Production, Maintenance and Quality Management for a two-product, Multi-warehouse Problem, with Multi-Level Statistical Process Chart of a Supply Chain under Service and Quality requirements.

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Abstract

In this work, we consider a joint model production system consisting of a single machine producing a multi-product connected to a multi-warehouse to satisfy changing and characterized customer demand of all the products and throughout the production horizon. The addition of quality control and maintenance to classical production would increase the reliability of the process, quality of the products and reduce the total cost of production due to the reduction in the non-conformal losses. The work will answer some managerial questions about which product to produce and how much each makes. Then the need for a maintenance strategy corresponding to the optimal production plan. Therefore, the production system is imperfect and subject to random failures, which directly affect quality. The system degradation is a function of use (duration) and production rate variation. The process control and product quality is with the aid of a multi-level statistical process control chart tool to simultaneously forecast the production planning with delivery quantities and the maintenance strategy involving corrective and perfect maintenance. The objective is to improve the reliability of the process, reduce the non-conformal products, thereby minimizing the total cost of production and satisfy different customers with varied random demand/Quality throughout the periods.

Keywords
Production control, Multi-product, Failure rate, Statistical Process Control and Maintenance.

1. Introduction
Characterized production planning coupled with corresponding maintenance tactics based on statistical control chart of quality will be an essential aspect of industrial engineering management. There is a need for industrial companies to remain competitive, satisfy multiple customer needs, and at the same time make profits. Presently, the competition in the global industrial scene is fast growing. The demands for a high-quality product at a reasonable price in dynamic global economic conditions coupled with the ever-changing markets, with a wide range of products, the reduced delivery time and costs have put more additional industry requirements in terms of both the customer's consciousness and demand (Hajej et al. 2021).
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1.1 Objectives
The objective of this study is to determine firstly the optimal plan of production to satisfy the varied demand of different customers under a given service and respective quality level. Secondly, according to the economic goal of production, the optimal parameters of the control chart are the sample size (n_1, n_2), the control interval (h_1, h_2), the control limits coefficient (A_1, A_2) and the optimal maintenance strategy characterized by the optimal number of preventive maintenance actions N^*. It's assumed that the horizon is proportioned equally into H periods with a length equal to Δt. Moreover, we assume that demand fluctuation is normal with mean $\mu_{di,j}(k)$ and variance $V_{di,j}(k)$, which are known, respectively. The demands are satisfied at the end of each period.

2. Literature Review

Production systems are becoming more complex and subject to multiple factors, while customers are becoming more exigent, demanding various products at random (Abubakar et al. 2020). Hence the need to develop integrated industrial strategies (maintenance, production, and quality). Unfortunately, production planning, maintenance, and quality control policies are hardly studied together in the literature. For industrial companies, determining the best production planning and maintenance strategy has always been a difficult task. According to Starbird (2002) the service or product quality varies depending on production, time, and cost. As a hierarchical decision-making process, the best maintenance and production plans minimize the entire cost, including production of all the products and inventory and maintenance costs for all periods. A statistical quality control process to make more interactive decisions about new collaborative maintenance strategies are employed to accomplish the task. The integration of optimal production, delivery planning and maintenance will be a good solution for lowering total production costs by boosting the production system's reliability and, as a result, reducing non-conformance items. Buzacott (1967) is one of the authors to accomplish the improvement of the production system. In his work "Automatic transfer lines with buffer stock," he studied the role of buffer stocks in boosting system productivity to solve the challenge of integrating maintenance into production plans. In the paper "An optimal production planning and maintenance policy for a multiple-product and single machine under failure rate dependency,"Mifdal et al. (2013) determined the optimal production planning followed by an optimal maintenance strategy for a production system involving a single machine, multiple-product, this study did not consider quality. Ghadimi and Aouam (2021) optimized the production jointly with safety stock in a serial production system, distributing multiple products under a service guarantee.

We observed that most works in the literature allowed the coordination of two fundamental capacities: production, maintenance, and quality. The integration of production and maintenance, then production with quality or maintenance with quality, and recently most comprehensive trends have jointly dealt with all three factors. Nourelfath et al. (2016) studied a multi-period multi-product capacitated lot-sizing context, integrates production, maintenance, and quality for an imperfect process. The machine inspection is carried out for each period, and imperfect preventive maintenance activities are simultaneously performed. The outcomes of the works determine the optimal values of the production plan and a preventive maintenance period where the machine is restored to control after producing non-conformal items. Guo et al. (2018) consider lot sizing, quality and maintenance for an imperfect production system. Rivera-Gomez et al. (2020) proposed an integrated production maintenance and quality control policy for an unreliable single product manufacturing system subject to degradation. Depending on the defectives proportion determined during the inspection, the inspector can decide on the appropriate preventive maintenance. A minimal repair is carried out at failure to restore the production system to its previous status (ABAO). Boushla et al. (2016) investigated production, preventive maintenance and quality control using a sampling method. The production lot size, the sampling plan, the safety stock, and overhaul planning are under quality constraints. Sett et al. (2017) studied production with inventory planning, which advanced the previous models, and integrated controls related to the minimum required machine availability. They obtained the optimal value of inventory level time at which buffer stock builds up the age of preventive maintenance action (safety stock)—also treated by Kenne et al. (2008) the latter determined the optimal buffer stock size of the production system subjected to periodic preventive maintenance (PM) action, thereby reducing the probability of failure reduce its age, proportional to the preventative maintenance. Nahas (2017) created a stochastic model of unstable equipment, which focused on joint optimization of economic production and aged-based preventive maintenance policy for deteriorating production systems using proportional hazard (PHM). Quality and quality management affects all aspects of production, inventory, delivery, maintenance, and customer requirements (cost and service satisfaction). Process improvement methods such as Six Sigma methods, System measurement analysis, Analysis of the causes and consequences applicable to stabilizing the production process, the accomplishment of quality which reduces the production of non-conformity, and the fulfilment of other customer necessities are explored in the literature. Six Sigma uses the base tools to improve the quality of products and processes by reducing the process variation and the associated high defect rate, such as the Control chart “x-bar” and the periodic preventive maintenance strategy as dealt with by several researchers. Ben-Daya and Rahim (2002) in their work “Effect of maintenance on the economic design of x-control chart”, and its extension by Zang and Zang (2017) suggested a framework of reliability and maintenance structure for a two-state process optimizing decision variable of a manufacturing system to determined discrete times of preventive maintenance activities, examined the impact of maintenance on cost. However, they ignored stock deficiency. Production and quality issues were attended to by Hajej et al. (2021) in their work which Integrated the model of production quality. The work studied a randomly failing manufacturing system that has to satisfy random demand during a finite production horizon and under a given service level. Other researchers who dealt with maintenance and quality are El cadi et al. (2021). They worked on integrated

