

# Modeling & Optimization of a Multi-Echelon Multi-Objective Humanitarian Logistics Model Using Branch & Cut Algorithm

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## Abstract

Humanitarian logistics management has recently become a trendy buzzword because of the number of natural disasters in recent years. Managing relief & logistics networks properly when disaster strikes are of great importance, as any negligence may result in a severe penalty such as loss of precious lives & public property. The unpredictable and unexpected nature of natural calamities is another concern. In a catastrophic event, decision-making usually involves choosing the best locations for aid distribution, distributing relief goods to different demand nodes, and managing the appropriate routes for transportation of relief goods, medicines, etc. This study proposes a multi-echelon multi-objective logistics model based on a multi-objective approach to finding optimal locations for setting up regional relief distribution centers with the minimum facility set up cost and to see the optimal quantity of goods to be flowed between nodes and stages of the relief supply chain to minimize distribution and supply costs, and the unmet Demand under uncertain conditions.

## Key Words

Humanitarian logistics, Emergency facility location, Relief distribution, Multi-objective optimization

## 1. Introduction

In humanitarian logistics, the goal is to ensure that supplies are delivered and stored promptly during natural disasters and compound emergencies to affected communities. While most often used in retail supply chains, logistics are essential tools in disaster relief operations. A crucial issue in humanitarian logistics is the kind and quantity of resources, the way of procuring and storing relief goods, the means of transportation to the affected regions, etc. (Zdamar and Ertem, 2015). Performance in humanitarian relief chains is very tough to measure due to some complex characteristics that humanitarian operations have, such as very unpredictable & uncertain nature of Demand, difficulty to obtain data from procedures, unpredictable working environment, the impact of unknown variables, like geography or weather, etc. (Holguín-Veras et al., 2013). This research attempts to measure and optimize the performance of a relief network considering the uncertainty that a disastrous event possesses.

A relief supply chain has three levels required for decision-making: supplier level, regional distribution center level, and affected area level. Very few researchers have considered the decision-making in all three tiers of humanitarian logistics networks, including facility location selection and relief distribution. Most of them took a deterministic approach to supply and Demand to avoid the complexity of model formulation and optimization. In other words, the development of a method that will address all these issues in a single model is still unexplored.

## 2. Literature Review

The humanitarian logistics field focuses on three phases of planning in the disaster lifecycle: preparation, response, and recovery (Özdamar and Ertem, 2015). Recently, the area of emergency supply chain management was reviewed (Jahre et al., 2007) in terms of practitioner contributions for humanitarian logistics, preparation, and reconstruction-related studies. According to Altay et al. (2006), disaster management includes three phases (pre, during & post). Using a multi-commodity, multi-modal network flow model for disaster relief operations, Haghani and Oh (1996) proposed a formulation and solution. Barbarosoglu et al. (2002) proposed a bi-level modeling framework to address routing and transportation issues during the initial response phase of disaster management.

The study by Ozdamar et al. (\*Zdamar, Ekinci, and Küçükyazici, 2004) addressed the distribution of multiple commodities from several supply centers to distribution centers near the affected areas during a disaster. A model was proposed by Yi and Ozdamar (2007) that integrated supply delivery with the evacuation of wounded people during disaster response activities. Researchers Beamon and Balcik (2008) examined the flexibility of humanitarian relief operations, specifically the ability to respond to different types and magnitudes of disasters, the ability to change output levels, the ability to change the variety of products. (To achieve a high level of effectiveness). Henteryck et al. (Van Henteryck, Bent, and Coffrin, 2010) suggested two-part distribution models. In disaster-affected localities, Nolz et al. (2010) developed a comprehensive model for water delivery systems. The model optimizes the physical placement of portable relief water reservoirs, including the route taken to get there in the least Amount of time. Taskin and Lodree (2010) devised a stochastic inventory dilemma for manufacturing and retail corporations challenging economic procurement and production decisions. The subsequent hurricane season demand distribution was predicted using a Markov chain. Victoriano et al. (2009; 2011) conducted a reliability method to analyze uncertainty about the magnitude of infrastructure damage. Rawls and others Rawls and Turnquist (2010) proposed a model for immediate post-disaster response under uncertainty in physical damage caused by the disaster. Similarly, Rawls and Turnquist (2011) treated Demand and infrastructure as stochastic parameters.

