researcher will not sure about the journal X and they will have no suggestions which depicts the indeterminacy/neutral thoughts about the journal X. Hence, the entire so obtained component is different and independent to each other's. Thus, Type-I Trapezoidal Single Valued Neutrosophic Number will be a better representative of the different researchers'' opinion regarding the selection of the journal X among the databases of the journal.

The Type-I Trapezoidal Single Valued Neutrosophic Number (T-I TSVNN) $\widetilde{N}_{\text{T-I TSVNN}}$ can be defined as $\widetilde{N}_{\text{T-I TSVNN}} = (p_1, p_2, p_3, p_4;\ q_1, q_2, q_3, q_4;\ r_1, r_2, r_3, r_4)$ with all the associated membership function such as truth $t_{\widetilde{N}_{\text{T-I TSVNN}}}(x)$, indeterminacy $i_{\widetilde{N}_{\text{T-I TSVNN}}}(x)$ and the falsity $f_{\widetilde{N}_{\text{T-I TSVNN}}}(x)$ membership function can be furnished as follows:

$$t_{\widetilde{N}_{\text{T-I TSVNN}}}(x) = \begin{cases} \dfrac{x-p_1}{p_2-p_1}, & if \;\; p_1 \le x \le p_2 \\ 1, & if \;\; p_2 \le x \le p_3 \\ \dfrac{p_4-x}{p_4-p_3} & if \;\; p_3 \le x \le p_4 \\ 0 & Otherwise \end{cases}$$

$$i_{\widetilde{N}_{\text{T-I TSVNN}}}(x) = \begin{cases} \dfrac{q_2-x}{q_2-q_1}, & if \;\; q_1 \le x \le q_2 \\ 0, & if \;\; q_2 \le x \le q_3 \\ \dfrac{x-q_3}{q_4-q_3} & if \;\; q_3 \le x \le q_4 \\ 1 & Otherwise \end{cases}$$

$$f_{\widetilde{N}_{\text{T-I TSVNN}}}(x) = \begin{cases} \dfrac{r_2-x}{r_2-r_1}, & if \;\; r_1 \le x \le r_2 \\ 0, & if \;\; r_2 \le x \le r_3 \\ \dfrac{x-r_3}{r_4-r_3} & if \;\; r_3 \le x \le r_4 \\ 1 & Otherwise \end{cases}$$

Where ${}^{-}0 \le t_{\widetilde{N}_{\text{T-I TSVNN}}}(x) + i_{\widetilde{N}_{\text{T-I TSVNN}}}(x) + f_{\widetilde{N}_{\text{T-I TSVNN}}}(x) \le 3^{+}, \quad x \in \widetilde{N}_{\text{T-I TSVNN}}$.

Remark 1: Generally, the trapezoidal single valued neutrosophic numbers are represented by four component (e.g.; a, b, c and, d) along with the three membership values, say $\alpha,\ \beta,\ \gamma \in [0,1]$. But this proposed approach, we have taken the TWELVE different components such as $(p_1, p_2, p_3, p_4;\ q_1, q_2, q_3, q_4;\ r_1, r_2, r_3, r_4)$. Both the numbers are same but the way of representation is different and generalized one. Here, we have used the asymmetric view of all the three membership functions solely. However, one can obtain the four component trapezoidal single valued neutrosophic numbers by taking the values $\left(p_1 = q_1 = r_1 = a\ ;\ p_2 = q_2 = r_2 = b\ ;\ p_3 = q_3 = r_3 = c\ ;\ p_4 = q_4 = r_4 = d\ \right)$ along with maximum values of the truth, indeterminacy and the falsity membership function as generalized values $\alpha,\ \beta,\ \gamma \in [0,1]$.

Definition 5: Type-II Trapezoidal Single Valued Neutrosophic Number (T-II TSVNN): Assuming that the quantity of truth, indeterminacy and falsity are dependent

This situation can also be realized by submitting a research paper to the journal for possible publication. After the submission, the author can get the three different decision such as acceptance, rejection or the under review. The entire three decision components are dependent to each other's. In this case, Type-II Trapezoidal Single Valued Neutrosophic Number will be a better representative of the status of the paper.

