Many industrial companies continue to face global uncertainties in demand and failures in supply. The main purpose of this research is to design and optimize a supply chain network (SCN) that performs under completely uncertain environment. In this paper, three advanced meta-heuristic algorithms based on Broyden-Fletcher-Goldfarb-Shanno (BFGS), POWELL, and Non-dominated Sorting Genetic Algorithm (NSGA-II) are used to solve the optimization problem. A real-life case study for a steel manufacturing integrated supply chain is used to demonstrate the efficiency of the model and the solutions obtained by meta-heuristic algorithms. The objective was to maximize the total profit of supply chain network under disruption conditions. The presented mathematical modeling provides an understandable overview of the system for managers to make appropriate decisions to achieve the maximum profit. Findings revealed that advanced meta-heuristic algorithms were the most efficient technique to solve the proposed model when compared with the traditional method.

**Keywords**

1. **Background**
Supply chain networks (SCNs) face many global difficulties in product delivery operations, including demand and delay uncertainties. Shen (2003) used a nonlinear optimization model to solve integrated supply chain problems. The objective was to minimize the total expected costs of the system associated with inventory, locations, and distributions operations during shipping. The design of supply chain network decisions can be affected by supply chain disruptions (SCD) (Qi et al. 2010). The authors discussed an integrated supply chain network design to determine retailers’ locations.

The objective considered also was to minimize the total expected costs associated with inventory, transportations, and locations. In addition, Liu et al. (2014) proposed a knowledge management framework to improve the competitiveness of organizations. The researchers identified critical knowledge for global supply chains to support different decisions.
simultaneously. The framework was categorized into three kinds of global knowledge including knowledge of global capacity, knowledge of global market, and knowledge of global supply networks. Gu et al. (2012) used system dynamic methodology models for integrated supply chains. The researchers proposed the dynamic methodology model to test the long-term decisions for integrated supply chains associated with remanufacturing. Hsieh et al. (2014) conducted a study of effect of ordering decisions on a multi-stages supply chain. The multi-stages supply chain considered consisting of multiple manufacturers and retailers under demand uncertainty (Hsieh et al. 2014). The results revealed that the decentralized system will gain the optimal profit of centralized integrated system once specific profit allocations are met. Li et al. (1999) analyzed two levels of the supply chain performance including operations and chains. The authors also discussed the effect of profit, delivery, elimination of waste to improve the efficiency and effectiveness of supply chains.

In this paper, we propose different integrated decisions to help the managers improving the chains of goods operations and minimizing the total expected costs. These decisions include shipping scheduling, time of selling or buying products, and production scheduling while the demand is unknown per seasons. Vermimmen et al. (2007) discussed the impact of scheduling online-shipping. They found that the performance of shipping can be affected by the decreasing schedule integrity. The authors presented a case study to show how the safety stock level can be affected by schedule unreliability. The results showed that the company can save money by the improvement of schedule reliability.

Meta-heuristic algorithms, such as Firefly Algorithm (FA), Genetic Algorithm (GA), Particle Swarm Optimization (PSO), Simulated Annealing (SA), and Ant Colony Optimization (ACO) are computational algorithms used to solve the optimization problems in many different applications. They have been used to solve the global optimization of supply chain networks associated with large-scale problems. For example, Olivares-Benitez et al. (2015) used a meta-heuristic algorithm combination of Greedy Function (GF) and Scatter Search (SS) to solve a multi-objective optimization problem associated with a two-echelon supply chain. Bottani et al. (2019) also formulated the problem as a bi-objective mixed-integer programming. The authors addressed the Resilient Food Supply Chain Design (RFSCD). They used a meta-heuristic algorithm based on ant colony optimization to solve the problem. The multi-objective considered to be optimized are total profit maximization and total lead time of supply chain minimization. Castillo-Villar et al. (2014) applied a meta-heuristic algorithm based on genetic algorithms to solve bioenergy supply chain problems. The authors found that meta-heuristic algorithms are considered an effective approach to solving large-scale problems associated with bioenergy supply chains. The objective was to design the best route of goods transportation. Another objective under consideration was to optimize the task and trucks scheduling for the distribution operations of biomass.