### 3. Branch and Cut Algorithm

A Branch and Cut (B&C) algorithm is a Branch and Bound (B&B) algorithm in which a cutting plane algorithm evaluates each sub-problem. It was developed as a fast and robust L.P. solver. But later, it was extended to BIP and MIP problems.

#### 3.1 B&C Algorithm in MIP Problems

While solving a MIP, cuts are classified into local and global. A cut that is generated by a sub-problem evaluation is called *international* if it does not fathom any feasible solutions of MIP, and *local* otherwise. Global cuts are stored in a cut pool. They are used in the subsequent sub-problem evaluation. In summary, a prototype of the B&C algorithm is described in Figure 1, which leaves some implementation flexibility. The algorithm contains items (a) Processing, (b) Node Selection, (c) Cutting Plane, (d) Heuristics, and (e) Variable Selection as described in Figure 1.

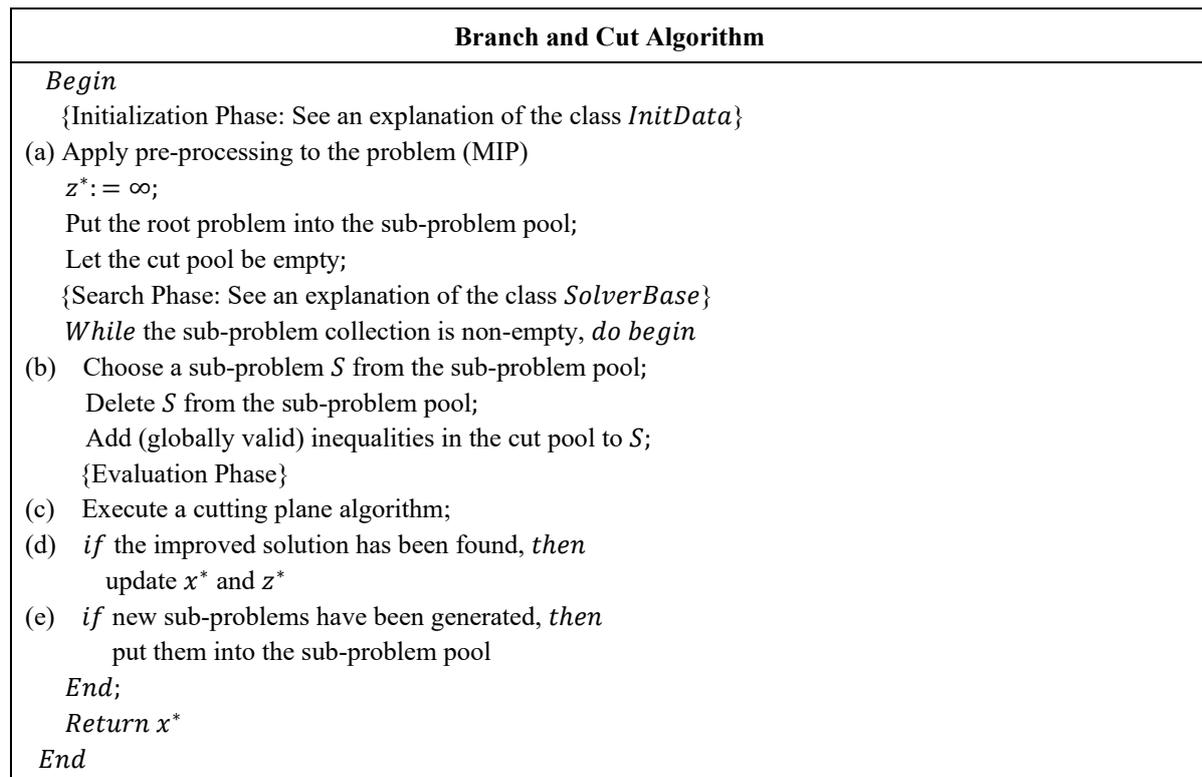


Figure 1. A prototype of the Branch and Cut Algorithm

## 4. Model Development

Natural disasters such as droughts, earthquakes, hurricanes, and floods have proven a global challenge due to their unpredictable nature and potential scale of impact represented by fatalities and social, environmental, and economic havoc. The degree by which a natural hazard causes devastation is quite related to the ability of the authority incumbent to cope with the given circumstances. In this research work, a multi-echelon multi-objective logistics network model was developed to address the following issues:

- First, attempts to find optimal locations for setting up regional relief distribution centers with minimum setup cost.
- An appropriate amount of goods must be procured from the supplier so that demands in the affected areas can be appropriately met.
- An attempt to find an optimal route for relief vehicles with minimum transportation cost.

### 4.1 Assumptions of the Study

- Demand and available supplier capacity will be obtained from historical data. In case of the absence of historical data, reasonable assumptions will be made to estimate those values.
- Per unit transportation cost from RRDC to A.A. is higher than per unit transportation cost from the supplier to RRDC as the post-disaster road condition in the affected area might worsen due to the impact of the catastrophic incident.
