Abstract

Net Present Value (NPV) is an important financial metric for the purposes of accepting or rejecting Projects in the Corporate Environment. The authors advocate limited free NPV applications available to the Management and Commerce Student or the Corporate Finance practitioner. The proposed application enables the analysis of NPV when factors such as Cost of Goods Sold to Sales Ratio; Capital Expenditure to Sales Ratio; Selling, General and Administrative Expense to Sales Ratio; Sales Growth Rate, Depreciation to Sales Ratio are available as inputs. Further, the parameters like Corporate Tax Rate with number of Years until Constant Growth and Firm Risk Level proxied by the Weighted Average Cost of Capital (WACC) are deemed available as inputs for the simulation. The approximate run-time for 5,000 simulations is less than one minute. Several useful outputs can be derived by the simulation of the proposed application. First, a graph of each of the distributions pertaining to the input variables is displayed. The User can select the type of distribution for each input variables, namely: Normal Distribution, Triangular Distribution and Uniform Distribution. In addition, a Base Case NPV is also computed for a set of Base Case input variables. By virtue of the attained cumulative probability NPV diagram, User can determine the probability whether NPV is less than or equal to a certain value. In addition, a sensitivity table is provided in the proposed application which displays the impact of input variables in the descending order based on the attained stochastic NPV results.

Keywords
Corporate Finance, WACC, Normal Distribution, Triangular Distribution, Uniform Distribution

1. Introduction

Fischer introduced Net Present Value (NPV) in 1907 when he discussed in the paper, the rate of Interest " where Capital was defined as an asset that produced income in various time frames or time periods. The notion of computing the value of a series of cash flows with a suitable discount rate was introduced as a concept. Discount rates were expected to account for inflation, as were the cash flows. According to Gaspars-Wieloch (2019), there are four techniques in accounting for Uncertainty in NPV measurement. Firstly, a technique is to amplify the discount rate, Secondly to use a range of values as the possible outcomes and this falls into a category called Sensitivity Analysis. Thirdly, Cash Flows are compared with Optimistic and Pessimistic values of Cash Flows and finally, the Cash flows are estimated with Scenario Planning and the use of Probability Distributions.

The states of Probability are estimated from prior data. Carnap (1950) argued in favor of premise-based conclusions or decision-making in the presence of frequencies of occurrence of events. Hau et al. (2010) investigated the gap between apriori and statistical probabilities as conceptualized by Knight (1921) and corroborated that rare events played a role in the relevance of the presence of the gap between apriori and statistical probabilities. In contrast, Guo and Ma (2014) analyzed newsvendor models for innovative products with a one-shot decision theory framework that focused on possibility rather than probability distribution and made use of scenario models as an alternative to decision making. NPV estimation with uncertainty with the help of Optimistic and Pessimistic coefficient values was done extensively in Gaspars-Wieloch (2019) and it applied scenario planning as well as decision maker’s attitude in relation to a certain problem related to computation of NPV. The motivation to write, and contribution from this particular paper comes from a desire to contribute to the computation of NPV when the input variables follow probabilistic distributions. Thus, the methodology deviates from Gaspars-Wieloch (2019) in that input variables are subject to various types of Probability Distributions and the resulting NPV is computed for each simulation. The empirical results, when tabulated, can generate interesting probability distributions, which in turn can be useful to the practitioner as well as Graduate Students in Finance and Commerce. In addition, Sensitivity Analysis can be conducted with each
input such as Cost of Goods Sold to Sales Ratio; Capital Expenditure to Sales Ratio; Selling, General and Administrative Expense to Sales Ratio; Sales Growth Rate, Depreciation to Sales Ratio are available as inputs. Mari (2020) considered stochastic revenue from energy sales and stochastic energy costs to arrive at the NPV computation and an efficient frontier was constructed for the maximum NPV for a given risk level. However, the various components such as current assets to sales, depreciation to sales and tax rates were not considered in the analysis. Further, no sensitivity analysis was done, in order to compare base case values with the minimum and maximum input values. Zhang (2022) arrived at NPV with the use of Brownian motion to model the interest rate, however did not subject any of the other ingredients of the NPV formulation to any probabilistic distribution neither did any Sensitivity analysis.