This research is engaged in designing a multi-objective supply chain network that performs under disruption conditions during the delivery of goods. We implemented advanced met-heuristic optimization algorithms and compared the results with traditional algorithms to test the performance of meta-heuristic algorithms. A real-life case study for a steel manufacturing company was conducted to test the effectiveness of our model using four advanced optimization algorithms, including BFGS, POWELL, and NSGA-II.

2. Related Work

The optimization techniques for product delivery have been used in many applications in many industries. For instance, Mukherjee and Ray (2006) reviewed the optimization techniques in the metal cutting process. The scholars classified the optimization techniques into two categories, including conventional and non-conventional techniques. The conventional techniques, such as Dynamic Programming (DP), Non-Linear Programming (NLP), and Linear Programming (LP) can get the optimal solutions, whereas the non-conventional techniques, such as Genetic Algorithm (GA), Simulated Annealing (SA), Tabu Search, and Firefly Algorithm (FA) can get only the near-optimal solutions. Medic and Radovanović (2014) introduced a deterministic direct search optimization named a Pattern Search algorithm (PS). The results showed that the meta-heuristic algorithms are considered the most powerful optimization tools to solve complex problems. The researchers applied the Pattern Search algorithm to get the optimal solutions for machining processes. The researchers discussed the optimization problems in statistics, focusing on quality control (Carlyle et al. 2000). They described several optimization techniques including simulated annealing and genetic algorithms. Yang and Chou (2005) proposed finding the solution of a multi-response optimization problem using a decision-making approach.
Researchers found that the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) is a powerful tool to solve a multi-response optimization problem. Venter (2010) presented the standard form of the general nonlinear and constrained problems. The researcher suggested how to select the appropriate optimization technique to solve the optimization problems. Jayal et al. (2010) presented optimization techniques for sustainable manufacturing processes. They found that using optimization techniques can determine the issues related to sustainable manufacturing.

Kobetski and Fabian (2009) presented a Mixed-Integer Linear Programming (MILP) model to optimize the time coordination of flexible manufacturing systems. They integrated the strength of Mixed-Integer Linear Programming for the scheduling of manufacturing systems. The results showed that the formulation of MILP is able to use the power of MILP to create schedules of time optimization. The researchers also used a nonlinear mixed-integer programming model to solve the production planning problems (Stecke 1983). Saxena and Jain (2011) presented the dynamic cell formation problem. They proposed a mixed-integer nonlinear programming model to solve the dynamic cellular manufacturing systems under stochastic demand. Zimmermann et al. (2001) solved large manufacturing system problems by using a two-phase optimization method including robust and efficient optimization techniques. The researchers proposed a metaheuristic based on simulated annealing to present the tool for automated optimization of Discrete Event Dynamic Systems (DEDS). Nouira et al. (2014) presented optimization models for manufacturing systems with a consideration for environmental impacts. The authors classified the optimization models into two models. For the first model, they assumed that the company offers the final product to ordinary and green customers. They formulated the problem as a MILP model. For the second model, they proposed that the company offers two separate kinds of products: one of them is offered to the ordinary customers, and the other one is offered to the green customers. The objective function of the model is to maximize the company’s profit. Table 1 summarizes the existing literature using meta-heuristic algorithms in supply chain applications.

Table 1. The summary of some literature using meta-heuristic algorithms in supply chain applications

<table>
<thead>
<tr>
<th>References</th>
<th>Application</th>
<th>Objective</th>
<th>Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Olivares-Benitez et al. 2013</td>
<td>A multi-Stage SC for single product. This system includes plants, distribution centers, and customers.</td>
<td>The objective was to minimize the transportation costs and transportation time from plants to customers.</td>
<td>Meta-heuristic based on Greedy Function (GF) and Scatter Search (SS).</td>
</tr>
<tr>
<td>Bottani et al. 2019</td>
<td>They addressed a Resilient Food Supply Chain Design (RFSCD).</td>
<td>The objective was to maximize the total profit and total lead time of supply chain network.</td>
<td>They used a meta-heuristic algorithm based on ant colony optimization to solve the problem.</td>
</tr>
<tr>
<td>Castillo-Villar et al. 2014</td>
<td>Bioenergy supply chain problems.</td>
<td>The objective was to design the best route of goods transportation.</td>
<td>They applied a meta-heuristic algorithm based on genetic algorithm to solve bioenergy supply chain problems.</td>
</tr>
<tr>
<td>Moncayo-Martinez and Zhang 2011</td>
<td>Supply chain design problem was to select the options that can minimize the total costs of supply chain. This system includes suppliers, plants, and products.</td>
<td>The objective function was to minimize the total costs of supply chain and lead time of deliveries simultaneously.</td>
<td>The authors used a meta-heuristic algorithm based on pareto ant colony optimization (PACO) to solve the problem.</td>
</tr>
<tr>
<td>Atabaki et al. 2019</td>
<td>They addressed a closed-loop supply chain with price sensitive demand with forward and reverse flows.</td>
<td>The objective was to maximize the profits associated with capacity and the constraints number of facilities opened.</td>
<td>They used a meta-heuristic algorithm based on firefly algorithm to solve large-sized problems.</td>
</tr>
</tbody>
</table>

