Determination of Optimal Control policy for a Stochastic Production-inventory Model in Segmented Market

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Abstract

Market segmentation is a key element of marketing and it enables interaction between marketing and production of a firm, which favors both the customers and the business. In this paper, a stochastic optimal control model is developed in which, the state of the inventory system is stated as *Ito* stochastic differential equation in segmented market approach. First, we consider a single source production and inventory problem with multi-destination demand where demand from all segments depends upon single inventory warehouse. Then, we consider a multi-destination production, inventory and demand problem involving segment-based production and inventory points corresponding to each market segment. This way demand from each segment can selectively reach each target inventory source. Both the problems are discussed and solved using Hamilton-Jacobi Bellman equation.

Keywords:
Market Segmentation, Production-Inventory System, Optimal Control Problem, Hamilton-Jacobi Bellman equation.

1. Introduction
The evolution of technology has resulted in a revolution in the industrial sector. Globalization has further resulted in production of goods on a large scale. In order to meet the demands of the customers in an effective way, the production and storage of products needs to be monitored properly.

Various production-inventory models have been proposed in the past. Bradshaw and Porter (1973) have also studied production-inventor systems by applying the concept of optimal control theory. Benjaafar et al. (2010) had studied a model in which the concept of customers' impatience is taken into account. In case the order is not delivered on time, the model incorporates a certain time limit which is customer dependent, after which the order stands cancelled. Benkherouf (2015) has given a production inventory model for products that are remanufactured or refurbished. Various other production inventory models have been given by Pooya and Pakdaman (2017) and Dizbin and Tan (2020).

In the past, authors have also analyzed models in the field of production and inventory planning for products that are perishable or those which deteriorate with time. Some such models have been studied by Hedjar et al. (2004), Tadj et al. (2006), Benhadid et al. (2008) and Chen (2018).
To facilitate the distribution of goods in the market, the process of partitioning the market has proved to be very successful. The market could be segmented on the grounds of geographical, demographical or behavioral characteristics of the population. This results in optimizing the effort of the producers and the satisfaction level of the

consumers. Some authors like Buratto et al. (2006), Jha et al. (2009) and Chaudhary et al. (2021) have dealt with dynamic advertising and inventory based mathematical models for segmented markets and analyzed them by application of optimal control theory. Various models in the field of production and inventory where the concept of market segmentation has been used is given by authors like Duran et al. (2007), Chen and Li (2009) and Mehta and Chaudhary (2021).

A more realistic approach in the formulation of such models has been the shift from being a deterministic one to a stochastic one. Kapuscinski and Tayur (1998), AlDurgam et al. (2017) and Duan et al. (2018) have given production inventor models with stochastic demand. Sethi and Thompson (2000) has given the production planning models which are characterized by Ito Stochastic differential equations. Various other authors have also worked in this direction. Karim and Huynh (2017) have considered a model in which the product considered is perishable in nature. The machine used for production faces a breakdown with stochastic repair time. Pal et al. (2017) have also studied a stochastic production-inventory model for deteriorating products having finite life cycle. Das et al. (2017) have considered a production-inventory model with defective items, where the buyer gets a stochastic credit period in order to compensate for defective items. The model considered by Azoury and Miyaoka (2020), provides solutions to an inventory-production model with stochastic and deterministic demands.

In this paper we have considered a production and inventory planning problem incorporating the concept of market segmentation. The segment wise demands of the population are analyzed and the model aims to minimize the costs incurred in the process of production and holding the produced goods. The state equation representing the evolution of inventory, is given by an Ito stochastic differential equation. The presence of white noise process in the system makes the model realistic, thereby increasing its applicability. The work also analyses the possibility of the warehouses being established at the respective market segments. This may further facilitate the distribution of goods to the consumers and reduce the inventory holding costs.

The paper is organized as follows. The second section gives the notations used in the paper. The third section explains the model development. In the fourth section two special cases are considered and in the fifth section a numerical illustration is taken. The paper is concluded in the sixth section.