The Type-II Trapezoidal Single Valued Neutrosophic Number (T-II TSVNN) $\widetilde{N}_{\text{T-II TSVNN}}$ can be defined as $\widetilde{N}_{\text{T-II TSVNN}} = \left(p_1, p_2, p_3, p_4;\ w_{\widetilde{N}_{\text{T-II TSVNN}}};\ u_{\widetilde{N}_{\text{T-II TSVNN}}},\ v_{\widetilde{N}_{\text{T-II TSVNN}}}\right)$ with all the associated membership function such as truth $t_{\widetilde{N}_{\text{T-II TSVNN}}}(x)$, indeterminacy $i_{\widetilde{N}_{\text{T-II TSVNN}}}(x)$ and the falsity $f_{\widetilde{N}_{\text{T-II TSVNN}}}(x)$ membership function can be furnished as follows:

$$t_{\widetilde{N}_{\text{T-II TSVNN}}}(x) = \begin{cases} w_{\widetilde{N}_{\text{T-III TSVNN}}} \dfrac{x - p_1}{p_2 - p_1}, & \mathit{if} \quad p_1 \le x \le p_2 \\ w_{\widetilde{N}_{\text{T-III TSVNN}}}, & \mathit{if} \quad p_2 \le x \le p_3 \\ w_{\widetilde{N}_{\text{T-III TSVNN}}} \dfrac{p_4 - x}{p_4 - p_3} & \mathit{if} \quad p_3 \le x \le p_4 \\ 0 & \mathit{Otherwise} \end{cases}$$

$$i_{\widetilde{N}_{\text{T-II TSVNN}}}(x) = \begin{cases} \dfrac{p_2 - x + u_{\widetilde{N}_{\text{T-II TSVNN}}}(x - p_1)}{p_2 - p_1}, & \mathit{if} \quad p_1 \le x \le p_2 \\ u_{\widetilde{N}_{\text{T-III TSVNN}}}, & \mathit{if} \quad p_2 \le x \le p_3 \\ \dfrac{x - p_3 + u_{\widetilde{N}_{\text{T-III TSVNN}}}(p_4 - x)}{p_4 - p_3} & \mathit{if} \quad p_3 \le x \le p_4 \\ 1 & \mathit{Otherwise} \end{cases}$$

$$f_{\widetilde{N}_{\text{T-II TSVNN}}}(x) = \begin{cases} \dfrac{p_2 - x + v_{\widetilde{N}_{\text{T-II TSVNN}}}(x - p_1)}{p_2 - p_1}, & \mathit{if} \quad p_1 \le x \le p_2 \\ v_{\widetilde{N}_{\text{T-II TSVNN}}}, & \mathit{if} \quad p_2 \le x \le p_3 \\ \dfrac{x - p_3 + v_{\widetilde{N}_{\text{T-II TSVNN}}}(p_4 - x)}{p_4 - p_3}, & \mathit{if} \quad p_3 \le x \le p_4 \\ 1 & \mathit{Otherwise} \end{cases}$$

Where $^{-}0 \le t_{\widetilde{N}_{\text{T-II TSVNN}}}(x) + i_{\widetilde{N}_{\text{T-II TSVNN}}}(x) + f_{\widetilde{N}_{\text{T-II TSVNN}}}(x) \le 2^{+},\quad x \in \widetilde{N}_{\text{T-II TSVNN}}$. The function $w_{\widetilde{N}_{\text{T-II TSVNN}}}, u_{\widetilde{N}_{\text{T-II TSVNN}}}, v_{\widetilde{N}_{\text{T-II TSVNN}}}$ satisfies the conditions $0 \le \alpha \le w_{\widetilde{N}_{\text{T-II TSVNN}}},\ u_{\widetilde{N}_{\text{T-II TSVNN}}} \le \beta \le 1,\ v_{\widetilde{N}_{\text{T-II TSVNN}}} \le \gamma \le 1$ and $^{-}0 \le \alpha + \beta + \gamma \le 2^{+}$ respectively.

Definition 6: De-neutrosophication/score function of Trapezoidal Single Valued Neutrosophic Number

De-neutrosophication/score function is the process of obtaining the crisp version of the neutrosophic number in the crisp logic corresponding to the neutrosophic number and its membership functions.