Reviewing the existing literature shows supply chain networks that perform under uncertainty can result in incurring high expected penalty cost and minimal profit. To overcome these difficulties, this study proposes a mathematical modeling to help managers improve chains of goods’ operations and maximize the total profit.

3. Problem Description
In this paper we introduce mathematical models using advanced meta-heuristic algorithms based on BFGS, POWELL, and NSGA-II. The objective of the model is to maximize the total profit of supply chain network when facing uncertainties in failures in supply and demand. This paper proposes a model considering demand uncertainty and failures in supply to meet customers demand and prevent any costs related to penalties due to failures in supply. In this study, we consider that supply chain networks consist of multi-suppliers, multi-manufacturers, and multi-markets, as shown in Figure 1. This model was developed to help managers determine which decisions can minimize the total expected cost of the system and improve the shipment of materials without disruptions and delays, preventing any related penalty costs. The raw materials are supplied by suppliers to manufacturers. The manager must determine the limited number of finished products that should be shipped from manufacturers to markets or final customers. Moreover, manufacturers have three programs of selling, buying, and stocking the finished products at a warehouse at a later time. The manager must achieve the optimized program of selling, stocking, and buying units. The manager also faces disruptions and delays in the production operations, which result in high penalty costs due to disruptions and delays. The finished-products will be stored at a warehouse, and the managers must decide when to sell or buy the finished-products. The manager also faces disruptions and delays in the shipment of finished products to the final customers or markets, which result in increasing penalty costs.

Figure 1. The stages of supply chain network considered in this research

4. Methodology

The main goal of this model is to develop a mathematical method to optimize a supply chain network that faces failure risks during the delivery of products using advanced optimization algorithms. The risks include shortage of raw materials and shortage of finished products. The proposed methodology is presented in Figure 2. We consider that supply chains work under uncertainty. The main purpose of the model is to maximize the profit and minimize the total expected costs associated with extra inventory and penalties for shortages. The model is implemented at a real-life case study for a steel manufacturing integrated supply chain for validation. The case study manages 13 supply chains work under uncertainty. All of them support the same product. Equation (1) calculates the total profit and costs associated with inventory and penalties for shortages, as follows:

\[ P \sum_{i} Q_{L,t} - C_{P,i} * P_{U,t} - C_{l,t} - I_{L,t} - U R_{L,t} * S F_{L,t} \]  

Where \( P_{E,i} \) and \( Q_{L,t} \) present the cost of product of supply chain \( i \) and quantity of products in supply chain \( i \) at time \( t \). Additionally, \( C_{P,i} \) and \( P_{U,t} \) indicate the cost of production at supply chain \( i \) and production of units at supply chain \( i \) at time \( t \). \( C_{l,t} \) and \( I_{L,t} \) present the carrying cost at supply chain \( i \) and inventory of units at supply chain \( i \) at time \( t \). The last term calculates supplier failure penalty cost at supply chain \( i \). Equation (2) determines that raw materials supplied by suppliers is considered the total production, as follows:

\[ P_{U,t} = \sum_{all\ suppliers} \left[ S F_{L,t} * Q_{r,i} - Q_{r,i} \right] \]  

Where \( S F_{L,t} \) and \( Q_{r,i} \) indicate the supplier failure penalty cost at supply chain \( i \) and quantity of raw materials in supply chain \( i \) respectively. Equation (3) indicates the demand of raw materials \( D_{r,i} \) and their inventories at the current time must equal raw materials’ inventory and production from last time.

\[ D_{r,i} + I_{L,t} = P_{U,t} + S F_{L,t} + I_{L,t-1} \]
In this research, we used advanced optimization algorithms based on BFGS, POWELL, and NSGA-II for optimizing a multi-objective supply chain network under failure risks. Literature indicates that the traditional mathematical models cannot achieve the optimal solution of large-scale models (Castillo-Villar 2014). So, we used advanced optimization algorithms based on meta-heuristic to get the near optimal solution of the model and compare the results with the traditional algorithm (SIMPLEX).