optimization of Production planning, maintenance and quality control policy without considering the effects of inventory control shortages.
Sequel to the relevant contributions in the existing literature, we identified some knowledge gaps. This study focuses on the statistical quality control method applied to integrated optimal maintenance and production plans to significantly impact overall system performance and satisfy customers and investors by improving quality and minimizing the total cost. The total price will include the total production of the multi-products, the entire inventory holding at the principal and multi-warehouse, and the comprehensive maintenance, which consists of the PM, CM, sampling inspection and the non-quality product cost. The integration of the control chart on the optimal production plan and maintenance strategy required the company to manufacture their products that satisfy a varying characterized demand over the entire period. In this context, the first actions of a hierarchical decision-making process - control of production rate and duration - the failure or dysfunction depends on the production system and is considered among the causes of non-conforming items. Hence, the integration of statistical process control on maintenance strategy, and collaboration with production and correlation the customer service level, different product production and quality bounds, regarded as the solution to reducing cost. A mathematical model will help determine the decision variables, which will minimize the total of production, maintenance, inventory and quality is developed. The remainder of the paper tests the solution's effectiveness using a numerical experiment to validate the solution's robustness, from which results are obtained and a conclusion drawn.

2.1 Production and maintenance problem

This work contributes to the overall performance optimization of a supply chain composed of a single machine (M) that produces two products (P1 & P2) stored in a principal manufacturing store. Appropriate delivery quantities ($\theta_{i,1}, \theta_{i,2}, \dots \theta_{i,L}$) will be delivered to multi-customer ($d_{i,1}, d_{i,2}, \dots d_{i,L}$) purchase ware-houses ($w_1, w_2, \dots w_L$) with a delivery time τ_i $(i: 1, \dots L)$. The system operates at a given service level θ_i, under the following constraints; $Prob[w_i(k) \geq 0] \geq \theta_i$, Production bounds: $u^1{}_{min} \leq u_1(k) \leq u^1{}_{max}$ for the product 1 and $u^2{}_{min} \leq u_2(k) \leq u^2{}_{max}$ for the product 2, over a finite horizon H, as shown in Figure 1 below. Machine M is prone to failure and repairs at random. The deterioration of the machine is influenced by both time and production rates. Consequently, the failure rate $\lambda(t)$ increases with time and production rate $u(k)$, affecting the reliability and capability of the production process responsible for non-conforming units.

2.2 Notations

The following parameters are used in the mathematical formulation of the model:

$u_1(k)$	:Production rate of product 1 during period $k, (k = 1, \dots, H)$ on machine M
$u_2(k)$	:Production rate of product 2 during period $k, (k = 1, \dots, H)$ on machine M
$u^1{}_{max}$	:maximal production rate of product 1 on machine M
$u^2{}_{max}$	:maximal production rate of product 2 on machine M
$u^1{}_{min}$	:minimum production rate of product 1 on machine M
$u^2{}_{min}$	:minimum production rate of product 2 on machine M
$S(k)$	:Inventory level of S at the end of period $k, (k = 0, 1, \dots, H-1)$
S	:Principal Store S (Manufacturing Stock).
$w_{1,i}(k)$	:Inventory level of the product 1 for each warehouse w_i and for period $k, (k = 0, 1, \dots, H-1)$
$w_{2,i}(k)$	:Inventory level of the product 2 for each warehouse w_i and for period $k, (k = 0, 1, \dots, H-1)$
τ_i	:Delivery time for all the products to all the warehouse w_i
$Q_{1,i}(k)$	:Delivery rate of the product 1, for each warehouse w_i during period $k, (k = 0, 1, \dots, H-1)$
$Q_{2,i}(k)$	:Delivery rate of the product 2, for each warehouse w_i during period $k, (k = 0, 1, \dots, H-1)$
L	:Number of ware-houses
Δt	:Length of a production period
$d_{1,i}(k)$	:Average demand of the product 1, for each customer, and during period $k, (k: 1, 2, \dots, H)$
$d_{2,i}(k)$	:Average demand of the product 2, for each customer, and during period $k, (k: 1, 2, \dots, H)$
$V_{di}(k)$	:Variance of demand during period $k, (k: 0, 1, \dots, H)$ for each customer
H	:Number of production periods in the planning horizon
$H.\Delta t$	:Length of the finite planning horizon
c_{p1}	:Unit production cost of product 1 on the machine M
c_{p2}	:Unit production cost of product 2 on the machine M
c_{hs}	:Inventory holding cost of unit product during one period at the principal store