1.1 Objectives

a. To create an application with various types of probability distributions of the Input Variables to NPV, so that the resultant NPV fluctuates with the varying inputs
b. To calculate the NPV with varying Input Variables and Obtain relevant Descriptive Statistics across Simulations
c. To run Sensitivity Analysis on the Input Variables and determine which Variable affects NPV most by observing NPV Range for each of the Input Variables

This paper is organized as follows: Section 2 discusses existing NPV formulation and associated literature, Section 3 discusses the methodology used in the estimation of NPV, Section 4 allows for the computing of base case NPV as well as NPV under varying input conditions, and Section 5 discusses NPV sensitivity to Input Variables is tested. Finally, Conclusions and potential for future development are discussed in Section 6.

2. Literature Review on Existing NPV formulation

The existing NPV formulation, attributed to several authors and publications, includes a left-hand side that computes the sum of the future value of cash flows at a pre-defined discount rate since cash flow in future periods is not the same as the cash flow in the current period where the calculation is being performed, Khan (1993) and Brealey and Myers (2020), where $\text{CF}_t$ is the Cash flow in time t and r is the risk level associated with the Cash flow as in Equation 1:

$$\text{NPV} = \sum_{0}^{T} \frac{\text{CF}_t}{(1+r)^t}$$

However, the equation (1) above does not offer adequate treatment, for varying the numerator $\text{CF}_t$ or the denominator r which is the risk level. It is noteworthy that when the time is zero, the cash flow is for the current period, or the Initial Cash Outlay (ICO), rest of the terms in Equation (2) below are zero, and as time varies to its ultimate year, T we are left with the Terminal Year Cash flow (TV). Various definitions exist for $\text{CF}_t$ and widely used value for $\text{CF}_t$, for example Fernandez (2019) as in Equation 2 is:

$$\text{CF}_t = \text{Sales}_t - \text{COGS}_t - \text{SGA}_t - \text{Tax}_t + \text{Depreciation}_t - \text{Change in Current Assets}_t - \text{Increase in Fixed Assets}_t + \text{Sale of Fixed Assets}_t + \text{Interest} \times (1-T)$$

where the subscript t is used for time and changes in current or fixed assets as computed for time t is the difference in value in time t and time t-1

2.1 Existing literature on Computation of NPV under Uncertainty

A fair level of NPV Analysis under Uncertainty has been done recently by Gaspars-Wieloch (2019) where the NPV has been estimated in a novel manner with the help of scenario planning (Method 4 in the Introduction has mentioned this Method). Coefficients of Optimism and Pessimism are used to account for the Attitude of the Decision Maker. Further, the paper discusses five possible Decision Making states, firstly, Decision Making Under Certainty (DMC), secondly Decision Making under Risk (DMR), where the possible states are known in the future with approximate probabilities, thirdly the Decision is Made with Partial Information (DMPH), where the future states are somewhat delineated, however the future state is modeled with probability intervals for each scenario, fourthly the author discusses situations where Decision Making is done under Complete Uncertainty (DMCU) and finally Decision Making under Total Ignorance (DMTI) where the future is completely unknown. In Gaspars-Wieloch (2019) the coefficients of optimism and pessimism are used in conjunction with the hybrid of the H+B rule (Hurwics and Bayes rules hybrid). The paper discusses the limitation of the Hurwics rule when used alone with optimism and pessimism coefficients in that only extreme values of payoff are considered while transitional values are ignored, however, the
H+B approach accounts for the rankings of transitional, symmetric as well as asymmetric distributions of payoffs. The paper applies appropriate optimist and pessimist coefficients to simulate NPV situations suitably for a range of projects and cash flow timings. However, the above paper does not derive the NPV from the raw inputs stated in Equation (2) of this paper. Therefore, this is a novel contribution to the literature, in that varying the inputs such as COGS, SGA, Capex/Sales, Depreciation/Sales to probability intervals, and importantly, allowing a selection of the distribution type for each of the input variables and then arriving at the NPV distributions with the help of a large number of simulations, can be constituted as an original contribution. In addition to Gaspars-Wieloch (2019), another paper by van Groenedaal (1998) was reviewed, and it states that much of NPV variability analysis is done with the methodology of changing one factor at a time and through scenario analysis. Our paper addresses that concern and performs a variability analysis of the NPV by changing all input factors at the same time, and measuring a deviation of the NPV from a Base Case, as determined by the Decision Maker. Further, Groenedaal (1998) takes the approach of using an experimental design in conjunction with regression models and elucidates that as a better approach than a conventional deterministic model. In Contrast, our approach is to use various probability distribution forms for the input variables for the NPV calculation and then arrive at the output NPV distribution itself.