![Diagram of methodology used in this research](image)

Figure 2. The proposed methodology used in this research

The Broyden–Fletcher–Goldfarb–Shanno (BFGS) algorithm, which is Quasi-Newton optimization algorithm, is used to find the total expected costs of holding and penalties for shortages, as follows:

\[ C_{pi} \cdot PU_{it} - C_{il} \cdot I_{it} - UR_i \cdot SF_{it} \]  

(4)

The optimality of the step size of BFGS algorithm \( (\alpha^{(i)}) \) is calculated in Equation (5), as follows:

\[ \alpha^{(i)} = \arg \min f(X^{(i)} + \alpha \cdot D^{(i)}) \]  

(5)

Where \( X^{(i)} \) represents the vector of solution of BFGS algorithm at stage \( i \). The descent direction \( (D^{(i)}) \) is determined in Equation (6), as follows:

\[ D^{(i)} = -W^{(i)} \nabla f(X^{(i)}) \]  

(6)

Where \( W^{(i)} \) represents the Hessian matrix. For validation, we used POWELL algorithm to find the local minimum function of the proposed supply chain network. Moreover, non-dominated sorting genetic algorithm (NSGA-II) is used to solve a multi-objective problem, which is maximization and minimization problems. We also used response surface methodology (RSM) algorithm based on statistical model (Kriging) to compare the test and validation of each
optimization algorithms using three performance indexes including Mean Absolute Error (MAE), R-squared and Akaike Information Criterion (AIC). The mean absolute error is used to get the best performance of model using meta-heuristic algorithms, which can be determined in Equation (7), as follows:

\[
MAE = \frac{\sum_{i=1}^{n}|x_i - \bar{x}|}{n}
\]  

(7)

Where \(n\) represents the number of error while the absolute value is the difference between predicted and observed values. Akaike Information Criterion can calculate the relative information that can be lost by a model and can be calculated in Equation (8), as follow:

\[
AIC = 2 \times I - 2 \ln(L)
\]  

(8)

Where \(I\) and \(L\) represent the number of independent variables and likelihood estimate respectively.

5. Results and Discussion

The main objective of this paper is to optimize a multi-objective supply chain network that perform under failure risks of shortages. The traditional optimization algorithm such as SIMPLEX cannot achieve the optimal solution of large-scaled models. So, to find the better findings, we used advanced optimization algorithms based on meta-heuristic to get the near optimal solution of the model and compare the results with the traditional algorithm (SIMPLEX). The model is implemented at a real-life case study for a steel manufacturing integrated supply chain for validation. The case study manages 13 supply chains, which work under uncertainty and face failure risks of shortage. The multi-objective is to maximize the profit and minimize the total expected costs of supply chains. We also trained RSM, which is trained upon a design of experiment (DOE) based on statistical models (Kriging), as shown in Table 2. This table illustrates that the best value of the selected validation criterion is represented by BFGS algorithm. Then, we compare the result of this algorithm with other powerful optimization algorithms to determine which algorithm has the best optimized solution and satisfactory results. The best algorithm is one with lowest MAE and AIC values and highest R-squared value. The result revealed that the best algorithm is BFGS, as shown in Table 2.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>MAE</th>
<th>R-SQUARED</th>
<th>AIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>BFGS</td>
<td>2.13E-07</td>
<td>1</td>
<td>-1831.92</td>
</tr>
<tr>
<td>NSGA-II</td>
<td>0.002029</td>
<td>0.999997</td>
<td>-151.658</td>
</tr>
<tr>
<td>POWELL</td>
<td>2.69335</td>
<td>0.0283381</td>
<td>927.573</td>
</tr>
<tr>
<td>SIMPLEX</td>
<td>0.017218</td>
<td>0.99997</td>
<td>12.2258</td>
</tr>
</tbody>
</table>

For BFGS algorithm, we ran the model for 400 iterations to get the optimal solution. Table 3 shows the best design iteration for the optimal solution using BFGS. The best optimal objective function was 44.7150 based on 98 iterations, as shown in Table 3.