- All the relief goods will be transported to the demand points in the affected area in a single trip to improve responsiveness.
- Relief quantity distributed to the different affected areas (A.A.) must be an integer as the relief goods will be dealt with as a consolidated unit load. Hence fraction or splitting will not be allowed.
- Facility setup cost is dependent on the Capacity and the location of the facility as land acquisition cost is different in different locations.
- All Demands of Affected Areas are properly met.

### 4.2 Stochastic Model

#### Sets

$i \in S \rightarrow$  Set of suppliers

$j \in D.C. \rightarrow$  Set of Regional Relief Distribution Centers (RRDC)

$k \in A.A. \rightarrow$  Set of Affected Areas (A.A.)

#### *Parameters*

(All volume/capacity parameters are given in cubic meter ( $m^3$ ) and all money amounts are in unit of 1000\$)

$n$ = number of potential Supplier plant locations/capacity (each level of Capacity will count as a separate location)

$m$ = number of demand points or RRDC

$K_i$ = potential Capacity of plant  $i$

$D_j$ = Capacity of RRDC  $j$

$d_k$ =Demand at A.A.  $k$

$f_i$ = annualized fixed cost of keeping Supplier plant  $i$  open

$W_j$ = annualized fixed cost of keeping RRDC  $j$  open

$C_{ij}$ = cost of producing and shipping one unit from plant  $i$  to RRDC  $j$  (cost includes production, inventory, transportation, and tariffs)

$C_{jk}$ = cost of producing and shipping one unit from plant  $i$  to RRDC  $j$  (cost includes production, inventory, transportation, and tariffs)

$x_{ij}$ = quantity shipped from Supplier Plant  $i$  to RRDC  $j$

$x_{jk}$ = quantity shipped from RRDC  $j$  to A.A.  $k$

$WC_r$  = Capacity of type  $r$  warehouse.

#### *Decision variables:*

$x_{ij}$  = Amount of commodities transported from supplier  $i$  to RRDC  $j$  (Decision variable 1)

$x_{jk}$  = Amount of commodities transported from RRDC  $j$  to A.A.  $k$  (Decision Variable 2)

$y_i$ = 1 if Supplier plant  $i$  is open, 0 otherwise (Decision variable 3)

$x_j$ =1 if RRDC  $j$  is open, 0 otherwise (Decision Variable 4)

Here scenario-based approach was used to form an effective stochastic model. The proposed multi-objective optimization model is as following:

*Objective function 1:*

$$\min \sum_{i=1}^n f_i y_i + \sum_{i=1}^n \sum_{j=1}^m c_{ij} x_{ij} + \sum_{j=1}^m W_j x_j + \sum_{j=1}^m \sum_{k=1}^r c_{jk} x_{jk}$$