### 3. Methods used in this Paper: Allowing Input Variables to Follow Various Types of Probability Distributions

In our paper, we want to state that the methodology used is done with Input Variables to the NPV following various types of Probability Distributions. The resulting NPV also comes with a Probability Distribution, which will be discussed in Section 5.

1. **The Uniform Distribution** is a simplistic Probability Distribution and plays an important role in the modeling of Random Variables, according to Jevremović (2014). The Discrete Uniform Probability Distribution Function (pdf) is defined as, according to Hogg et al. (2005), Dekking (2005), Johnson (1994), Johnson et al. (1995), L’Ecuyer (2011). The nist web-site states the definition of the Continuous Uniform Distribution as having equal probability for all values between two real numbers $A$ and $B$ and in Mathematical form as in Equation 3:

$$ f(x) = \begin{cases} \frac{1}{B-A} & \text{for} \quad A \leq x \leq B \\ 0 & \text{otherwise} \end{cases} \quad (3) $$

Where $A \leq x \leq B$ and $x \in R$

In our paper, any or all of the Input Variables for NPV computation are allowed to have a Uniform Distribution as defined in Equation (3) above, and the required elements for the distribution are minimum and maximum values. It is important to note that the range of values being used for the input variables must exceed zero. As an example, it is impossible for COGS, SGA, Capex, and Depreciation Ratios to be negative. Our paper assumes random numbers for the $B$ and $A$ in order to simulate the Input Variable selected, and each iteration assigns a number between the low and high values $B$ and $A$ respectively. None of the existing literature reviewed at the time of writing the paper allows for Input variables for NPV to have a Uniform Distribution.

2. **The Normal Distribution** is possibly the most widely used distribution in Analysis, and is due to Gauss (1809), and over two centuries later, this distribution has found application in countless situations. Ahsanullah et al. (2014) state that the normal distribution is extremely important and widely used in many scientific applications as well as in statistics. Casella and Berger, (2001), Kruskal et al. (1997), and McPherson (1990) all discuss the Normal Distribution and its application in various domains. None of the existing literature reviewed at the time of writing the paper allows for Input variables for NPV to have a Normal Distribution. The nist web-site states the definition of the Normal Distribution as having the following Mathematical form as in Equation 4:

$$ f(x) = \frac{e\frac{-(x-\mu)^2}{2\sigma^2}}{\sigma \sqrt{2\pi}} \quad (4) $$

Where $-\infty \leq x \leq \infty$

Our paper assumes a value for $\mu$ and $\sigma$ as inputs from the user and generates values for $f(x)$ for each simulation. Since the Input Variables cannot have a mean or standard deviation less than zero, these restrictions are imposed via the
software. Once the user selection is valid, normal distribution value of f(x) is generated for the mean and standard deviation selected.

iii) Triangular Distributions are ones where a minimum, maximum and likely outcome is known.
The Triangular distribution pdf used in our paper is defined as follows for \( a \leq x \leq b \), \( a \in (-\infty, \infty) \), \( a \leq c \leq b \) thus a is the minimum, b the maximum and c is the mode in the pdf and defined in Ma et al. (2015) and in Evans et al. (2000), as in Equation 5

\[
\begin{align*}
0 & \quad \text{for } x < a \\
\frac{2(x-a)}{(b-a)(c-a)} & \quad \text{for } a \leq x \leq c \\
\frac{2}{b-a} & \quad \text{for } x = c \\
0 & \quad \text{for } x > b \\
1 & \quad \text{for } x \leq \text{To} \\
2 & \quad \text{for } x \geq \text{To}
\end{align*}
\]  