From **Remark 1**, we can write the parametric form of trapezoidal neutrosophic number and can be given as follows:

$$t_{\widetilde{N}_{\text{T-I TSVNN}}}(\alpha) = p_1 + \alpha\left(p_2 - p_1\right)$$
$$t_{\widetilde{N}_{\text{T-II TSVNN}}}(\alpha) = p_4 - \alpha\left(p_4 - p_3\right)$$
$$i_{\widetilde{N}_{\text{T-I TSVNN}}}(\beta) = q_2 - \beta\left(q_2 - q_1\right)$$
$$i_{\widetilde{N}_{\text{T-II TSVNN}}}(\beta) = q_3 + \beta\left(q_4 - q_3\right)$$
$$f_{\widetilde{N}_{\text{T-I TSVNN}}}(\gamma) = r_2 - \gamma\left(r_2 - r_1\right)$$
$$f_{\widetilde{N}_{\text{T-II TSVNN}}}(\gamma) = r_3 + \gamma\left(r_4 - r_3\right)$$

Here, $0 \le \alpha, \beta, \gamma \le 1$ and $0 \le \alpha + \beta + \gamma \le 3$ respectively. Now, the de-neutrosophication can be defined as follows:

$$d\left(\widetilde{N}_T\right) = \frac{\int_{\alpha=0}^{1} \frac{\left(t_{\widetilde{N}_{\text{T-I TSVNN}}} + t_{\widetilde{N}_{\text{T-II TSVNN}}}\right)}{2} d\alpha + \int_{\beta=0}^{1} \frac{\left(i_{\widetilde{N}_{\text{T-I TSVNN}}} + i_{\widetilde{N}_{\text{T-II TSVNN}}}\right)}{2} d\beta + \int_{\gamma=0}^{1} \frac{\left(f_{\widetilde{N}_{\text{T-I TSVNN}}} + f_{\widetilde{N}_{\text{T-II TSVNN}}}\right)}{2} d\gamma}{3}$$

$$= \frac{\int_{\alpha=0}^{1} \frac{\left(p_1 + \alpha(p_2 - p_1) + p_4 - \alpha(p_4 - p_3)\right)}{2} d\alpha + \int_{\beta=0}^{1} \frac{\left(q_2 + \beta(q_2 - q_1) + q_3 + \beta(q_4 - q_3)\right)}{2} d\beta + \int_{\gamma=0}^{1} \frac{\left(r_2 - \gamma(r_2 - r_1) + r_3 + \gamma(r_4 - r_3)\right)}{2} d\gamma}{3}$$

$$= \frac{p_1 + p_2 + p_3 + p_4 + q_1 + q_2 + q_3 + q_4 + r_1 + r_2 + r_3 + r_4}{12}$$

3.2. Neutrosophic Process Capability Indices C_p and C_{pk}

The process capability analysis (PCA) becomes more complex when there is existence of uncertainty in the input parameters. The modeling of the PCIs leads to the flexible structure under uncertainty and turns into the different from the traditional structure of the PCIs. The possibilities of getting uncertainty in the SLs are very high because of the different perceptions and subjective view of thinking of quality engineers. Moreover, in order to determine the SLs of a product, due to measurement errors caused by measuring system or inspector factor during quality control of parts, the processes monitoring may not be performed effectively in such situations. So, dealing with the uncertainty and vagueness the conventional PCA measures are not worthy to use. The use of fuzzy and its extensions can be used to tackle the uncertainty. Sometimes, fuzzy set also unable to capture the required degrees of uncertainty because it contains only the belongingness degrees of the parameters in to the set and does not talk about the indeterminacy and non- belongingness degrees which is also an integral part of the uncertain parameters. Thus, when the SLs are not precise, one can get benefitted by incorporating the NSs concept to make more realistic information and more accurate decisions. The NSs concept enables the quality engineer to consider the truth as well as the indeterminacy and falsity degrees while defining the SLs and then increases the efficiency of the PCA. Hence, the presented works discuss the PCIs using the different flexible NSs and analyze the effect on process capability indices by depicting the structure of SLs based on the neutrosophic numbers. It also explores the NSs concept in the domain of PCA and its related field of process monitoring when the processes are normal and non-normal.