c_{hi} : Inventory holding cost of unit product during one period at the ware-house w_i, $(i = 0,..L)$ all product.
c_i :Unit cost of inspection
c_r :Unit cost of one defective unit
ACQ :Average cost of quality
c_{si} :Cost of Sampling inspection
c_{NCr} :Cost of non-Conformal items
θ_i : Probability index related to each customer I service level satisfaction.
$AOP_{i,j}$:Average number of samples to detect the 'out of control' state for $i = Products = \{1, 2\}$.
A_1 : Number of standard deviations between the center line of the control chart and the control limits P1.
A_2 : Number of standard deviations between the center line of the control chart and the control limits P2.
h :Sampling interval
c_1 :The magnitude of the shift to the 'out of control' state compared to the centerline in the case of P1
c_2 :The magnitude of the shift to the 'out of control' state compared to the centerline in the case of P2
$P(S_{1,1})$:The probability of non-detection of the shift to the control limits for product 1 sample
$P(S_{1,2})$:The probability of non-detection of the shift to the control limits for product 2 sample
$P(S_{2,1})$:The probability to trigger a false alarm for product 1 sample
$P(S_{2,2})$:The probability to trigger a false alarm for product 2 sample
C_{pm} :Cost of preventive maintenance action
C_{cm} : Cost of corrective maintenance action
C_M : Total maintenance cost
mu : Monetary unit.

2.3 Description of the problem

The production machine M linked to the supply chain as shown below in the figure 1, is subject to random failure. The production rates influence the degradation degree of the machine. Consequently, the failure rate $\lambda(t)$ increases with time and production rate $u(k)$ and affects the production process's reliability, resulting in the production of non-conforming units. We assured that the machine is controlled at every h time unit during production period Δt by a quantitative quality statistic X with n measurements for the sample t It is assumed that the individual measures of all the products ($P1$ & $P2$) must all be between $UCL1$ upper control limit product 1 and $LCL1$ lower control limit product 1. Similarly, product 2 must also be between $UCL2$ upper control limit product 2 and $LCL2$ more lower control limit product 2. The measurement results are saved in a measurement card with a risk of the first species. It's assumed that the production process is stable if the law respectively of $X_{1,t}$ and $X_{2,t}$ are Gaussian constant with mean $(\mu_{1,0}, \mu_{2,0})$ and standard deviation σ_1, σ_2 known or well estimated for specific products. The process is under control when all statistics are within control limits: between the lower and upper control limits.

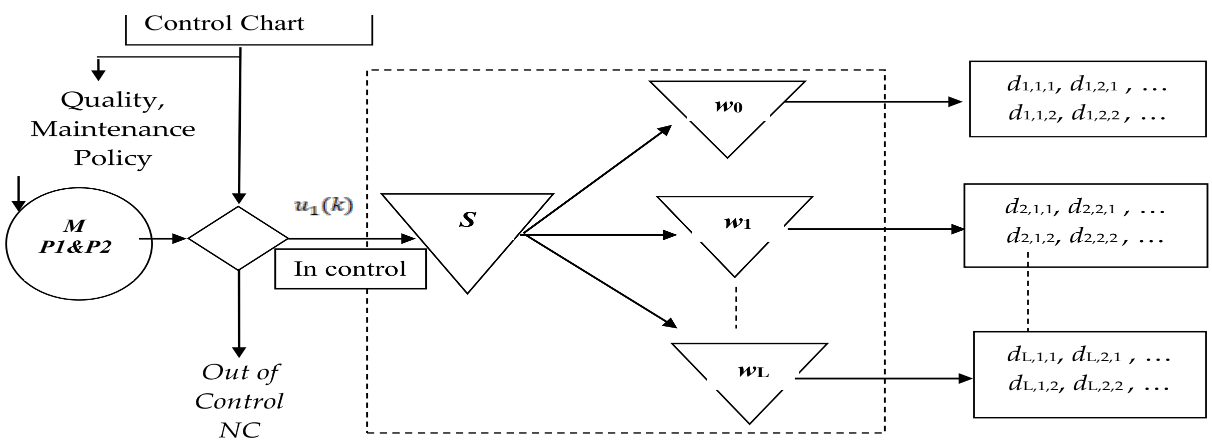

Figure 1. Production System linked to Supply chain management.

The control chart defines four possible scenarios where two scenarios happen in the control state while the other two occur in the out of control state. The control state for product 1 and product 2 are scenario I/1, scenario I/2,

respectively and the out of control states for products 1 and 2 are scenario II/1, scenario II/2, respectively, as clearly shown in Figure 2 below.
A maintenance strategy (PMMR: perfect preventive maintenance actions with minimal repair) is performed during the finite production horizon to reduce the degradation and failure rate that depends on the machine's production rate variation. Perfect Preventive maintenance actions negligible durations periodically planned $T = \alpha.\Delta t = \alpha.\beta.h$ $(\alpha\ nd\ \beta\ are\ integer\ numbers)$ and a minimal repair for each failure between two successive preventive maintenances during the production horizon when the production process is "under control" state between two preventive maintenance actions (Scenario 1). in the case where any of the products production process is "out of control" (Scenario 2) between the j^{th} and $(j+1)^{th}$ sampling due to occurrence of an assignable cause, and consequently the sample average of the quality indicator measurements located beyond the control limits.