**Subject to:**

$$\sum_{i=1}^n x_{ij} = D_j \text{ for } j=1, \dots, m$$

$$\sum_{j=1}^m x_{ij} \leq K_i y_i \text{ for } i=1, \dots, n$$

$$y_i, x_j \in \{0, 1\} \text{ for } i=1, \dots, n \text{ \& } j=1, \dots, m$$

$$\sum_{j=1}^m x_{jk} = d_k \text{ for } k=1, \dots, r$$

$$\sum_{k=1}^r x_{jk} \leq D_j x_j \text{ for } j=1, \dots, m$$

$$\sum_{i=1}^n x_{ij} \geq \sum_{k=1}^r x_{jk} \text{ for } j=1, \dots, m$$

$$x_{ij}, x_{jk} \geq 0$$

The Objective Function Minimizes facility set up cost, warehouse cost and transportation cost from Supplier to Relief Distribution centers & Relief Distribution Centers to Affected Area.

### 4.3 Vehicle Routing Model

*Parameters & Decision Variables*

$d_{jk}$  = Distance from node  $j$  to node  $k$  Affected Areas

$z_{jk} = 1$  if a truck goes from node  $j$  to node  $k$  (binary)

$e$  = Number of Points (1- Depot, 2...  $e$  - Affected Areas)

**Objective Function 2:**

$$\text{Minimize } \sum_{j=1}^e \sum_{k=1}^e d_{jk} \cdot z_{jk}$$

**Subject to,**

$$\sum_{j=1}^e z_{jk} = 1 \text{ for } k=1 \dots e$$

$$\sum_{k=1}^e z_{jk} = 1 \text{ for } j=1 \dots e$$

$$u_j - u_k + e z_{jk} \leq e - 1, j, k = 2 \dots e$$

$$z_{jk} \in \{0, 1\}$$

The Objective Function Minimizes Total distance travelled by the supplier vehicle and this model suggests a pathway for the vehicle to minimize distance as well as travel cost. The Constraints deal with Flow Conservation rate, subtour elimination using Miller-Tucker-Zemlin formulation. All nodes must be visited according to the constraints.

### 5. Numerical Example

This study focused on developing an effective relief network for a particular district in Bangladesh. So the tentative Demand and supplier capacity data was collected from local relief authority of the concerned district.

There are three types of the warehouse for setting up Regional Relief Distribution Centre (RRDC) – small, medium and large. Three tentative locations were selected for constructing Regional Relief Distribution Centre (RRDC). Five tentative locations were selected for setting up distributing relief directly to the affected population. There were three types of relief goods – food ( $I_1$ ), water ( $I_2$ ), and medicine ( $I_3$ ) (Table 1-8).

Table 1. Warehouse capacity at RRDC

Sl. No.	Warehouse type	Warehouse capacity (in cubic meter)
1	Small	1000
2	Medium	1200
3	Large	1500

Table 2. Unit Transportation Cost from Supplier to RRDC

Supplier	RRDC1	RRDC2	RRDC3
$S_1$	81	92	101
$S_2$	117	77	108
$S_3$	102	105	95

RRDC	AA1	AA2	AA3	AA4	AA5
RRDC1	91	102	161	123	185
RRDC2	187	177	105	98	100
RRDC3	106	108	95	189	111

Table 4. Supplier Capacity

Sl. No.	Suppliers	Capacity (in cubic meter)
1	S <sub>1</sub>	1631
2	S <sub>2</sub>	1735
3	S <sub>3</sub>	1352

Table 5. Demand at Affected Areas

Sl. No.	Affected Area	Warehouse capacity (in cubic meter)
1	AA1	266
2	AA2	292
3	AA3	56
4	AA4	248
5	AA5	20

Table 6. Facility Set up Cost

Supplier Region	Capacity (in cubic meter)	Cost
1	1352	475
2	1631	500
3	1735	675

Warehouse type	Capacity (in cubic meter)	Cost
Small	1000	175
Medium	1200	295
Large	1500	400