Based on NS theory, the process capability indices C_p and C_{pk} have been presented. We have used the neutrosophic fuzzy notations for lower and upper specifications limits such as LSL^N and USL^N respectively. The corresponding neutrosophic process capability indices are represented by C_p^N and C_{pk}^N, and can be defined as follows:

$$C_p^N = \frac{USL^N - LSL^N}{6\sigma}$$

$$C_{pk}^N = \min\left\{\frac{USL^N - \mu}{3\sigma}, \frac{\mu - LSL^N}{3\sigma}\right\}$$

Case I. When the specification limit is T-I TSVNN: Assuming that the quantity of truth, indeterminacy and falsity are not dependent

We have considered that all the three membership functions such as truth, indeterminacy and the falsity are not dependent to each other's. To represent such situations, we have taken the advantage of Type-I Trapezoidal Single Valued Neutrosophic Number (T-I TSVNN). For this purpose, the SLs have been defined as the neutrosophic numbers. Suppose that the lower neutrosophic specification limit is defined by using the flexible structure of neutrosophic number as $LSL^N = \left\langle (lsl_1, lsl_2, lsl_3, lsl_4), (lsl_1^*, lsl_2, lsl_3, lsl_4^*), (lsl_1^{**}, lsl_2, lsl_3, lsl_4^{**}) \right\rangle$ and $USL^N = \left\langle (usl_1, usl_2, usl_3, usl_4), (usl_1^*, usl_2, usl_3, usl_4^*), (usl_1^{**}, usl_2, usl_3, usl_4^{**}) \right\rangle$.

Using the above neutrosophic SLs, the corresponding neutrosophic process capability indices C_p^N and C_{pk}^N can be represented as follows:

Index C_p^N :

$$C_p^N = \frac{USL^N - LSL^N}{6\sigma}$$

$$C_p^N = \frac{\left\langle (usl_1, usl_2, usl_3, usl_4), (usl_1^*, usl_2, usl_3, usl_4^*), (usl_1^{**}, usl_2, usl_3, usl_4^{**}) \right\rangle \langle - \rangle \left\langle (lsl_1, lsl_2, lsl_3, lsl_4), (lsl_1^*, lsl_2, lsl_3, lsl_4^*), (lsl_1^{**}, lsl_2, lsl_3, lsl_4^{**}) \right\rangle}{6\sigma}$$

$$C_p^N = \left\langle \begin{matrix} \left(\frac{usl_1 - lsl_4}{6\sigma}, \frac{usl_2 - lsl_3}{6\sigma}, \frac{usl_3 - lsl_2}{6\sigma}, \frac{usl_4 - lsl_1}{6\sigma}\right) \\ \left(\frac{usl_1^* - lsl_4^*}{6\sigma}, \frac{usl_2 - lsl_3}{6\sigma}, \frac{usl_3 - lsl_2}{6\sigma}, \frac{usl_4^* - lsl_1^*}{6\sigma}\right) \\ \left(\frac{usl_1^{**} - lsl_4^{**}}{6\sigma}, \frac{usl_2 - lsl_3}{6\sigma}, \frac{usl_3 - lsl_2}{6\sigma}, \frac{usl_4^{**} - lsl_1^{**}}{6\sigma}\right) \end{matrix} \right\rangle$$

Index C_{pk}^N :

$$C_{pk}^N = \min\left\{ \begin{matrix} \frac{\left\langle (usl_1, usl_2, usl_3, usl_4), (usl_1^*, usl_2, usl_3, usl_4^*), (usl_1^{**}, usl_2, usl_3, usl_4^{**}) \right\rangle \langle - \rangle \mu}{3\sigma}, \\ \frac{\mu \langle - \rangle \left\langle (lsl_1, lsl_2, lsl_3, lsl_4), (lsl_1^*, lsl_2, lsl_3, lsl_4^*), (lsl_1^{**}, lsl_2, lsl_3, lsl_4^{**}) \right\rangle}{3\sigma} \end{matrix} \right\}$$