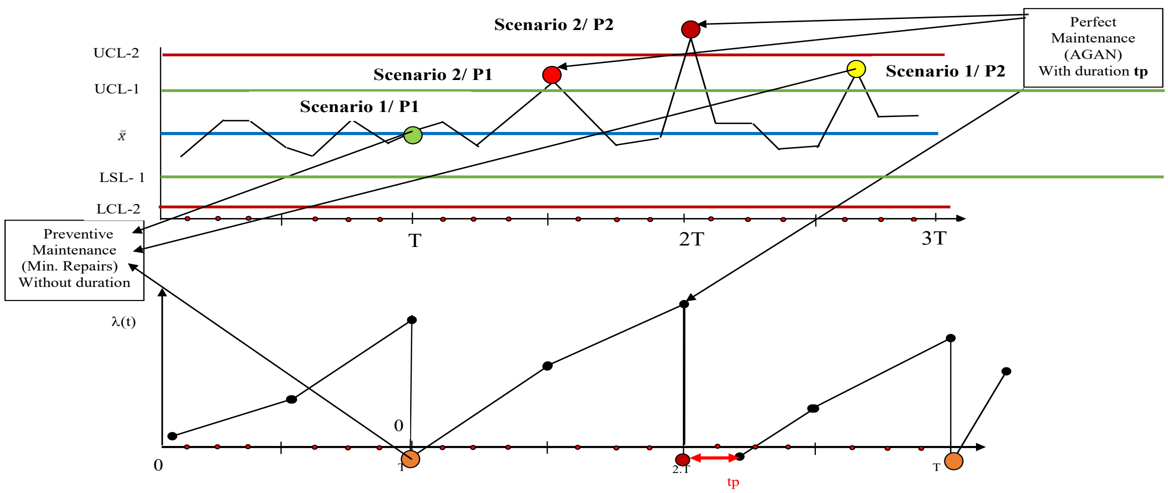

Figure 2. Classified Integrated quality control and maintenance strategy

The control limit for product 1 is $(LCL^1 \geq \bar{X}_{1,t} \geq UCL^1)$, and product 2 is $\left(LCL^2 \geq \bar{X}_{2,t} \geq UCL^2\right)$.

Where $LCL^1 = \mu_{1,0} - \frac{A_1}{\sqrt{n_1}} \times \sigma_1$ (1)

And $UCL^1 = \mu_{1,0} + \frac{A_1}{\sqrt{n_1}} \times \sigma_1$ (2)

Similarly $LCL^2 = \mu_{2,0} - \frac{A_2}{\sqrt{n_2}} \times \sigma_2$ (3)

And $UCL^2 = \mu_{2,0} + \frac{A_2}{\sqrt{n_2}} \times \sigma_2$ (4)

The mean value of the quality characteristic may change from $\mu_{1,0}\ to\ \mu_1$ or $\mu_{2,0}\ to\ \mu_2$ the production has to stop. Perfect maintenance action to restore the machine to as good as the new state with a duration (t_p) is applied until the end of the production period when the latest plans of the preventive maintenance, according to cumulative failure rate is re-launch again from the end of this period as indicated in Figure 2 above. Our main goal is the sustainability of the production system through reliability enhancement, the satisfaction of classified customers with various product and quality demands while ensuring profitability in the present-day dynamic global market forces. We minimize the sum the all products production cost, the total inventory holding costs at all the principal stores, and the multi-warehouses, the delivery costs, and the costs associated with the maintenance strategy, sampling inspection, and non-conformal products lost cost to carry out this collaborative production, delivery, quality and maintenance optimization

3. Methods

The objective of this study is to determine firstly the optimal plan of production to satisfy the varied demand of different customers under a given service and respective quality level. Secondly, according to the economic goal of production, the optimal parameters of the control chart are the sample size (n_1, n_2), the control interval (h_1, h_2), the control limits coefficient (A_1, A_2) and the optimal maintenance strategy characterized by the optimal number of preventive maintenance actions N^*. It's assumed that the horizon is proportioned equally into H periods with a length equal to Δt. Moreover, we assume that demand fluctuation is normal with mean $\mu_{di,j}(k)$ and variance $V_{di,j}(k)$, which are known, respectively. The demands are satisfied at the end of each period.

3.1 Assumptions

The following hypotheses are considered based on some industrial reality and constraints. Concerning the maintenance actions duration, it is not always that the repair and overhaul duration can be regarded as negligible in the industrial fact. We designed that no time is needed for each PMMR proposed maintenance action in the control state for this study. The plan is only to give us an idea of the necessary tools, resources, and technicians to plan the repair and reduce the intervention durations for a forecasting maintenance planning characterized by the optimal number of preventive maintenance actions and the intervals between two successive preventive maintenances. On the other hand, we consider duration (t_p) for each activity of perfect maintenance corresponds to each out of control point due to assignable causes. This duration (t_p) is necessary to find and avoid with the continuous improvement tools (AMDEC, DMAIC, ishikawa etc) to dealt with all the assignable causes that can disrupt the process.

Considering the following hypotheses:

1. The random customer demand follows the normal law, and every request unsatisfied in a period causes a delayed penalty.
2. Each delivery time τ_i from S to each warehouse w_i is constants, same for all products (P1 & P2) and is multiple of Δt.
3. The Products (1 & 2) minimal and maximal production rates $(u^1{}_{min}, u^1{}_{max})$ and $(u^2{}_{min}, u^2{}_{max})$ are known and constant.
4. The repair and preventive activity duration are negligible for the maintenance strategy performed when the production process is under control.
5. The times of perfect maintenance performed for each failure caused by the quality indicator which happened to be located out of control is t_p*, and a new maintenance cycle is started again based on the cumulative breakdowns.
6. The non-conformal products are lost.
7. Defective products are due to the process degradation of the machine and the other assignable causes.
8. all unit costs are constant and known.
9. All resources to carry out maintenance activities are still available.
10. The preventive maintenance activities are perfect and restore the equipment to "as good as new" condition.

