Imperfect Preventive Maintenance Planning Optimization for Leased Machines

Zied Hajej, Mohamed Ali Kammoun and Nidhal Rezg
Laboratoire de Génie Informatique, de Production et de Maintenance
(LGIPM), Metz, France
{mohamed-ali.kammoun, zied.hajej, nidhal.rezg}@univ-lorraine.fr

Abstract

We aim in this paper to investigate a preventive maintenance scheduling with imperfect repair for a manufacturing system. The considered system is composed of several parallel leased machines. It has to satisfy a random demand over a finite horizon. Machines are subject to failures that are strongly correlated to the production rates of each machine. The objective is to study the impact of imperfect preventive maintenance on maintenance plan for a leased system characterized by a variable number of machines. We propose hereinafter a model which allows minimizing the total maintenance cost by obtaining the optimal number of imperfect maintenances for each machine. A numerical example is given in order to validate the robustness of the developed approach.

Keywords
Imperfect maintenance, Leased machines, Random demand, Equipment degradation, Maintenance cost.

1. Introduction

Production policy and maintenance strategy of manufacturing systems are subjects to different uncertainties such as failures, variability in machines availability, random customers' demand, etc. In order to face these uncertainties, the firms are oriented to policies of collaboration and cooperation with other companies. In this context, leasing industrial equipment rather than purchasing it is considered as one of the best solution in term of cost. Pongpech et al. (2006) show the importance of the leasing and presented that the cause of equipment leasing is the increase of the cost of purchase of a new one.

Concerning maintenance while leasing equipment, Chang et al. (2009) considered a maintenance policy and a mathematical model for the expected total profit. At failure, minimal repair brings the equipment back to the operating state. Imperfect preventive maintenances (PM) are carried out to avoid possible failures when the age of the equipment reaches a certain threshold value. A numerical example was given in order to illustrate the impact of the number of lease periods and the maintenance policy on the expected total cost. Chang et al. (2011) proposed a mathematical model and a numerical example in order to obtain the optimal maintenance schedule and the length of the lease period by illustrating the effects of the optimal lease period length on the maintenance plan.

Shalaby et al. (2004) developed a new optimization problem for multi-component and multi-state systems by proposing an optimization model for preventive maintenance scheduling. Rezg et al. (2004) proposed a jointly stock and maintenance optimization in a production system of numerous machines. Cui et al. (2014) presented jointly optimized the production and the maintenance plans for a manufacturing system made of a single unit taking into account the quality and the solution robustness. A three-phase heuristic approach is used to solve the problem. Fakher et al. (2015) considered a lot-sizing solving problem with the preventive maintenance scheduling taking into account the quality of the equipment. A genetic algorithm was used in order to determine the production and maintenance schedules for a system made of M fixed machines. Yalaoui et al. (2014) studied the case of a production system which has a decreasing capacity in order to determine its production and maintenance plans. Zied et al. (2014) studied a single machine system producing multi-products and subject to random failures. The total production and maintenance costs were minimized taking into account several parameters such as inventory, backorder, set-up, preventive and corrective maintenance.

In the same context, the study of Feng and Yan (2000) considered the variability of demand, the production capacity and processing time in a discrete-state version of Kimemia and Gershwin (1983). A production system composed by a serial machines with random demand is introduced in the work of Song and Sun (1998). These works are extended by taking into account the maintenance actions optimization (Nguyen and Murthy, 1981; Panagiotidou and Tagaras, 2007; Park et al., 2018; Wang et al., 2019). Kibouka et al. (2018) introduced the setup times and costs in the optimization model. In addition, Kenné et al. (2012) concentrated on supplier reliability in the supply chain. Gouiaa et al. (2018) discussed the integrated model of economic production, obtained quantity and maintenance policy. Recently, Kammoun et al. (2020) proposed new approach to deal with dynamic lot-sizing and maintenance problem under aspect energetic. They introduced a novel maintenance strategy to determine a common preventive maintenance plan for several machines to minimize the total maintenance cost. Also, the authors proposed a risk analysis to unforeseen disruption of availability machines with the aim of helping the production stakeholders to achieve the obtained forecasting lot-size plan.

