

# A Comparative Study between Linear and Non-linear Control of Induction Motor

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## Abstract

Induction motor is one of the most common motors in industrial applications because of its advantages related to its durability and low cost compared to other motors. In order to design the drive system of the induction motor, either scalar or vector methods are used. The vector control has a high dynamic performance in terms of response time and overcoming external disturbances. In this paper, a comparative study was presented between two control methods for regulating motor speed based on the concept vector control. The paper focuses on the importance of regulating both components of the magnetic flux vector, in order to avoid the magnetic saturation that can occur when the system parameters change. The simulation results in Matlab-Simulink showed that back stepping controller has a robust performance compared to the PI regulator, where it achieves lower settling time and greater ability in overcoming external disturbances despite the change of system parameters. The both control systems were tested for the presence of a change in values of the stator and rotor resistances, the simulation results showed that a magnetic saturation has occurred in the machine in order to use PI regulator, but for a back stepping controller, this problem was eliminated.

## Key words

Induction motor, magnetic saturation, Back Stepping, Vector control.

## 1. Introduction

Induction motors are classified as the most widely used motors due to their reliability, low cost and rigidity compared to other types of electric motors (Ebrahim et al. 2010; Nasir Uddin et al. 2002; Chen and Zhang, 2015). Despite that, induction motors were not commonly used in the past due to the difficulty of controlling their speed. Rather, DC motors were relied upon for ease of controlling their speed. However, developments in electronic converters technologies effectively contributed to an increase in reliance on induction motors, until they became the most widely used in the world Industry (Chen and Zhang, 2015; Diyoke and Okeke, 2016).

In order to control the speed of the induction motor, various methods have been developed. The method of changing the frequency and voltage (V/F) is the simplest method [Karthik and Chelliah, 2016; Peña and Díaz, 2016.], the more complex methods are Vector control methods, but these methods have high dynamic performance, so we rely in this article on Vector control to design the control unit (Ebrahim et al., 2010; Chen and Zhang, 2015; Diyoke and Okeke 2016).

In order to design the control system, it is possible to rely on traditional regulators (PID) because they are simpler and easier to implement (Chen and Zhang 2015), but the performance of the control system in this case will be bad in terms of response speed and the system's ability to overcome external disturbances and change system parameters (Nasir Uddin et al., 2002, Chen and Zhang 2015), so advanced control systems such as the linear quadratic controller (Ebrahim et al., 2010; Swargiary et al., 2015), sliding mode controller (Liu and Hao, 2006; Benchaib et al., 1999), back stepping controller are relied upon [Tan et al., 2003; Trabelsi, et al., 2012.]. It can also rely on intelligent and expert control systems to design the control system (Fu and Li 2015; Kar et al. 2011; Kwan and Lewis 2000; Bhushan et al., 2011).

In our research, we will apply linear and non-linear control systems, but what distinguishes this research is conducting a test of the proposed control systems for the presence of a change in the parameters of the system, and studying the

impact of this on the presence of magnetic saturation in the motor, and clarifying the importance of regulating each of the two components of the Magnetic flux vector in a general reference d-q frame.

## 2. Dynamic Model of the Induction Motor

The mathematical model of a three-phase induction motor that uses two components of a rotor flux vector and two components of a stator current vector as state variables, is given in a general reference d-q frame ( $\omega_k = \omega_s$ ), as follows (Chen and Zhang, 2015; Trabelsi et al., 2012; Fereka et al., 2018):

$$\dot{i}_{sd} = -a_5 i_{sd} + \omega_k i_{sq} + a_3 \Phi_{rd} + a_4 \omega \Phi_{rq} + b v_{sd} \quad (1-1)$$

$$\dot{i}_{sq} = -\omega_k i_{sd} - a_5 i_{sq} - a_4 \omega \Phi_{rd} + a_3 \Phi_{rq} + b v_{sq} \quad (1-2)$$

$$\dot{\Phi}_{rd} = a_2 i_{sd} - a_1 \Phi_{rd} + \omega_r \Phi_{rq} \quad (1-3)$$

$$\dot{\Phi}_{rq} = a_2 i_{sq} - \omega_r \Phi_{rd} - a_1 \Phi_{rq} \quad (1-4)$$

$$\dot{\omega} = (p/J)(G \Phi_{rd} i_{sq} - G \Phi_{rq} i_{sd} - T_d) - (f/J)\omega \quad (1-5)$$