3.2 Control Chart

The control chart can measure the effectiveness of a control card by using the probability of not detecting an adjustment when taking an n sample of size. The card's significance is inversely proportional to the possibility, implying that all the better when this probability is low and converse. When the process does not work correctly, the average can vary, and it takes as value. We note the expression of the adjustment of the mean in a number of standard deviations:

$$\delta_{i,j} = \frac{\mu_i - \mu_{i,0}}{\sigma_{i,0}} \quad \text{where i = Products = } \{1,2\} \tag{5}$$

Let $P_{i,j}(c)$ be the probability of not detecting an adjustment of c standard deviations when taking a sample of c pieces.

$$P_{i,j}(\delta_{i,j}) = F(-\delta_{i,j}.\sqrt{n_i} + A_i) - F(-\delta_j.\sqrt{n_i} - A_i), \quad j = \{1,2,\dots,\}, \text{ i} = \{1,2\} \tag{6}$$

With F: distribution function of the reduced centered normal law.

Hence, the average operational period (AOP) characterized the average number of successive samples leading to the first point out of range, for a given adjustment and on any given product i, i = $\{1,2\}$.

$$AOP_{i,j} = \frac{1}{1-P_{i,j}(\delta_{i,j})} = \frac{1}{1-F(-\delta_{i,j}.\sqrt{n_i}+A_i)-F(-\delta_{i,j}.\sqrt{n_i}-A_i)} \tag{7}$$

3.3. Production, inventory, and delay penalty costs formulation.

Our goal is to simultaneously establish the optimum production rates, the optimal inventory quantities based on the demand and the quality control chart's optimal parameters, which minimizes the total cost. The total expected cost is including the production cost, the inventory cost, the delay penalties, the quality cost (sampling inspection, false alarm & non-conformal products) and the total maintenance cost (preventive, minimal repairs and corrective maintenance).

3.3.1 Production Cost

The production cost at period k is equal to the sum of production with respect to all products

$$PC_T = \sum_{k=1}^{H}\{C_{p_1} \times u_1(k) + C_{p_2} \times u_2(k)\}.\Delta t \tag{8}$$

3.3.2 Holding cost at the main production Store (S)

Each unit product put in stock anywhere and at any time will generates a unit cost of holding it, therefore to minimized the total cost, the need to carefully plan for an optimal stock quantity according to the inventory management system to satisfy customer demand at some given service requirements. The inventory level balance equation characterizes the progress of the principle inventory for periods $(k = 1, 2 \dots, H)$, which is defined by:

$$S(k) = S(k-1) + \big(u_1(k) + u_2(k)\big).\Delta t - \sum_{i=1}^{L}\sum_{p=1}^{2} Q_{p,i}(k-\tau_i), \text{ with } k = \{0, 1 \dots, H\} \tag{9}$$

$Z_S(k)$ is the area generated by the evolution of the inventory level at period k, $(k = 1, 2, \dots, H)$

$$Z_S(k) = Max\{S(k-1), 0\}.\Delta t + \frac{1}{2}.\big(u_1(k) + u_2(k)\big).\Delta t^2 \tag{10}$$

Consequently, the holding cost is expressed by:

$$HC_S = \sum_{k=1}^{H} C_h \times Z_S(k) \tag{11}$$

Where c_h = The unit cost of holding inventory.

3.3.3 Holding cost at the retails warehouses ($\mathbf{w_i}$)

The inventory holding cost for the product at all the warehouses is given by the following expression;

$$HC_W = \sum_{k=1}^{H} C_h \times Z_{w_i}(k) \tag{12}$$

$Z_{w_i}(k)$ is the area generated by the inventory level evolution during the period k, $(k = 1, 2, \dots, H)$

The inventory level of each warehouse inventory w_i at the period k is given by the following equation:

$$w_i(k) = w_i(k-1) + \sum_{p=1}^{2} Q_{p,i}(k-\tau_i) - \sum_{p=1}^{2} d_{p,i}(k) \tag{13}$$

The inventory level of each product, in any warehouse w_i at any given period k equals to the inventory level of w_i at period $(k-1)$ plus the quantity of the sum of all products that arrives to w_i (i.e. $Q_{1,i}(k-\tau_i)$ $+Q_{2,i}(k-\tau_i)$) minus all the customer demands $d_{1,i}$ & $d_{2,i}$ at period k.

The generated area of the inventory level evolution for each warehouse (w_i) during any given period k is given as follows:

$$Z_{w_i}(k) = Max\{w_i(k-1), 0\}.\Delta t + \frac{1}{2}(Q_{1,i}(k-\tau_i) + Q_{2,i}(k-\tau_i)).\Delta^2 \tag{14}$$

Consequently,

3.3.4 Total Inventory Holding cost (HC)

This consists of the inventory holding cost at the main production store (S), and all the costs of inventories from the retail multiple warehouses (w_i {i = 1, ... L}) expressed as follows:

$$HC_T = C_h \times \sum_{k=1}^{H}\begin{pmatrix} Max\{S(k-1), 0\}.\Delta t + \frac{1}{2}.\big(u_1(k) + u_2(k)\big).\Delta t^2 \\ Max\{w_i(k-1), 0\}.\Delta t + \frac{1}{2}(Q_{1,i}(k-\tau_i) + Q_{2,i}(k-\tau_i)).\Delta^2 \end{pmatrix} + \tag{15}$$

3.3.5 The service level

The service level requirement constraint for each warehouse at each period k is expressed by the following constraint. For (k= 1, …, H-1) and (i=1, …, L)

$$\text{Pro } [w_i(k) \geq 0] \geq \theta_i \tag{16}$$

3.3.6 Production bounds

The following constraint defines an upper and lower bounds on the production levels of all the products, during each period k.