$$C_{pk}^{N} = \min\left\{ \begin{array}{l} \left\langle \begin{array}{l} \left(\frac{usl_1-\mu}{3\sigma}, \frac{usl_2-\mu}{3\sigma}, \frac{usl_3-\mu}{3\sigma}, \frac{usl_4-\mu}{3\sigma}\right), \\ \left(\frac{usl_1^{*}-\mu}{3\sigma}, \frac{usl_2-\mu}{3\sigma}, \frac{usl_3-\mu}{3\sigma}, \frac{usl_4^{*}-\mu}{3\sigma}\right), \\ \left(\frac{usl_1^{**}-\mu}{3\sigma}, \frac{usl_2-\mu}{3\sigma}, \frac{usl_3-\mu}{3\sigma}, \frac{usl_4^{**}-\mu}{3\sigma}\right) \end{array} \right\rangle \\ \left\langle \begin{array}{l} \left(\frac{\mu-lsl_4}{3\sigma}, \frac{\mu-lsl_3}{3\sigma}, \frac{\mu-lsl_2}{3\sigma}, \frac{\mu-lsl_1}{3\sigma}\right), \\ \left(\frac{\mu-lsl_4^{*}}{3\sigma}, \frac{\mu-lsl_3}{3\sigma}, \frac{\mu-lsl_2}{3\sigma}, \frac{\mu-lsl_1^{*}}{3\sigma}\right), \\ \left(\frac{\mu-lsl_4^{**}}{3\sigma}, \frac{\mu-lsl_3}{3\sigma}, \frac{\mu-lsl_2}{3\sigma}, \frac{\mu-lsl_1^{**}}{3\sigma}\right) \end{array} \right\rangle \end{array} \right\}$$

Then one-sided capability indices C_{pk}^{N} based on the T-I TSVNN can be defined as follows:

$$C_{pkl}^{N} = \left\{ \left\langle \begin{array}{l} \left(\frac{usl_1-\mu}{3\sigma}, \frac{usl_2-\mu}{3\sigma}, \frac{usl_3-\mu}{3\sigma}, \frac{usl_4-\mu}{3\sigma}\right), \\ \left(\frac{usl_1^{*}-\mu}{3\sigma}, \frac{usl_2-\mu}{3\sigma}, \frac{usl_3-\mu}{3\sigma}, \frac{usl_4^{*}-\mu}{3\sigma}\right), \\ \left(\frac{usl_1^{**}-\mu}{3\sigma}, \frac{usl_2-\mu}{3\sigma}, \frac{usl_3-\mu}{3\sigma}, \frac{usl_4^{**}-\mu}{3\sigma}\right) \end{array} \right\rangle \right\}$$

$$C_{pku}^{N} = \left\{ \left\langle \begin{array}{l} \left(\frac{\mu-lsl_4}{3\sigma}, \frac{\mu-lsl_3}{3\sigma}, \frac{\mu-lsl_2}{3\sigma}, \frac{\mu-lsl_1}{3\sigma}\right), \\ \left(\frac{\mu-lsl_4^{*}}{3\sigma}, \frac{\mu-lsl_3}{3\sigma}, \frac{\mu-lsl_2}{3\sigma}, \frac{\mu-lsl_1^{*}}{3\sigma}\right), \\ \left(\frac{\mu-lsl_4^{**}}{3\sigma}, \frac{\mu-lsl_3}{3\sigma}, \frac{\mu-lsl_2}{3\sigma}, \frac{\mu-lsl_1^{**}}{3\sigma}\right) \end{array} \right\rangle \right\}$$

The minimum value of C_{pk}^{N} can be calculated by using the Definition 7 (score function).

Case- II: When the specification limit is T-II TSVNN: Assuming that the quantity of truth, indeterminacy and falsity are dependent

Here we assume that all the three membership functions such as truth, indeterminacy and the falsity are dependent to each other's. To represent such situations, we have taken the advantage of Type-II Trapezoidal Single Valued Neutrosophic Number (T-II TSVNN). For this purpose, the SLs have been defined as the neutrosophic numbers. Suppose that the lower neutrosophic specification limit is defined by using the flexible structure of neutrosophic number as $LSL^{N} = \left\langle (lsl_1, lsl_2, lsl_3, lsl_4);\ w_{LSL^N};\ u_{LSL^N};\ v_{LSL^N} \right\rangle$ and $USL^{N} = \left\langle (usl_1, usl_2, usl_3, usl_4);\ w_{USL^N};\ u_{USL^N};\ v_{USL^N} \right\rangle$.