Where:  $a_1 = R_r / L_r$ ,  $a_2 = L_m R / L_r$ ,  $a_3 = L_m R_r / (\sigma L_s L_r^2)$ ,  $a_4 = L_m / (\sigma L_s L_r)$ ,  
 $a_5 = (L_r^2 R_s + L_m^2 R_r) / (\sigma L_s L_r^2)$ ,  $b = 1 / (\sigma L_s)$ ,  $G = P L_m / L_r$ ,  $\sigma = 1 - L_m^2 / (L_s L_r)$   
 $\omega = P \Omega$ ,  $\omega_r = \omega_k - \omega$

Where:  $\omega_k$  reference frame speed.  $\omega_s$ : Synchronous speed.  $\omega$ : Rotor electric angular speed,  $\Omega$ : Rotor mechanical angular speed,  $v_{sq}$ ,  $v_{sd}$ : stator voltage components,  $i_{sq}$ ,  $i_{sd}$ : stator current components.  $\Phi_{rq}$ ,  $\Phi_{rd}$ : rotor flux components. P: the number of pole pairs,  $L_m$ : magnetizing inductance,  $L_s$ : stator inductance,  $L_r$ : rotor inductance, J: rotor inertia, f: friction coefficient,  $T_d$ : load torque.

### 2.1 Vector Control of Induction Motor

The dynamic model of an induction motor is a highly non-linear model, so it is difficult to control its speed, To facilitate the design of the control system, Vector control is used, which is the most common technique for controlling the speed of an induction motor, The vector control has greatly improved the dynamic performance of the induction motor (Ebrahim et al. 2010; Nasir Uddin et al. 2002, Karthik and Chelliah 2016).

The main objective of vector control is to obtain a linear relationship of the electromagnetic torque, as the case in independent excitation DC motors, and thus achieve possibility of decoupling control of flux and torque (Ebrahim et al. 2010; Karthik and Chelliah 2016; Liu and Hao 2006). To achieve the possibility of decoupling, the rotor flux vector is attached with d-axis, where the rotor flux component  $\Phi_{rq}$  is always equal to zero as shown in in Figure1 (Nasir Uddin et al., 2002; Chen, and Zhang, 2015; Liu and Hao, 2006).

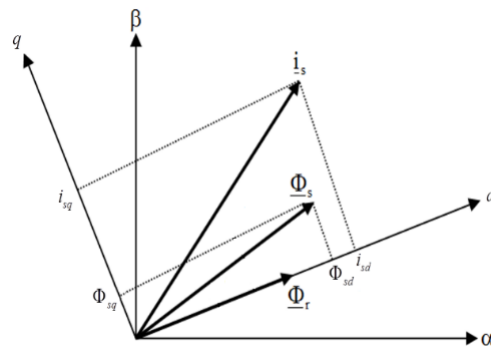


Figure 1. the reference (d-q) frame

### 3. Control system design

The control system consists of two control loops. In the inner loop, the currents are regulated using hysteresis regulators in order to obtain the pulses to be applied to the inverter feeding the motor. Therefore, the current regulating loop can be equivalent to a first-order transfer function with a gain factor equal to one.

In the outer control loop, the flux and speed will be regulated using three different control methodologies:

1- PI controllers.

2- Back Stepping controllers.

#### 3.1 PI controllers:

PID controllers are the easiest and simplest types of controllers, and the most commonly used in electrical machine driving systems such as motors, generators, and control systems, but they have some disadvantages such as their sensitivity to changing system parameters. Also, the dynamic model of the system must be approximated to a linear model in order to implement these controls (Nasir Uddin et al., 2002; Chen, and Zhang, 2015; Peña and Díaz, 2016).

#### Controlling of the rotor flux component $\Phi_{rd}$ using PI controller:

If the vector control is fulfilled, then

$$\underline{\Phi}_r = \Phi_{rd} \quad (2-1)$$

$$\Phi_{rq} = 0, \quad (2-2)$$

$$\dot{\Phi}_{rq} = 0, \quad (2-3)$$

Then equation (1-3) becomes as follows:

$$\dot{\Phi}_{rd} = a_2 i_{sd} - a_1 \Phi_{rd} \quad (2-4)$$

Applying the Laplace transforms, and taking into account the current regulating loop, it is possible to draw a block diagram to obtain the transfer function between  $i_{sd-ref}$  and  $\Phi_{rd}$  as shown in Figure 2.