In our work, based on the works of Medhioub et al. (2014) and Zied et al. (2015) we propose maintenance strategy integrated to production policy by considering the impact the variation of the production rates and imperfect preventive maintenance on the degradation degree of each machine and consequently the influence on the average number of failure and the optimal maintenance strategy. The contribution of this paper is that the decision about performing a PM activity is related to the average number of failures of the machine contrarily to previous works which considered in most cases the reliability level as a threshold. In this work we aim to determine the optimal production and maintenance plans characterized by the optimal combination of number of leased machines, the quantity of products and the PMs number for each machine and that by minimizing the total costs.

This paper is organized as follow: We describe in Section 2 the problem and its industrial context. An analytical study of the used policy and its deterministic equivalent problem are given in Section 3. Section 4 is devoted to a numerical example. A conclusion is proposed in section 5.

2. Problem description and industrial context

2.1 Industrial context

The production system under study is inspired by an existing industrial case of a manufacturing firm specialized in producing steel parts. The policy of this firm consists in leasing machines in order to produce specific types of products ordered by customers. The firm receives a random demand of one type of product which fluctuates according to a normal distribution and that must be satisfied. The number of machines to lease is strongly dependent to the customer's demand and may vary from a production period to another. Each machine can perform $X_{\max}^{N}$ hours in regular time, and $X_{\max}^{S}$ hours in overtime. The following figure illustrates the studied production system.

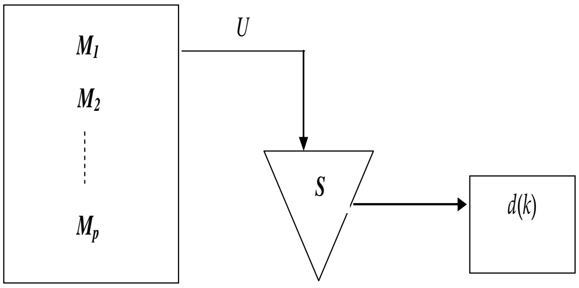

Figure 1. Production system description

The objective of this work is to determine the economical production plan characterized by the optimal number of machines for each production period as well as the production quantities to be produced per period and the then to determine the optimal maintenance strategy characterized by the optimal number of preventive maintenance to be achieved on the machines.

2.2 Notations

We use the following notations in order to describe our model

H: time horizon;
Δk: length of each production period; d_k : random demand for each period k
S_k : stock level at the end of a period k ; u: production rate of machines
p_{ik} : quantity produced by machine i ($i = 1,\ldots, M_k$) in period k ($k = 1, 2, \ldots$, H);
$\lambda_{ik}(t)$: failure rate of a machine i during a production period k;
A_i : average number of failures;

Decision Variables
M_k: leased machines' number during k (k= 1, 2, …, H);
X_{ik}^N: operating time that a machine i ($i = 1,\ldots, M_k$) perform during period k as regular time ($k = 1, 2, \ldots, H$);
X_{ik}^S: operating time that a machine i ($i = 1,\ldots, M_k$) perform during period k as overtime ($k = 1, 2, \ldots, H$);
N_i: number of imperfect preventive maintenance actions during time horizon;

Production and maintenance costs
c_1 : cost of leasing a single unit; c_2 : fixed costs of leasing (eg: administration costs);
c_3 : adding/removal's cost of a machine from period *k-1* to period *k;*
c_4 : constant of asymmetry in costs (HMMS); c_o : Operating cost per overtime hour;
c_n : Cost of unused regular time hour; c_h : holding cost;
$\Gamma(t)$: total maintenance cost; M_p : single preventive maintenance cost;
M_c : single corrective maintenance cost;

3. Production policy

In order to easy the problem's resolution, we aim in this section at developing an analytical study and establishing a deterministic equivalent formulation of our problem.