$$u^1_{min} \le u_1(k) \le u^1_{max} \tag{17}$$

Also

$$u^2_{min} \le u_2(k) \le u^2_{max} \tag{18}$$

3.3.7 Total Production Cost

The production cost of all the products and for any period k

$$PC_T = \sum_{k=1}^{H} \{C_{p_1} \times u_1(k) + C_{p_2} \times u_2(k)\}.\Delta t \tag{19}$$

3.3.8 Delay Penalties Cost

The delay penalties are characterized by the consequence of a delay to satisfy all the demands. If a delay situation occurred at the end of period k caused a shortage recovered during the next period (k+1):

The penalties are determined as a function of the required duration dw(.) to produce the missed quantity at the end of each period, given by the following expression:

$$PC_T = (C_{p_1} + C_{p_2}) \times (\sum_{k=1}^{H}(\sum_{i=1}^{L} dw_i)), \qquad dw_i = \frac{|\min(w_i(k),0)|}{Q(k+1-\tau_i)} \tag{20}$$

3.4. Maintenance Policy

Maintenance cost consists of the false preventive and corrective maintenance costs, and also depends on the scenario "scenario 1: in-control" and "scenario 2: out-control" that happens. The resolution of maintenance planning problem consists of minimizing costs related to preventive and corrective maintenance. The maintenance strategy considered in this work is preventive maintenance with minimal repair in the 'in-control' state of the system. Preventive actions scheduled over the finite horizon H which is divided equally into N parts of duration T. We suppose that performing a preventive action corresponds to times $\mathrm{i.T}$ $(\mathrm{I} = 1, 2, .\mathrm{N})$ consists to restore the machine to an as good as new condition. However, between successive preventive interventions, breakdowns may happen, the minimal repair is performed. It is assumed that the repair and overhaul durations are negligible.

Essentially the status of the machine depends on its average failure rate influenced by the total production rates of the multi-products and delivery plan variation. Whereas the maintenance strategy is characterized by the optimal number of preventive maintenance actions N* and the most adequate spacing between them noted T*.

$$CM_T = cm_1 \times \left(P(S_{1,1}) * P(S_{1,2})\right) + cm_2 \times \left(2 - P(S_{1,1}) - P(S_{1,2})\right) \tag{21}$$

$$P(S_{1,1}) = Prob(LCL^1 \ \ge \bar{X}_t \ge UCL^1) = F\left(\mu_{1,0} + \frac{A_1}{\sqrt{n_1}} \times \sigma_1\right) - F\left(\mu_{1,0} + \frac{A_1}{\sqrt{n_1}} \times \sigma_1\right) \tag{22}$$

$$P(S_{1,2}) = Prob(LCL^2 \ \ge \bar{X}_t \ge UCL^2) = F\left(\mu_{2,0} + \frac{A_2}{\sqrt{n_2}} \times \sigma_2\right) - F\left(\mu_{2,0} + \frac{A_2}{\sqrt{n_2}} \times \sigma_2\right) \tag{23}$$

$$cm_1 = C_{pm} \times \left\lfloor \frac{H}{T} \right\rfloor + C_{cm} \times \varphi_M(U,N) \tag{24}$$

Where $\varphi_M(U,N)$ is the average number of failures as a function of the production plan defined by the vector U and the number of preventive maintenance actions N^*, note also $\frac{H}{T} = N$.

$$\varphi_M(U,N) = \sum_{j=0}^{N-1}\left[\sum_{k=In\left(j\times\frac{N}{\Delta t}\right)+1}^{In\left((j+1)\times\frac{N}{\Delta t}\right)\times\Delta t} \int_0^{\Delta t} \lambda_k(t)dt + \right.$$

$$\left. \int_1^{(j+1)\times N - In\left((j+1)\times\frac{N}{\Delta t}\right)\times\Delta t} \lambda_{In\left(j+1)\times\frac{N}{\Delta t}\right)+1}(t) + \int_{(j+1)\times N}^{\left(In\left((j+1)\times\frac{N}{\Delta t}\right)+1\right)\times\Delta t} \frac{\left(\left(In\left(j+1)\times\frac{N}{\Delta t}\right)+1\right)\right)}{\left(\frac{u_1(k)+u_2(k)}{u^1{}_{\max+2max}}\right)} \times \lambda_n(t)dt\right] \tag{25}$$

With $\lambda(t)$ represents the linear failure rate function at production period k expressed as follows:

$$\lambda_k(t+1) = \lambda_k(\Delta t) + \left(\frac{u_1(k)+u_2(k)}{u^1{}_{\max+2max}}\right).\lambda_n(t)dt \qquad \forall t \in [0, \Delta t] \tag{26}$$

$\lambda_n(t)$: failure rate for nominal conditions (maximal production during the all horizon H.Δt).

$$cm_2 = \sum_{j=0}^{M}\left(C_{pm} \times \left\lfloor\frac{AOP_{j+1}-AOP_j}{N_j}\right\rfloor + C_{cm} \times \varphi_{Mj}(U,N_j)\right) + C_{pm} \times \left\lfloor\frac{H-AOP_M}{N_j}\right\rfloor + C_{cm} \times \varphi_M(U,N_M) \tag{27}$$

M: number of detection of the out of control

$$\varphi_{Mj}(U,N_j) = \sum_{z=0}^{N_j}\sum_{k=\left\lfloor\frac{AOP_j}{N_j}\right\rfloor+\left\lfloor z\times\frac{N_j}{\Delta t}\right\rfloor+1}^{\left\lfloor\frac{AOP_j}{N_j}\right\rfloor+\left\lfloor(z+1)\times\frac{N_j}{\Delta t}\right\rfloor} \int_0^{\Delta t}\lambda_k(t)dt \qquad N_j = \left\lfloor\frac{AOP_{j+1}-AOP_j}{N_j}\right\rfloor \tag{28}$$