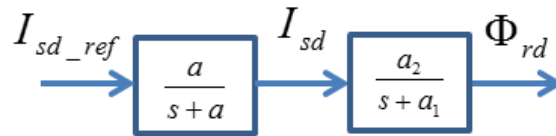


Figure 2. The block diagram of an induction motor on d-axis

#### Controlling of the electric speed using PI controller:

Likewise, and by ignoring the external disturbances in equation (1-5), it becomes as follows:

$$\dot{\omega} = (p/J)(G\Phi_{rd}^* i_{sq}) - (f/J)\omega \quad (2-5)$$

Applying the Laplace transforms, and taking into account the current regulating loop, it is possible to draw a block diagram to obtain the transfer function between  $i_{sq-ref}$  and  $\omega$  as shown in Figure 3. Where:  $\Phi_{rd}^* = 0.945$ .



Figure 3. the block diagram of an induction motor on q-axis

#### Deduce a value of $\omega_r$ :

In order to obtain  $\omega_r$ , based on the relationships (2-2)-(2-3), relationship (1-4) can be fixed to be as follows (Nasir Uddin et al. 2002; Trabelsi et al., 2012).

$$\omega_r = i_{sa} a_2 / \Phi_{rd}^* \quad (2-6)$$

The block diagram of the control system using PI is shown in Figure 4.

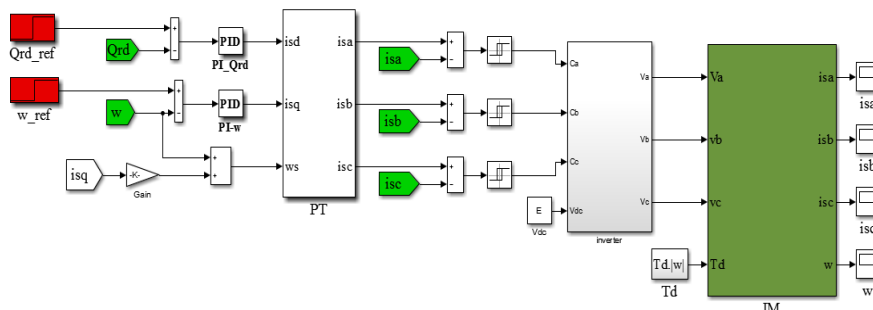


Figure 4. the block diagram of the control system using PI controllers

### The Back steeping controller:

The controller design method based on back stepping was first proposed by Koditschek in 1987. The common back-step design methodology is one of the commonly used control methods in non-linear control systems. The main idea of this technique is to design control loops iteratively by assuming that some state variables are "virtual controls" and each control loop gives a reference to the inner loop until the desired control signal is reached. (Li and Khajepour, 2004). The control signal for each control loop is determined by a condition on Lyapunov for stability, which states that the Lyapunov function must be defined positively and its derivative must be defined negatively (You et al., 2011, Nikdel et al., 2016).

### 3.2 Controlling of the rotor flux component $\Phi_{rd}$ using Back steeping controller

Let's assume:

$$e_1 = \Phi_{rd-ref} - \Phi_{rd} \quad (3-1)$$

By deriving the previous relationship and after substituting the relationship (1-3) we find:

$$\dot{e}_1 = -a_2 i_{sd} + a_1 \Phi_{rd} - \omega_r \Phi_{rq} \quad (3-2)$$

We now suppose the  $z_1$  function as follows:

$$z_1 = \gamma_1 e_1 + \int \gamma_2 e_1 dt \quad (3-3)$$

By deriving the previous function, we find:

$$\dot{z}_1 = \gamma_1 \dot{e}_1 + \gamma_2 e_1 \quad (3-4)$$

By substituting the relationship (3-2), we get:

$$\dot{z}_1 = -\gamma_1 a_2 i_{sd} + \gamma_1 a_1 \Phi_{rd} - \gamma_1 \omega_r \Phi_{ra} + \gamma_2 e_1 \quad (3-5)$$

We now suppose the Lyapunov function according to the following formula:

$$v_1 = 0.5z_1^2 \quad (3-6)$$

By derivation of the Lyapunov function, and by substituting the relation (3-5), we find:

$$\dot{\mathbf{v}}_1 = -z_1 \gamma_1 a_2 \mathbf{i}_{sd} + z_1 \gamma_1 a_1 \Phi_{rd} - z_1 \gamma_1 \omega_r \Phi_{ra} + z_1 \gamma_2 \mathbf{e}_1 \quad (3-7)$$