The equivalent deterministic problem is characterized by maintaining the same properties of the original problem and setting some variables to their means and variances. We introduce at first the following notations used to describe the deterministic problem:

$$E\{S_{i,k}\} = \hat{S}_{i,k} \ ; \ E\{M_k\} = M_k \ ; \ E\{X_{ik}^S\} = X_{ik}^S \ ; E\{X_{ik}^N\} = X_{ik}^N$$
$$V_{P_k} = 0;\ V_{M_k} = 0;\ V_{X_{ik}^N} = 0;\ V_{X_{ik}^S} = 0$$

With $E\{x\}$ is the mean of x and $V\{x\}$ is the variance.

Making the required transformations, we can obtain the following simplified cost

$$\underset{\left(M_k, X_{ik}^N, X_{ik}^S\right)}{Min} \left(\begin{array}{l} \sum_{k=1}^{H} \left\{ \begin{array}{l} \left(c_1 \cdot M_k + c_2\right) + \left(c_3 \cdot \left(M_k - M_{k-1} - c_4\right)^2\right) \\ + c_o \cdot \theta_k \cdot \Delta t \cdot \sum_{i=1}^{M_k} X_{ik}^S \\ + \left(1-\theta_k\right) \cdot c_n \cdot \Delta t \cdot \sum_{i=1}^{M_k} \max\left[0, \left(X_{\max}^N - X_{ik}^S\right)\right] \\ + c_h \cdot \left(\hat{S}_k - \left(a_1 + a_2 \cdot \hat{d}_k\right)\right)^2 \end{array} \right\} \\ + \left(1+a_2\right)^2 \cdot V_d \cdot \frac{H}{2} \cdot \left(H+1\right) + a_2^2 \cdot V_d \\ \cdot \frac{H}{2} \cdot \left(H-1\right) \end{array} \right) \tag{1}$$

The deterministic transformation of the service level constraint leads [10] to a specific minimum cumulative production quantity which is strongly correlated to the service level requirements. This constraint can be expressed as follows:

$$\Pr ob\left(S_k \geq 0\right) \geq \beta \;\; \Rightarrow \;\; \sqrt{k} \times \left(\sigma_d\right) \times \varphi^{-1}\left(\beta\right) - \hat{S}_{k-1} + \hat{d}_k$$

φ : Cumulative Gaussian distribution function with mean $\hat{d}_k$ and finite variance $V_{d(k)}$.

4. Maintenance policy

The objective of this maintenance strategy is to find the optimal number of preventive maintenance actions by minimizing the total cost of preventive and corrective maintenance actions. Recall that the total maintenance cost during the finite horizon of production is expressed by the following equation:

$$\min_{(N)} CM = Mp.(N_i - 1) + Mc.A_i$$

With :

$$A_{ij} = \sum_{j=1}^{k} \prod_{\alpha=0}^{j-1} b_\alpha \int_0^{T_\alpha} \left(\begin{array}{l} \lambda_{i(j-1)}(t) + \frac{p_{ik} / \Delta k}{u_{\max}} . \lambda_n(t). \\ \left(\sum_{\alpha=0}^{j-1} a_\alpha T_\alpha + t \right) \end{array} \right) dt$$

To determine the optimal number of preventive maintenance actions Ni for each machine, we used the following algorithm by considering the production plan established in the production policy as an inputs variables.