Using the above neutrosophic SLs, the corresponding neutrosophic process capability indices C_p^N and C_{pk}^N can be represented as follows:

Index C_p^N :

$$C_p^N = \frac{USL^N - LSL^N}{6\sigma}$$

$$C_p^N = \frac{\left\langle (usl_1, usl_2, usl_3, usl_4); w_{USL^N}; u_{USL^N}; v_{USL^N} \right\rangle \langle - \rangle \left\langle (lsl_1, lsl_2, lsl_3, lsl_4); w_{LSL^N}; u_{LSL^N}; v_{LSL^N} \right\rangle}{6\sigma}$$

$$C_p^N = \left\langle \left(\frac{usl_1 - lsl_4}{6\sigma}, \frac{usl_2 - lsl_3}{6\sigma}, \frac{usl_3 - lsl_2}{6\sigma}, \frac{usl_4 - lsl_1}{6\sigma} \right); w_{USL^N} \wedge w_{LSL^N};; u_{USL^N} \wedge u_{LSL^N};\ v_{USL^N} \wedge v_{LSL^N} \right\rangle$$

Index C_{pk}^N:

$$C_{pk}^N = \min\left\{ \frac{USL^N - \mu}{3\sigma}, \frac{\mu - LSL^N}{3\sigma} \right\}$$

$$C_{pk}^N = \min\left\{ \frac{\left\langle (usl_1, usl_2, usl_3, usl_4); w_{USL^N}; u_{USL^N}; v_{USL^N} \right\rangle \langle - \rangle \mu}{3\sigma}, \frac{\mu \langle - \rangle \left\langle (lsl_1, lsl_2, lsl_3, lsl_4); w_{LSL^N}; u_{LSL^N}; v_{LSL^N} \right\rangle}{3\sigma} \right\}$$

$$C_{pk}^N = \min\left\{ \begin{array}{l} \left\langle \left(\frac{usl_1 - \mu}{3\sigma}, \frac{usl_2 - \mu}{3\sigma}, \frac{usl_3 - \mu}{3\sigma}, \frac{usl_4 - \mu}{3\sigma} \right); w_{USL^N}; u_{USL^N}; v_{USL^N} \right\rangle, \\ \left\langle \left(\frac{\mu - lsl_4}{3\sigma}, \frac{\mu - lsl_3}{3\sigma}, \frac{\mu - lsl_2}{3\sigma}, \frac{\mu - lsl_1}{3\sigma} \right); w_{LSL^N}; u_{LSL^N}; v_{LSL^N} \right\rangle \end{array} \right\}$$

Furthermore, the one-sided capability indices C_{pk}^N based on the T-III TSVNN can be defined as follows:

$$C_{pkl}^N = \left\langle \left(\frac{\mu - lsl_4}{3\sigma}, \frac{\mu - lsl_3}{3\sigma}, \frac{\mu - lsl_2}{3\sigma}, \frac{\mu - lsl_1}{3\sigma} \right); w_{LSL^N}; u_{LSL^N}; v_{LSL^N} \right\rangle$$

$$C_{pku}^N = \left\langle \left(\frac{usl_1 - \mu}{3\sigma}, \frac{usl_2 - \mu}{3\sigma}, \frac{usl_3 - \mu}{3\sigma}, \frac{usl_4 - \mu}{3\sigma} \right); w_{USL^N}; u_{USL^N}; v_{USL^N} \right\rangle$$

The minimum value of C_{pk}^N can be calculated by using the Definition 7 (score function).