We add and subtract  $k_1 z_1^2$ , so we find:

$$\dot{v}_1 = -k_1 z_1^2 + z(-\gamma_1 a_2 i_{sd} + \gamma_1 a_1 \Phi_{rd} - \gamma_1 \omega_r \Phi_{ra} + \gamma_2 e_1 + k_1 z_1) \quad (3-8)$$

The derivative of the Lyapunov function must be a negative identifier, so we find:

$$i_{sd\ ref} = (\gamma_1 a_1 \Phi_{rd} - \gamma_1 \omega_r \Phi_{rq} + \gamma_2 e_1 + k_1 z_1) / (\gamma_1 a_2) \quad (3-9)$$

### 3.3 Controlling of the electric speed using Back Stepping controller

Let's assume:

$$e_2 = \omega_{ref} - \omega \quad (3-10)$$

By deriving the previous relationship and after substituting the relationship (1-5) and ignoring Td, we find:

$$\dot{e}_2 = -PGi_{sq} \Phi_{rd} / J + PGi_{sd} \Phi_{rq} / J + f\omega / J \quad (3-11)$$

We now suppose the function  $z_2$  according to the following formula:

$$z_2 = \gamma_3 e_2 + \int \gamma_4 e_2 dt \quad (3-12)$$

By deriving the previous function, we find:

$$\dot{z}_2 = \gamma_3 \dot{e}_2 + \gamma_4 e_2 \quad (3-13)$$

By substituting the relationship (3-11), we find:

$$\dot{z}_2 = -\gamma_3 PGi_{sq} \Phi_{rd} / J + \gamma_3 PGi_{sd} \Phi_{rq} / J + \gamma_3 f\omega / J + \gamma_4 e_2 \quad (3-14)$$

We now suppose the Lyapunov function according to the following formula:

$$v_2 = 0.5 z_2^2 \quad (3-15)$$

By derivation of the Lyapunov function, and by substituting the relation (3-14), we find:

$$\dot{v}_2 = -z_2 \gamma_3 PGi_{sq} \Phi_{rd} / J + z_2 \gamma_3 PGi_{sd} \Phi_{rq} / J + z_2 \gamma_3 f\omega / J + z_2 \gamma_4 e_2 \quad (3-16)$$

We add and subtract  $k_2 z_2^2$ , so we find:

$$= -k_2 z_2^2 + z_2 (-\gamma_3 PGi_{sq} \Phi_{rd} / J + \gamma_3 PGi_{sd} \Phi_{rq} / J + \gamma_3 f\omega / J + \gamma_4 e_2 + k_2 z_2) \quad (3-17)$$

The derivative of the Lyapunov function must be a negative identifier, so we find:

$$i_{sq_{ref}} = (\gamma_3 PGi_{sd} \Phi_{rq} / J + \gamma_3 f\omega / J + \gamma_4 e_2 + k_2 z_2) / (\gamma_3 PG\Phi_{rd} / J) \quad (3-18)$$

**Controlling of the rotor flux component  $\Phi_{rq}$  using Back Stepping controller:**

Let's assume:

$$e_3 = \Phi_{rq-ref} - \Phi_{rq} \quad (3-19)$$

By deriving the previous relationship and after substituting the relationship (1-4), we find:

$$\dot{e}_3 = -a_2 i_{sq} + \omega_r \Phi_{rd} + a_1 \Phi_{rq} \quad (3-20)$$

We now suppose the  $z_3$  function as follows:

$$z_3 = \gamma_5 e_3 + \int \gamma_6 e_3 dt \quad (3-21)$$

By deriving the previous function, we find:

$$\dot{z}_3 = \gamma_5 \dot{e}_3 + \gamma_6 e_3 \quad (3-22)$$

By substituting the relationship (3-20), we find:

$$\dot{z}_3 = -\gamma_5 a_2 i_{sq} + \gamma_5 \omega_r \Phi_{rd} + \gamma_5 a_1 \Phi_{rq} + \gamma_6 e_3 \quad (3-23)$$

We now suppose the Lyapunov function according to the following formula:

$$v_3 = 0.5 z_3^2 \quad (3-24)$$

By derivation of the Lyapunov function, and by substituting the relation (3-23), we find:

$$\dot{v}_3 = -z_3 \gamma_5 a_2 i_{sq} + z_3 \gamma_5 \omega_r \Phi_{rd} + z_3 \gamma_5 a_1 \Phi_{rq} + z_3 \gamma_6 e_3 \quad (3-25)$$