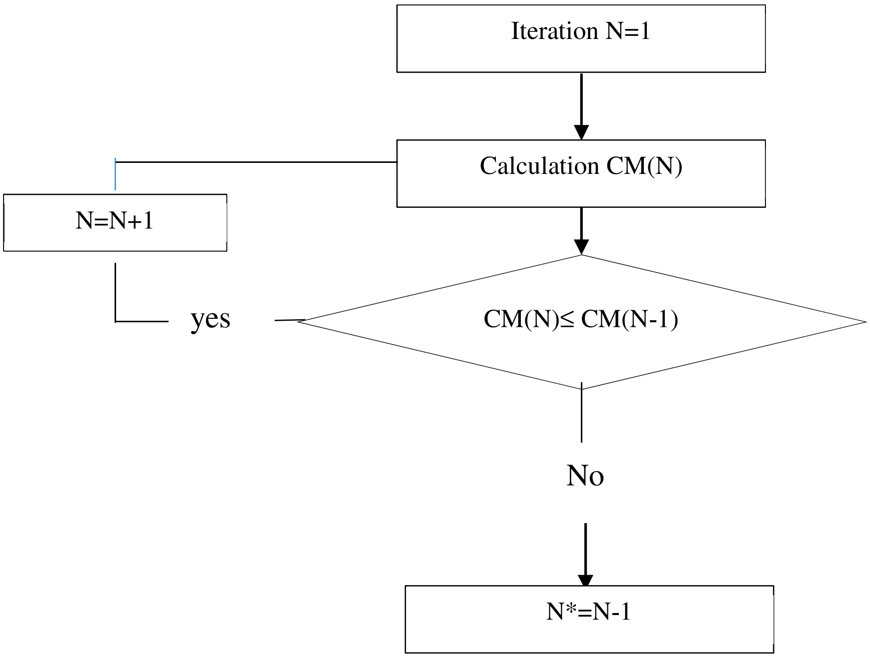

Figure 2. Numerical procedure for maintenance strategy

5. Numerical results

The following arbitrarily chosen input data are considered: $\Delta k = 1$; H=15; c_1=500; c_2=10; c_3=50; c_4=2.5; c_h=15; c_n=20; c_o=30; M_p=100; M_c=5000. Service rate: β=0.95 and initial inventory level: I_0=300. Random Demand: $\sigma_d = 25$ and μ_k^d=1000.

The nominal degradation follows a Weibull distribution given by:

$$\lambda_n(t) = \frac{\gamma}{\mu} \cdot \left(\frac{t}{\mu}\right)^{\gamma-1}$$

The nominal failure rate function corresponds to a nominal processing rate when machines perform at their maximum level.

As inputs we assume that: $\gamma = 2$ and μ =100.

Using all this input data we are now able to find the economic production plan which consists in the optimal number of leased machines and the quantity to produce in each period using a numerical procedure method. The results are exhibited in table 1.

Table 1. Production planning

$\boldsymbol{Period}$	$\boldsymbol{D_k}$	$\boldsymbol{P_k}$	$\boldsymbol{M_k}$
$Period$ 1	962	1000	5
$Period$ 2	1005	840	5
$Period$ 3	1001	920	6
$Period$ 4	1044	1040	6
$Period$ 5	994	1320	5
$Period$ 6	1022	760	5
$Period$ 7	935	920	6

$Period\ 8$	998	1240	5
$Period\ 9$	992	800	5
$Period\ 10$	1004	1000	5
$Period\ 11$	1013	1360	6
$Period\ 12$	1007	1360	5
$Period\ 13$	1010	1200	6
$Period\ 14$	1026	840	6
$Period\ 15$	1005	1240	5
$\boldsymbol{Total\ cost}$	$1{,}014.10^7 m.u$		

For the maintenance plan, we use the results of the production planning as inputs. The maintenance scheduling can be obtained considering M_p=100; M_c=5000; $b_\alpha = \frac{3k+1}{2k+1}$; $a_\alpha = \frac{k}{2k+1}$.

In order to obtain the optimal PM plan, the cost parameters, the Weibull parameters and the adjustment factors must be determined. These parameters can be found using the failure and maintenance database. Concerning adjustment parameters, the users need to observe the failure rate function before and after each PM**.** In our numerical example, we will arbitrarily select these parameters' values and introduce them in our model in order to illustrate the results.

Recall that our contribution for maintenance policy is to establish the maintenance plans for each machine. Also, this study assume that failure rate of machine depends on its produced quantity. So, first, the optimal production plan presented by Table 1 is used as inputs to establish the maintenance plans for each machine, as shown in Table 2. We can conclude that for similar production rates between machines leads to very close maintenance plans, which is explained by the strong correlation between failure rate and production rate. The total maintenance cost of six machines is equal to $2478{,}8\ m.u$.