4. Numerical example

The proposed neutrosophic process capability indices are implemented on the real-life application in the manufacturing company. The firm manufactures the piston and liner which is located in Konyas Industrial Area (Kahraman, C et al. (2017), Kaya, I., (2009)). For implementing the proposed neutrosophic PCIs, the production process of the piston is analyzed and the total diameter lengths of the piston are tackled by using the proposed methodology. The processes are monitored regularly and the defective and non-defective products are sorted out. Since the process capability analysis of any products require a flexible and sensitive dimension, the particular SLs may not be defined precisely well in advance. The quality improvement department decided to assign the flexible SLs to the diameter of the piston and used the proposed neutrosophic numbers with the different cases discussed in the Section 3. The measure of central tendency such as mean and variance of the ideal dimension values are determined as 175.20 and 0.00009580, respectively. The neutrosophic PCIs such as C_p^N and C_{pk}^N can be calculated for the different cases. The Table 1 and Figure 1 represent the solution results with the comparative study.

Case I: The SLs is considered as T-I TSVNN and can be defined as

$$LSL^N = \left\langle \begin{pmatrix} (175.164;175.165;175.166;175.167); \\ (175.163;175.165;175.166;175.168); \\ (175.162;175.165;175.166;175.169) \end{pmatrix} \right\rangle$$

and

$$USL^N = \left\langle \begin{pmatrix} (175.234;175.235;175.236;175.237); \\ (175.233;175.235;175.236;175.238); \\ (175.232;175.235;175.236;175.239) \end{pmatrix} \right\rangle.$$

The process capability of this process within the defined SLs is approximately 1.17. Neutrosophic process capability shows that considering the indeterminacy membership function, the capability of the process cannot be less than 1.12 and more than 1.26, and considering the falsity-membership function, it cannot be less than 1.09 and more than 1.29.

$$C_p^N = \langle (1.14;1.17;1.21;1.24);(1.11;1.17;1.21;1.28);(1.07;1.17;1.21;1.31) \rangle$$

$$C_{pk}^N = \min \begin{Bmatrix} \langle (1.16;1.19;1.22;1.26);(1.12;1.19;1.22;1.29);(1.09;1.19;1.22;1.33) \rangle \\ \langle (1.12;1.16;1.19;1.22);(1.09;1.16;1.19;1.26);(1.05;1.16;1.19;1.29) \rangle \end{Bmatrix}$$

$$C_{pk}^N = \{ \langle (1.16;1.19;1.22;1.26);(1.12;1.19;1.22;1.29);(1.09;1.19;1.22;1.33) \rangle \}$$

Case II: The SLs is considered as T-II TSVNN and can be defined as

$$LSL^N = \langle (175.234;175.235;175.236;175.237); 0.950; 0.055; 0.070) \rangle$$

and

$$USL^N = \langle (175.164;175.165;175.166;175.167); 0.950; 0.055; 0.070 \rangle.$$

The process capability of this process within the based on crisp SLs is approximately 1.19. However, the quality inspector may not be able to define these SLs precisely. Consequently, the indeterminacy and falsity degrees of the process are expressed as 0.05 and 0.075.

$$C_p^N = \langle (1.14;1.17;1.21;1.24); 0.950; 0.055; 0.070 \rangle$$

$$C_{pk}^N = \min \begin{Bmatrix} \langle (1.16;1.19;1.22;1.26); 0.950; 0.055; 0.070) \rangle \\ \langle (1.12;1.16;1.19;1.22); 0.950; 0.055; 0.070) \rangle \end{Bmatrix}$$

$$C_{pk}^N = \{ \langle (1.12;1.16;1.19;1.22); 0.950; 0.055; 0.070) \rangle \}$$

Table 1. Comparative study of the numerical examples

PCIs	**Proposed Approach**		Yalcin S. and Kaya I. (2022)	
	Case - I	**Case II**	**Case - I**	**Case II**
C_p^N	1.17	1.19	1.11	1.12
C_{pk}^N	1.17	1.19	1.11	1.12