We add and subtract  $k_3 z_3^2$ , so we find:

$$\dot{v}_3 = -k_3 z_3^2 + z_3 (-\gamma_5 a_2 i_{sq} + \gamma_5 \omega_r \Phi_{rd} + \gamma_5 a_1 \Phi_{rq} + \gamma_6 e_3 + k_3 z_3) \quad (3-26)$$

The derivative of the Lyapunov function must be a negative identifier, so we find:

$$\omega_r = (\gamma_5 a_2 i_{sq} - \gamma_5 a_1 \Phi_{rq} - \gamma_6 e_3 - k_3 z_3) / (\gamma_5 \Phi_{rd}) \quad (3-27)$$

The block diagram of the control system using Back Stepping controllers is shown in Figure 5.

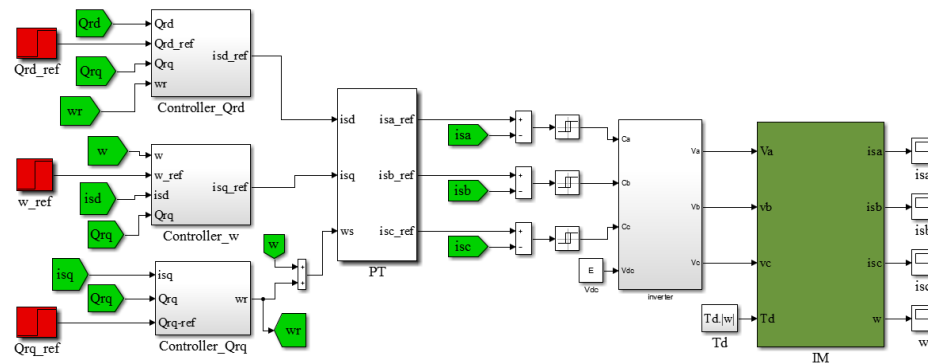


Figure 5. the block diagram of the control system using Back Stepping controllers

#### 4. Simulation results

The speed is regulated at 250rad/sec, the rotor flux component Qrd is regulated at 0.945wb, while the rotor flux component Qrq must be equal to zero. The load torque is applied at the moment 1sec. Figure 6 shows the response time for speed regulation, Figure.7 shows the response time for component Qrd regulation and Figure 8 shows the response time for component Qrq regulation.