<table>
<thead>
<tr>
<th>Iterations</th>
<th>Algorithm</th>
<th>Phase</th>
<th>Objective Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>88</td>
<td>BFGS</td>
<td>Evolution</td>
<td>43.8064</td>
</tr>
<tr>
<td>90</td>
<td>BFGS</td>
<td>Evolution</td>
<td>43.8064</td>
</tr>
<tr>
<td>93</td>
<td>BFGS</td>
<td>Evolution</td>
<td>43.8064</td>
</tr>
</tbody>
</table>
We also ran the model for 400 iterations to get the optimal solution using NSGA-II algorithm. Table 4 shows the best design iteration for the optimal solution. The best optimal objective function was 41.7237 based on 383 iterations, as shown in Table 4. We also ran the model for 400 iterations to get the optimal solution using POWELL algorithm. Table 5 shows the best design iteration for the optimal solution. The best optimal objective function was 44.7150 based on 126 iterations, as shown in Table 5.

### Table 4. The optimal solution of model using NSGA-II algorithm

<table>
<thead>
<tr>
<th>Iterations</th>
<th>Algorithm</th>
<th>Phase</th>
<th>Objective Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>373</td>
<td>NSGA-II</td>
<td>Evolution</td>
<td>41.0207</td>
</tr>
<tr>
<td>375</td>
<td>NSGA-II</td>
<td>Evolution</td>
<td>40.9793</td>
</tr>
<tr>
<td>377</td>
<td>NSGA-II</td>
<td>Evolution</td>
<td>41.2199</td>
</tr>
<tr>
<td>379</td>
<td>NSGA-II</td>
<td>Evolution</td>
<td>41.2796</td>
</tr>
<tr>
<td>380</td>
<td>NSGA-II</td>
<td>Evolution</td>
<td>41.2351</td>
</tr>
<tr>
<td>381</td>
<td>NSGA-II</td>
<td>Evolution</td>
<td>41.3677</td>
</tr>
<tr>
<td>383</td>
<td>NSGA-II</td>
<td>Evolution</td>
<td>41.7237</td>
</tr>
</tbody>
</table>

The feasible solution of SIMPLEX, BFGS, and NSGA-II algorithms is shown in Figure 3, Figure 4, and Figure 5 respectively. The training and validation of model using NSGA-II algorithm are shown in Figure 6.

### Table 5. The optimal solution of model using POWELL algorithm

<table>
<thead>
<tr>
<th>Design ID</th>
<th>Algorithm</th>
<th>Phase</th>
<th>Objective Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>111</td>
<td>POWELL</td>
<td>Evolution</td>
<td>44.7140</td>
</tr>
<tr>
<td>113</td>
<td>POWELL</td>
<td>Evolution</td>
<td>44.7146</td>
</tr>
<tr>
<td>119</td>
<td>POWELL</td>
<td>Evolution</td>
<td>44.7138</td>
</tr>
<tr>
<td>122</td>
<td>POWELL</td>
<td>Evolution</td>
<td>44.7149</td>
</tr>
<tr>
<td>125</td>
<td>POWELL</td>
<td>Evolution</td>
<td>44.7146</td>
</tr>
<tr>
<td>126</td>
<td>POWELL</td>
<td>Evolution</td>
<td>44.7150</td>
</tr>
</tbody>
</table>
Figure 3. The feasible solutions of using SIMPLEX

Figure 4. The feasible solutions of using BFGS
6. Conclusion
Global Supply Chain Networks (GSCNs) experiencing failures in supply require investigation to discover the cause of supply delays. In this study, a mathematical model was developed to predict the delays and failures in supply. The problem was developed as a multi-objective function to optimize multi-stage supply chain networks under risks of failure in order to achieve the least cost and greatest profit. The model considered in this paper presented integrated supply chain decisions to optimize the delivery of products under demand uncertainty and failure risks (disruptions,
delays, etc.). The results of the proposed model can help managers achieve optimal solutions for inventory operations and determine when they can apply make-to-order policies. This research can be extended by considering different objectives and methodologies. This research can also be extended by using different meta-heuristic algorithms to achieve near optimal solutions and compare the results of each algorithm based on their accuracies.

References

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