After computing these parameters, the optimal number of preventive maintenance actions N_i* is obtained by minimizing the total maintenance cost. The maintenance plan is illustrated in the following table.

Table 2. Maintenance planning

$Machine$	N_i	$CM\ (m.u)$
1	5	612.6
2	4	510,2
3	4	455,3
4	4	425,8
5	5	595,2
6	4	492,3

From tables above, we can conclude that the number of preventive maintenance activities for machines are nearly the same. This is due to very similar usage rates for machines which affect their degradation rate. The similarity in usage rate is due to very close customer's demand over the horizon.

6. Conclusion

We presented in this work an integrated preventive maintenance problem for a parallel leased machines system. We studied the impact of the quantity of goods produced on the degradation rate of the machines as well as the influence of imperfect preventive maintenance activities on the maintenance plan. The number of machines and the quantity produced at each production period which are strongly correlated to the random demand were also determined and used as inputs data to determine the maintenance schedule.

In the future, an important extension of this work will be addressed by considering the ecological aspect by the management of the returned products with bad quality. In this context, the returned product can be reused as a raw material in the production process or for remanufacturing, which allows us to optimize consumed energy and reduce the waste that can be created. Another application is possible by extending the proposed model on production system that produces more than one item. In this case, the set-up time of machines and consequently the incurred start-up energy can be considered. In addition, several random requests corresponding to the multi-product produced should be met.

References

Pongpech, J., & Murthy, D. N. P. (2006). Optimal periodic preventive maintenance policy for leased equipment. *Reliability Engineering & System Safety*, *91*(7), 772-777.

Chang, W. L., Yeh, R. H., & Lo, H. C. (2009). Optimal number of leased periods and maintenance policy with threshold value on age for leased equipment. *APIEMS*, 14-16.

Shalaby, M. A., Gomaa, A. H., & Mohib, A. M. (2004). A genetic algorithm for preventive maintenance scheduling in a multiunit multistate system. *Journal of Engineering and Applied Science*, *51*(4), 795-811.

Rezg, N., Xie, X., & Mati, Y. (2004). Joint optimization of preventive maintenance and inventory control in a production line using simulation. *International Journal of Production Research*, *42*(10), 2029-2046.

Cui, W. W., Lu, Z., & Pan, E. (2014). Integrated production scheduling and maintenance policy for robustness in a single machine. *Computers & Operations Research*, *47*, 81-91.

Fakher HB, Nourelfath M, Gendreau M, 2015. Joint production-maintenance planning in an imperfect system with quality degradation, *6th IESM Conference*, October 2015, Seville, Spain

Yalaoui A, Chaabi K, Yalaoui F., 2014. Integrated production planning and preventive maintenance in deteriorating production systems, *Information Sciences*, vol. 278, pp. 841-861.

Zied, H., Sofiene, D., & Nidhal, R. (2014). Joint optimisation of maintenance and production policies with subcontracting and product returns. *Journal of Intelligent Manufacturing*, *25*(3), 589-602.

Zied, H., Nidhal, R., & Tarek, A. (2015, March). A production planning optimization for multi parallel machine under withdrawal right. In *2015 International Conference on Industrial Engineering and Operations Management (IEOM)* (pp. 1-5). IEEE.

Medhioub, F., Hajej, Z., & Rezg, N. (2014, June). A stochastic production planning optimization for multi parallel machine under leasing Contract. In *Probabilistic Safety Assessment & Management conference*.

Chang, W. L., & Lo, H. C. (2011). Joint determination of lease period and preventive maintenance policy for leased equipment with residual value. *Computers & Industrial Engineering*, *61*(3), 489-496.

Feng, Y., & Yan, H. (2000). Optimal production control in a discrete manufacturing system with unreliable machines and random demands. *IEEE Transactions on Automatic Control*, *45*(12), 2280-2296.