And

$$\varphi_M(U,N_M) = \sum_{z=0}^{N_M}\sum_{k=\left\lfloor\frac{AOP_M}{N_M}\right\rfloor+\left\lfloor z\times\frac{N_M}{\Delta t}\right\rfloor+1}^{\left\lfloor\frac{AOP_M}{N_M}\right\rfloor+\left\lfloor(z+1)\times\frac{N_M}{\Delta t}\right\rfloor} \int_0^{\Delta t}\lambda_k(t)dt \qquad N_M = \left\lfloor\frac{H-AOP_M}{N_M}\right\rfloor \tag{29}$$

3.5. The average total quality cost

The average total cost of quality corresponds to the sum of the sampling inspection cost and the cost of rejection of non-conforming Products.
The expression for the average total cost of sampling is:

$$C_{Si} = \sum_{j=0}^{M}\left(\left(C_{1,i} \times n_{1,i} \times (AOP_{1,(j+1)} - AOP_{1,j})\right) + \left(\left(C_{2,i} \times n_{2,i} \times (AOP_{2,(j+1)} - AOP_{2,j})\right)\right)\right) \tag{30}$$

While the average total cost of rejection (c_{NCr}) products is corresponding to the production non-conforming quantity during the out of control period expressed by:

$$c_{NCr} = \sum_{j=0}^{M}\left(\left(C_{1,r} \times u_1 \times \left(\left\lfloor\frac{AOP_{1,j}}{\Delta t}\right\rfloor\right) \times \left(\frac{AOP_{1,j}}{\Delta t} - h\right)\right) + \left(C_{2,r} \times u_2 \times \left(\left\lfloor\frac{AOP_{2,j}}{\Delta t}\right\rfloor\right) \times \left(\frac{AOP_{2,j}}{\Delta t} - h\right)\right)\right) \tag{31}$$

Subsequently, the average total cost of quality ACQ is given by:

$$ACQ = \sum_{j=0}^{M}\Bigg(\left(C_{1,i} \times n_{1,i} \times (AOP_{1,(j+1)} - AOP_{1,j})\right) + \left(\left(C_{2,i} \times n_{2,i} \times (AOP_{2,(j+1)} - AOP_{2,j})\right)\right) +$$
$$\left(C_{1,r} \times u_1 \times \left(\left\lfloor\frac{AOP_{1,j}}{\Delta t}\right\rfloor\right) \times \left(\frac{AOP_{1,j}}{\Delta t} - h\right)\right) + \left(C_{2,r} \times u_2 \times \left(\left\lfloor\frac{AOP_{2,j}}{\Delta t}\right\rfloor\right) \times \left(\frac{AOP_{2,j}}{\Delta t} - h\right)\right)\Bigg) \tag{32}$$

Where; $AOP_{1,0} = 0,\ AOP_{2,0} = 0.$

4. Data Collection

A supply chain composed of a production system that produces two type of product to meet the delivery for two ware house (*L*=2) to satisfy random varied demands over a finite planning horizon: *H*=12 months each of period length Δ*t*=1 month. Assumed that the standard deviation of each demand of a product is the same for all periods and each demand $\sigma_{di(\{i:1,2\})}$ *=1.2* and 1.5 for product 1 and 2 respectively. We the initial inventory level assume that *S* (0) = 0. The average demand for product 1, customers of warehouse 1 and warehouse 2: $d_{1,1}(k)= d_{1,2}(k)$=150 while the average demand for product 2, customers of warehouse 1 and warehouse 2: $d_{2,1}(k)= d_{2,2}(k)$ =15, {*k*: 0, …, *H*-1}.
- Lower and upper boundaries of product 1 production capacities: $u^1{}_{min} = 0$ *and* $u^1{}_{max} = 500$
- Lower and upper boundaries of product 2 production capacities: $u^2{}_{min} = 0$ *and* $u^2{}_{max} = 25$.
- $C_{p,1}$ *=2 mu*, $C_{p,2}$ *=5 mu*, $S_{i,1}(0) = 0$, $S_{i,2}(0) = 0$, C_{hs}*=0.2 mu/k,*
- $w_{i,1}(0) = 10$, $w_{i,2}(0) = 10$, Cw_i*=0.2 mu/k* {i:1,….*L*=2}.
- $\mu_{0,1} = 5$, $\sigma_1 = 1.2$, $\mu_{0,2} = 6$, $\sigma_2 = 1.5$ and $\delta_1 = 0.4$, $\delta_2 = 0.8$
- *Ci* = 15 *mu* /product, *Cr* = 70 *mu* /defective item.
- customers satisfaction degree, is equal to 95% (θ_i=0.95 (*i*=1, 2)).
- The degradation law of the production system characterized by a Weibull distribution with scale and shape parameters are respectively $\beta_1=100$ and $\alpha_1=2$.
- The costs of corrective and preventive maintenance actions are respectively Ccm =75000 *mu* and Cpm=500 *mu*

The characterized varied demand, optimal production of products (P1 & P2) and delivery plans for each warehouse 1 and 2 according to $(Q_{1,i}, Q_{2,i})$ are given in Table 1, and Table 2 respectively:

Table 1. Characterized Varied demand

Period (k)	$d_{1,1}$	$d_{1,2}$	$d_{2,1}$	$d_{2,2}$
1	150	160	15	16
2	170	130	17	13
3	150	150	15	15
4	150	150	15	15
5	150	140	15	14
6	140	160	14	16
7	160	160	16	16
8	140	160	14	16
9	160	140	16	14
10	150	150	15	15
11	150	150	15	15
12	150	140	15	14

5. Results and Discussion.

To validates the proposed integrated control of production, inventory, quality and maintenance approach for the developed multi-product production system, the above described numerical example was experimented, and a sensitivity analysis conducted. Below are relevant numerical, graphical results represented in Tables 2 and Figure 3.