Table 8. Relative Distances (km) of different nodes from Vehicle Depot to Affected Areas

Node	1	2	3	4	5
1	0	17	20	22	24
2	17	0	32	18	40
3	20	32	0	22	38
4	22	18	22	0	24
5	24	40	38	24	0

## 6. Results and Discussions

The numerical example has been solved by Branch and Cut Algorithm. The algorithm was coded in CPLEX 20.1. The computer where the code was run had Intel(R) Core(TM) i5-6300HQ CPU 2.30GHz processor and 8 GB RAM.

Solution (optimal) with objective function 1 = 404034 (**Minimized Total Cost**)

Solution (optimal) with objective function 2 = 117 (**Minimized Total Distance for supplier vehicle**)

**Decision Variable 1** =  $\begin{bmatrix} 1000 & 0 & 0 \\ 0 & 1200 & 0 \\ 0 & 0 & 1500 \end{bmatrix}$ ;

**Decision Variable 2** =  $\begin{bmatrix} 266 & 292 & 0 & 0 & 0 \\ 0 & 0 & 0 & 248 & 20 \\ 0 & 0 & 56 & 0 & 0 \end{bmatrix}$ ;

**Decision Variable 3** = [1 1 1];

**Decision variable 4** = [1 1 1];

**Decision Variable 5** =  $\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$ ;

**Solving CPU Elapsed Time in (Seconds): 3.21875 (Objective Function 1)**

**Solving CPU Elapsed Time in (Seconds): 7.140625 (Objective Function 2)**

Decision variable 3 & 4 Suggest to keep all the Supplier Plants and Warehouses to keep open to tackle the scenario. Decision variable 1 suggests to keep the warehouses open of 1000,1200 & 1500 cubic meter in three RRDC Locations Respectively to satisfy the Demand in Affected Areas. Decision variable 2 suggests the following: in table 9.

Table 9. Amount of Commodities Transported From RRDC to Affected Area

RRDC	AA1	AA2	AA3	AA4	AA5
RRDC1	266	292	0	0	0
RRDC2	0	0	0	248	20
RRDC3	0	0	56	0	0

Decision Variable 5 Suggests the following travel path:

1 → 2 → 4 → 5 → 3 → 1

In the travel path, this model suggests to follow the above nodes and pathway and finally return to depot. This model minimizes the total distances for the supplier vehicle. We have considered node 1 as our Depot (figure 2).

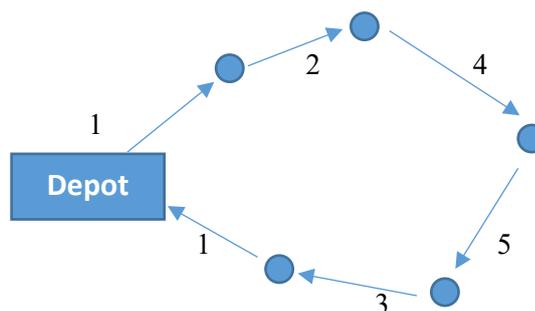


Figure 2. Flow diagram of supplier vehicle

## 7. Conclusions

A scenario-based technique was utilized to turn the deterministic model into a stochastic model in order to deal with the unknown components of the relief chain. The created Multi-objective Mixed Integer Linear Programming (MILP) issue was solved using the Branch and Cut Algorithm.

This study implies that a more effective and appropriate relief supply logistics model may be constructed if it is as near to the real-world scenario as feasible by integrating all potential risks. The more adaptable the model, the closer it will be to actual situations and the greater the efficiency in terms of lowering both logistical costs and property loss.

There are several ways in which this study might be expanded and refined. Reliability concept can also be incorporated in the model to minimize associated risk in the logistics relief network. This model can also be extended to make the decision to evacuate the wounded people and transfer them to a safer place. Fuzzy Rule-Based System can be incorporated into the model to improve its flexibility.

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