Kimemia, J., & Gershwin, S. B. (1983). An algorithm for the computer control of a flexible manufacturing system. *AIIE Transactions*, *15*(4), 353-362.

Song, D. P., & Sun, Y. X. (1998). Optimal service control of a serial production line with unreliable workstations and random demand. *Automatica*, *34*(9), 1047-1060.

Nguyen, D. G., & Murthy, D. N. P. (1981). Optimal preventive maintenance policies for repairable systems. *Operations Research*, *29*(6), 1181-1194.

Panagiotidou, S., & Tagaras, G. (2007). Optimal preventive maintenance for equipment with two quality states and general failure time distributions. *European journal of operational research*, *180*(1), 329-353.

Park, M., Jung, K. M., & Park, D. H. (2018). Optimization of periodic preventive maintenance policy following the expiration of two-dimensional warranty. *Reliability Engineering & System Safety*, *170*, 1-9.

Wang, Y., & Li, L. (2013). Time-of-use based electricity demand response for sustainable manufacturing systems. *Energy*, *63*, 233-244.

Kammoun, M. A., Hajej, Z., & Rezg, N. (2020). Risk analysis for lot-sizing and maintenance optimization problem under energy constraint with subcontractor solution. *International Journal of Quality & Reliability Management*.

Kibouka, G. R., Nganga-Kouya, D., Kenné, J. P., Polotski, V., & Songmene, V. (2018). Maintenance and setup planning in manufacturing systems under uncertainties. *Journal of Quality in Maintenance Engineering*.

Kenné, J. P., Dejax, P., & Gharbi, A. (2012). Production planning of a hybrid manufacturing–remanufacturing system under uncertainty within a closed-loop supply chain. *International Journal of Production Economics*, *135*(1), 81-93.

Gouiaa-Mtibaa, A., Dellagi, S., Achour, Z., & Erray, W. (2018). Integrated Maintenance-Quality policy with rework process under improved imperfect preventive maintenance. *Reliability Engineering & System Safety*, *173*, 1-11.

Biographies

Hajej Zied is an Associate professor (HDR) at the University of Lorraine, Metz platform since September 2012. It operates research and responsible for the **RiAD (Risk Analysis on Decision Making)** team in the laboratory LGIPM Metz and responsible of master for industrial engineering system (ISC-GSI) delocalized in Wroclaw-Poland. After obtaining his doctorate at the University of Paul Verlaine - Metz in 2010, he was employed at the University of Metz as research engineer until August 2012. His main areas of research on the optimization of maintenance policies coupled to production and the development of methods and support the design and control tools in the production systems of goods and services. He is the author of numerous articles in international community of industrial engineering. Her teaching areas include Reliability/Maintenance, modeling and organization of manufacturing and logistics systems, the practice of simulation, automation, and quality system production.

Mohamed Ali Kammoun is a research engineer in the laboratory of computer engineering, production and maintenance (LGIPM) and work at the University of Lorraine since 2009. He obtained his PhD in automatics, signal and image processing, computer engineering in 2013, master in science and technology in 2009 and electrical engineering in 2008. Dr. Kammoun's main research interest is in control synthesis, integrated maintenance in production and data mining integrated in maintenance strategies. Recent applications include air traffic flow management, controller synthesis for industrial systems and hospital supply chain management.

Nidhal Rezg is a Doctor of Industrial Automatic from the National Institute of Applied Sciences (INSA) in Lyon in 1996. Accreditation to supervise research at the University of Metz in 2003. he was Professor at the Faculty of Engineering of the University of Moncton, New Brunswick Canada from 1997 to 1999 and Associate professor at the University of Metz until 2004, and currently holds the position of Professor of University. He is director of LGIPM laboratory since October 2006 and scientific responsable of the INRIA CusTom team from 2007 to 2011. His research interests is the optimization of maintenance policies coupled to production, the optimal control SED. He is the author of sixty papers in international journals, directors of 12 theses and 4 Accreditation to supervise research. Keywords research are modeling, simulation and optimization of stochastic processes, reliability and maintenance and Petri nets.