5.1 Numerical Results

Table 2. Optimal production and delivery Quantities plan.

K	$u_{1,1}$	$u_{1,2}$	$u_{2,1}$	$u_{2,2}$	$Q_{1,1}$	$Q_{1,2}$	$Q_{2,1}$	$Q_{2,2}$
1	292	292	10	10	17	133	3	1
2	288	288	16	16	393	302	0	23
3	206	206	24	24	50	356	15	13
4	265	265	6	6	220	26	20	19
5	185	185	8	8	328	278	11	7
6	420	420	15	15	137	354	21	3
7	289	289	18	18	260	44	19	1
8	60	60	8	8	32	39	4	21
9	359	359	5	5	16	251	3	2
10	44	44	13	13	31	270	11	12
11	134	134	17	17	356	30	22	4
12	50	50	11	11	340	401	15	2

5.2 Graphical Results

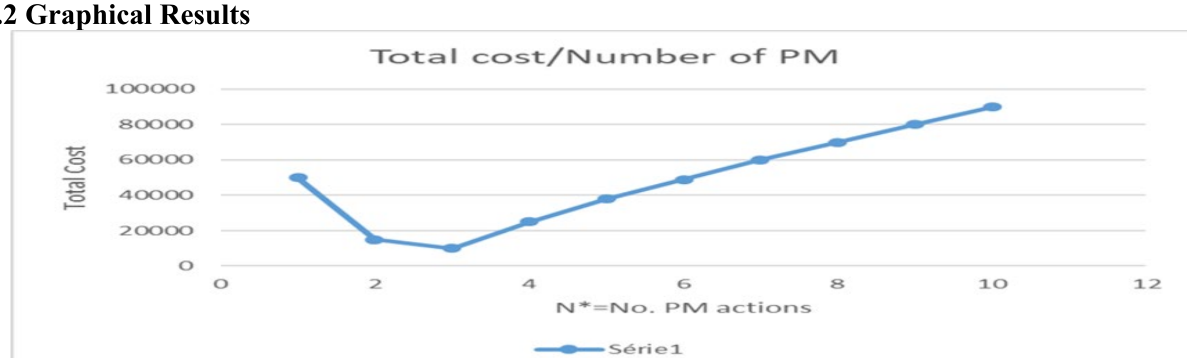

Figure 3. Optimal Number of maintenance action.

5.3 Validation

Hence, the obtained strategy consists of taking one sample size ($n_1 = 60$) of product 1 every 96 hours = 4 days (h_1*). Furthermore, according to the control chart parameters associated with product 1, the optimal number of standard deviations between the center line and the control limits is 3.5 (A_1*). similarly for product 2 by taking one sample size ($n_2 = 4$) of product 2 every 48 hours = 2 days (h_2*). Furthermore, according to the control chart parameters associated with product 2, the optimal number of standard deviations between the center line and the control limits is 2.5 (A_2*). The first shift of the product 1 to the first 'out of control' state occurs on average operational period $AOP_1 = 4.\Delta t = 24$ samples, while that of the Product 2 happens much later due to its wide tolerance limits. The cumulative effects of the total production strategy on the single machine, as indicated in the Figure 3 above, requires that 3PM actions are carried out every T = 4. Δt = 16.*h* and 3T = 12.Δt = 48.*h*.

6. Conclusion

This paper studied a joint approach of production, maintenance, delivery, and quality problem of a single machine multi-product production system, satisfying optimally several varied, random demands as presented in the Table 1, considering the impacts of the production rate, inventory quantities on the machine degradation. Based on the collaborative production plan, inventory management, shown in Table 2, and maintenance strategy (PMMR) formulations. We developed a mathematical model which determines the optimal number of PM actions that minimize the total cost of production, inventory, maintenance, and quality by obtaining the different decision variables characterized by the sample size, the sampling interval, the control limits coefficients and the PM period. We established a good combination between the degradation of the machine depending on production rates, the impact of inventory delivery quantities, and the control chart parameters to reduce the non-conforming items and guarantee the production system's reliability.

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Biography

Aminu Sahabi Abubakar, R.Eng. is currently a PhD student of Industrial Engineering. He is developing new maintenance strategies for a multi-product production system under operational and quality constraints at the LGIPM laboratory, UFR MIM University of Lorraine-France. With varied working experience, he served as a maintenance study and methods superintendent Peugeot Automobile LTD Nigeria, Operation & Maintenance Engineer at the Kebbi State water board 2003-2010, Senior Power System Engineer Nigerian Air Space Management Agency 2010-2018, and Lecturer I with the Mechanical Engineering Department, Kebbi State University of Science & Technology Aleiro 2018-2020. He earned a Bachelor of Engineering Degree, M.Sc. Mechanical Engineering from Ahmadu Bello University Zaria-Nigeria, and a Higher National Diploma Production Engineering. In addition to academic qualifications, he belongs to various professional bodies with many certifications; Registered Mechanical Engineer by the Council for the Regulation of Engineering in Nigeria (COREN), corporate member of the Nigerian Society of Engineers, Nigerian Institution of Mechanical Engineers, member of British Project professionals, a fellow to the Nigerian Association of Technologists in Engineering, and Fellow Institute of Management Consultants. He has published scientific articles in rated journals, such as the Journal of Applied Science (MPDI), FUDMA Journal of Science, FUOYE Journal of Engineering and Technology, and many international conferences.

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