4. Data Generation: Base Case NPV, Analysis of Sensitivity of NPV to Input Variables
The Base Case NPV is computed with Variables related to User entered values in the Base Case column in Figure 1. The Base Case values are entered for each of the following Variables: i) Sales Growth %, ii) COGS to Sales Growth %, iii) SGA to Sales %, iv) Depreciation to Sales %, v) Capex to Sales %, vi) Terminal Year Growth rate, vii) WACC% and viii) Tax %. Number of years for the cash-flow computation and finally x) Base year Sales. Each Variable can be probabilistically estimated with the type U for Uniform, T for Triangular, and N for Normal. The number of Simulations Parameter should be set to a number preferably less than 5,000 in order to get statistically improved results. A number higher than 5,000 can slow down the process, therefore the number selected for number of simulations should be between 1,000 and 5,000 both included. Once the data is entered, a series of validations are made. The first set of validations include a check on range of values as required for a Uniform distribution and the To value should be at least equal to the To value, and they should both exceed zero. In case the type of validation is Normal, the mean and standard deviation should both be selected and should be non-zero values. In a Triangular Distribution, the Minimum, Maximum and Mode parameters are all required and should be greater than zero. The Mode should lie in between Minimum and Maximum values. As soon as the parameters are selected and are valid, the NPV computation proceeds in a step wise manner as follows:

4.1 Steps in Varying Input Variables and Obtaining NPV
i) Compute NPV with Base Input Parameter Values
ii) For the Next Iteration (Until the number of Simulations are Complete)
iii) Obtain the Simulated Values of the Iteration for each Input Variable for the Type of Distribution Selected
iv) Compute the NPV for the Value of the Input Variables in this Simulation
v) If NPV Calculation has been requested, Record the NPV for the Simulation, as well as the Simulation Parameters
vi) Go to the Next Iteration (Step ii)
 vii) Plot Graphs for Input Parameter and its type of Distribution
viii) Compute the Deviation from the Base NPV and Obtain the Most Relevant Input Parameter

5. Results and Discussion
5.1 Numerical Results
1. NPV is computed for the Base Year if the selection is made against NPV in Figure 1. The Base Year NPV as well as the NPV for each simulation are calculated and Figure 6 shows the NPV Distribution for each simulation, and displays the fluctuation of the NPV for each simulation. It is clear from the diagram, the NPV rarely exceeds 3,000 or rarely goes below 2,000. Figure 7 shows the cumulative distribution of the resulting NPV distribution, which looks approximately like a normal distribution. Figure 8 shows the cumulative probability distribution and it has an S-shaped curve. From the S-shaped cumulative pdf, it is easy to determine the percentage of data points below or above a selected NPV value in the X-axis. Since the number of simulations is vast in number, it is not possible to show all the simulation results, therefore Base NPV and Descriptive Statistics are being shown in Table 1.
Table 1. Descriptive Statistics for Simulated Values of NPV in INR

<table>
<thead>
<tr>
<th>Simulation Statistic</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base NPV</td>
<td>4,091.9</td>
</tr>
<tr>
<td>Mean</td>
<td>3,889.60</td>
</tr>
<tr>
<td>Median</td>
<td>3,820.83</td>
</tr>
<tr>
<td>St. Dev</td>
<td>632.55</td>
</tr>
<tr>
<td>Variance</td>
<td>400,122.15</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.56</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>-0.06</td>
</tr>
<tr>
<td>1st Q</td>
<td>3,403.37</td>
</tr>
<tr>
<td>3rd Q</td>
<td>4,284.53</td>
</tr>
<tr>
<td>Min</td>
<td>2,530.44</td>
</tr>
<tr>
<td>Max</td>
<td>6,147.01</td>
</tr>
<tr>
<td>number of simulations</td>
<td>3,000.00</td>
</tr>
</tbody>
</table>

Sensitivity Analysis is performed for each of the Input Variables. The method of performing the sensitivity analysis is to obtain the NPV at the minimum as well as maximum values of that variable, when other input variables are at the Base Value selected (ceteris paribus).