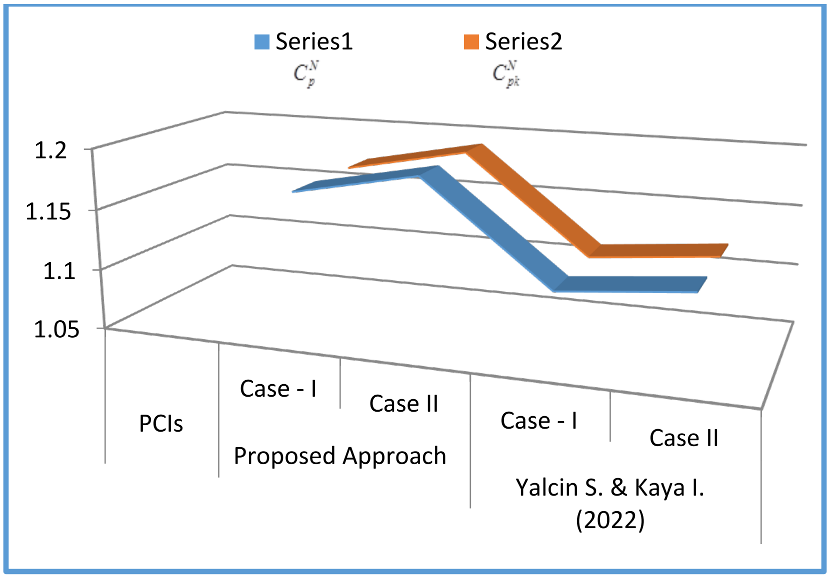

Figure 1. Graphical representation of the solution results.

4.1. Results and Discussion

The results are obtained based on the nature of neutrosophic numbers used such as TI-SVNN and TII-SVNN. It can be observed that with help of fuzzy approach the maximum membership degree attained is 1, whereas the truth, indeterminacy and falsity membership degrees of neutrosophic C_p^N are determined 0.95, 0.05 and 0.075, respectively. Looking at the results, one can conclude that the obtained results contain more information about process capability indices. All the membership functions are flexible in nature and contain the information based on the different perceptions and views of the decision-makers. It has also integrated the PCA with the indeterminacy degrees. More precisely, the neutrosophic process capability measurement approach are based on more logical aspects of human decision-making behavior and also can define the instability of human decisions in a better way. Additionally, the NSs provide an advantage over fuzzy sets while applying various operations by the quality engineers.

The obtained results of TII-SVNN are also compared with FSs. It is observed that in addition to membership and non-membership functions of IFSs, indeterminacy function in NSs provides an opportunity to examine process capability in more detail. According to the results, it is seen that the crisp value of neutrosophic C_p^N is 1.19, but at the same time, it cannot be less than 1.09 and greater than 1.29. The result shows that the biggest advantage of NSs over other new fuzzy set extensions is the uncertainty function.

5. Conclusion

Process capability analysis (PCA) is very important and integral part of the manufacturing system to timely monitor the processes. For the PCA, various statistical based measures are available which are capable to analyze the processes. The PCIs such as C_p and C_{pk} are the most commonly used methods which are available in the literature. The presented study discusses the PCIs under the NSs environment dealing with the uncertainty and incomplete information more accurately and efficiently. The NSs is the generalization and extension of the traditional fuzzy sets. It contains the three different forms of assessing the uncertainty such as independency truth, indeterminacy and falsity functions that enhance the capability of the processes related to the uncertain model. Most often, it seems that the quality engineer may not have the enough ideas or complete information for making the perfect and fruitful decisions. For instance, if the quality engineer shares the views or opinion with 0.4, 0.8, 0.2 as true, indeterminacy, and falsity value, respectively then only the NSs can be successfully used to tackle this situation. On analyzing the literature on PCAs, it is found that the use of the conventional fuzzy set is discussed tremendously and a narrow part of the literature or the research work is dedicated to the NSs uncertainty. Therefore, this research fills this gap and led to a concrete base for PCAs under NSs.

The NSs provides the flexible state-of-art while applying to the engineering problems. The conventional PCIs such as C_p and C_{pk} are discussed under the neutrosophic environment and re-structured based on the NSs. Aiming to this purpose, the SLs are represented as trapezoidal single-valued neutrosophic numbers under different condition and PCAs are analyzed by investigating the neutrosophic SLs. We have also explored the advantage and the merits of NSs with the PCAs. The two neutrosophic process capability indices C_p^N and C_{pk}^N is investigated and implemented on the manufacturing system problems. The obtained neutrosophic PCIs exhibits the wholesome and effective knowledge about the capability of process as compared to the conventional fuzzy set approaches. In the future, the different extensions of fuzzy set such as intuitionistic fuzzy, hesitant fuzzy, spherical fuzzy etc. can be applied to analyze the PCAs.

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