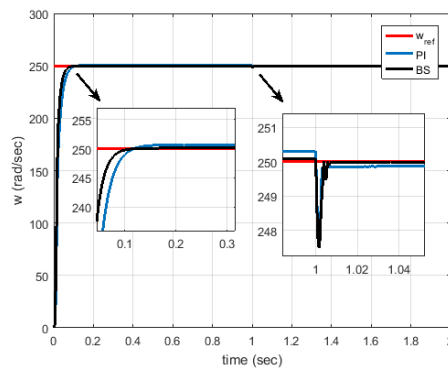


Figure 6. Response time for speed regulation

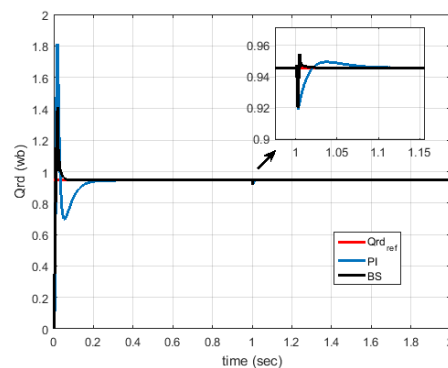


Figure7. Response time for Qrd regulation

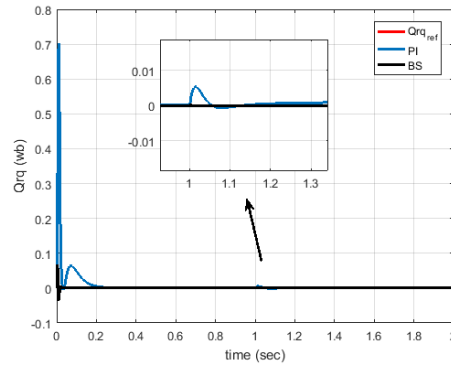


Figure 8. Response time for Qrq regulation

We note from Figure 6 that the settling time for speed regulation is the same for both control systems ( $t_s=0.15$  sec). From Figure 7, we note that the settling time for Qrd regulation is 0.05sec for back-stepping controller, while it is equal to 0.2sec for PI regulators. Static error is equal to zero for both control systems. From Figure 8, we note that for Qrq regulation. The settling time is 0.01sec for back-stepping controller, while it is 0.2 sec for PI regulators. Static error is equal to zero for both control systems. From the above we find that Back-Stepping controller achieves a higher response speed and if we also consider the moment of loading, we find that it achieves greater ability in overcoming the external disturbances represented by the load torque. The performance of both control systems was tested for the presence of a change in both stator and rotor resistances ( $R_s=1.3 \cdot R_{sn}$ ,  $R_r=1.5 \cdot R_{rn}$ ), Figure 9 shows the response time for speed regulation. Figure 10 shows the response time for Qrd regulation. Figure 11 shows the response time for Qrq regulation.

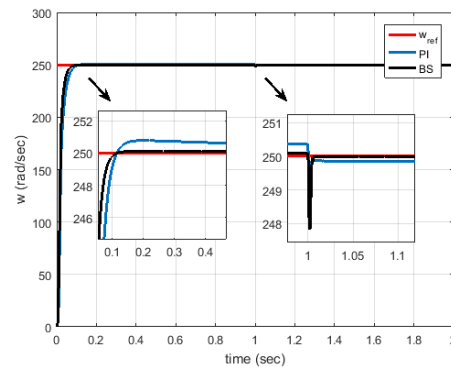


Figure 9. Response time for speed regulation after changing stator and rotor resistances

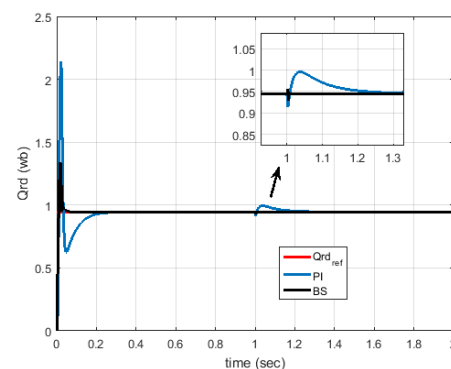


Figure10. Response time for Qrd regulation after changing stator and rotor resistances

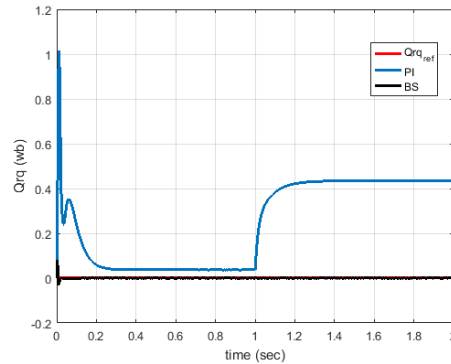


Figure 11. Response time for  $Q_{rq}$  regulation after changing stator and rotor resistances

We note from Figure 9 that both controllers maintain a constant settling time for speed regulation, ( $t_s=0.15\text{sec}$ ). From Figure.10 we note that the settling time for  $Q_{rd}$  regulation is 0.05sec for back-stepping controller, while it is 0.2sec for PI regulator. Static error is equal to zero for both control systems. From Figure.11, we note that for PI regulator, the static error equals 0.03wb before applying the load torque and increases to 0.42wb after applying the load torque, this causes a magnetic saturation to occur in the machine, resulting in an increase in magnetic losses, a decrease in efficiency and an increase in the thermal heating of the machine. While we note that the static error is equal to zero for back-stepping controller, which contributes to the absence of magnetic saturation in the machine.

## 5. Conclusions and Future Research Direction

In this research, a comparative study was presented between two control methods for regulating the motor speed based on the concept of vector control. The results of the paper indicate the importance of regulating the both components of the rotor flux vector, in order to avoid the occurrence of magnetic saturation in the machine when there is a change in the values of the stator and rotor resistances.

The simulation results showed the superiority of the Back-Stepping controller compared to the PI in terms of response time characteristics (settling time and static error) and overcoming external disturbances. The using of back-stepping controller has clearly contributed to avoiding the problem of magnetic saturation and improving the dynamic performance of the induction motor. For future study, the control system should be implemented with a magnetic flux estimator design.

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