5.2. Graphical Results
Graphs are generated for Input Variable Simulations. For Sales Growth % and COGS/Sales % the selection has been T for Triangular and a simple visual inspection of Figure 2 and Figure 3 show that the Triangular distribution does seem to hold, for the selection of Mode = 10% for Sales Growth % and Mode = 20% for COGS/Sales %. Figure 4 shows the Normal Distribution of the SGA/Sales % Input Variable Figure 5 shows the Uniform Distribution for the Input Variables Depreciation to Sales %. Further, the impact of each input Variable is plotted on the Sensitivity Analysis Graph. Figure 6 shows the NPV fluctuations and it is observable that the NPV values follow a stationary process. Figure 7 shows an approximately normal distribution of the NPV and Figure 8 shows the S-shaped curve for cumulative frequency of the NPV.

5.3 Proposed Improvements

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1 The currency chosen for this paper is Indian Rupee or INR. However, the currency can be modified in the software as required by the user and is fully configurable in MS Excel 2019
2 For base year sales, currency chosen is Indian Rupee or INR
Further development in this application could include the inclusion of Scenarios, associated probabilities for each scenario and a comparison and weighted average NPV of the Scenarios. Probability Distributions for input variables could be auto-detected.

5.4 Validation
The model is validated by the following means
i) The visually observed graphs of the Uniform, Triangular and Normal Distributions are testimony to the actual values used in the simulation (Visual Verification).
ii) The Base Value NPV in Table 1 (4,091) is close to the Average Value of the NPV (3,889), when the distribution values of each input variable approximately centers around the base value.
iii) The NPV distribution of simulations is approximately normal, as evidenced by Figure 7, Figure 8.

Figure 2. Sales Growth % Triangular Distribution Representation

Figure 3. COGS to Sales % Triangular Distribution Representation

Figure 4. SGA to Sales % Normal Distribution Representation
Figure 5. Depreciation to Sales % Uniform Distribution Representation

Figure 6. NPV Simulations

Figure 7. NPV Frequency Distributions

Figure 8. NPV Cumulative Frequency Distributions
Table 2. Range of NPV with Various Input Parameters

<table>
<thead>
<tr>
<th>Input Parameter</th>
<th>Low Value</th>
<th>High Value</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>WACC% NPV Range</td>
<td>3,488.12</td>
<td>1,626.78</td>
<td>1,861.34</td>
</tr>
<tr>
<td>Sales Growth% NPV Range</td>
<td>1,056.57</td>
<td>2,081.57</td>
<td>1,025.00</td>
</tr>
<tr>
<td>TY Growth% NPV Range</td>
<td>1,690.58</td>
<td>2,609.65</td>
<td>919.07</td>
</tr>
<tr>
<td>SGA Sales% NPV Range</td>
<td>2,495.59</td>
<td>1,676.34</td>
<td>819.25</td>
</tr>
<tr>
<td>COGS Sales% NPV Range</td>
<td>2,561.43</td>
<td>1,911.86</td>
<td>649.57</td>
</tr>
<tr>
<td>Depr Sales% NPV Range</td>
<td>1,946.21</td>
<td>2,178.55</td>
<td>232.34</td>
</tr>
<tr>
<td>Capex Sales% NPV Range</td>
<td>2,124.24</td>
<td>2,059.55</td>
<td>64.69</td>
</tr>
<tr>
<td>CA Minus CL to Sales %Range</td>
<td>2,091.80</td>
<td>2,043.29</td>
<td>48.51</td>
</tr>
</tbody>
</table>

6. Conclusion
The NPV pdf is a normal distributional though the underlying Input Variables have different types of distributions, ranging from Normal to Triangular to Uniform. Our paper is different from prior work in NPV calculation under Uncertainty e.g., Gaspars-Wieloch, H. (2019), who modeled NPV under Scenario based optimistic and pessimistic coefficients. The Ranges of the Input Variable are selected by the user, and future development should focus on the detection of the type of distribution from historical information. From Table 2 and Figure 9, it is clear that the WACC % has the highest impact on the NPV fluctuation while the CA-CL to Sales % has the least impact on NPV fluctuation. Sales growth%, TY Growth % have high impact while COGS/Sales %, Depr/Sales % and Capex to Sales % have lower impact on NPV fluctuation.

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