2. Model Notations

Here we assume that a manufacturer is producing a single product which is further sold in a market which is partitioned into n segments. The notations used for the development of the model are mentioned below:

T : Total time period considered for planning
P_t : Production rate at time t
P_{it} : Production rate at time t in i^{th} segment
I_t : Inventory level at time t
I_{it} : Inventory level at time t in i^{th} segment
D_{it} : Demand rate at time t in i^{th} segment
$K(P_t)$:Cost rate corresponding to the production rate at time t
$H(I_t)$:Inventory holding cost rate at time t
k : Production cost coefficient
h : Inventory holding cost coefficient
$K_i(P_{it})$: Cost rate corresponding to the production rate at time t in i^{th} segment
$H_i(I_{it})$: Inventory holding cost rate at time t in i^{th} segment
k_i : Production cost coefficient in i^{th} segment
h_i : Inventory holding cost coefficient in i^{th} segment
σ : Constant diffusion coefficient

3. Model Development

The production-inventory planning is the most crucial task for the manufacturers. Post production, the goods are stored in warehouses followed by their distribution in the market as per the demand. Here we are assuming that the market is sectioned into segments of population with similar interests. In order to efficiently manage the availability of the goods to the consumers, we have considered two ways in which their production and storage can be done. One way is to produce and then store all the products in a common warehouse from where they can be sent to the various segments.

The second way is to produce and store them at warehouses situated in the different market segments. For both the above cases we assume that there are n segments. (i.e., i = 1, …, n).

3.1 Single Source Inventory and Multi-Segmented Demand

In this case we consider a situation when the production and storage occurs at a single source. The goods are then sold in the different market segments depending on the respective demands.

The stock-flow equation for the problem is given by the $It\hat{o}$ stochastic differential equation as follows:

$$dI_t = (P_t - \sum_{i=1}^{n} D_{it})dt + \sigma dZ_t \tag{1}$$

with I_0 denoting the initial inventory level. Here Z_t denotes the standard Weiner process, whose derivative represents the white noise which denotes some random fluctuations in the inventory level. Since the manufacturer aims to find the optimal production rate that minimizes the total expected costs, the objective function is given as:

$$\underset{P_t}{\text{Min}}\ \text{E}\left(\int_0^T [K(P_t) + H(I_t)]dt\right)$$

Taking $K(P_t) = kP_t^{\,2}$ and $H(I_t) = hI_t^{\,2}$, we can rewrite the objective function as:

$$\underset{P_t}{\text{Max}}\ \text{E}\left(\int_0^T \left[- kP_t^{\,2} - h\, I_t^{\,2}\right]dt\right) \tag{2}$$

Let V(I,t) be the expected value of the objective function from time t to T. This function is known as the value function and it satisfies the following Hamilton-Jacobi-Bellman equation:

$$\underset{P_t}{\text{Max}}\left(- kP_t^{\,2} - h\, I_t^{\,2} + \frac{\partial V}{\partial t} + \frac{\partial V}{\partial I}\left(P_t - \sum_{i=1}^{n} D_{it}\right) + \frac{1}{2}\sigma^2 \frac{\partial^2 V}{\partial I^2}\right) = 0 \tag{3}$$

with boundary conditions

$$\text{V(I,T)} = 0 \tag{4}$$

We now differentiate the expression in equation (3) with respect to P_t and equate it to zero, to get the optimal value of the production rate which minimizes the cost as follows

$$P_t = \frac{1}{2k}\frac{\partial V}{\partial I} \tag{5}$$

In order that the production rate is non-negative, the optimal production rate is given by

$$P_t = Max\left\{0, \frac{1}{2k}\frac{\partial V}{\partial I}\right\} \tag{6}$$

On substituting the value of production rate P_t from equation (5) in equation (3) we get:

$$-k\left(\frac{1}{2k}\frac{\partial V}{\partial I}\right)^2 - hI_t^{\,2} + \frac{\partial V}{\partial t} + \frac{\partial V}{\partial I}\left(\frac{1}{2k}\frac{\partial V}{\partial I} - \sum_{i=1}^{n} D_{it}\right) + \frac{1}{2}\sigma^2\frac{\partial^2 V}{\partial I^2} = 0 \tag{7}$$

Equation (7) can be further simplified as:

$$\frac{1}{4k}\left(\frac{\partial V}{\partial I}\right)^2 - hI_t^{\,2} + \frac{\partial V}{\partial t} - \sum_{i=1}^{n} D_{it}\frac{\partial V}{\partial I} + \frac{1}{2}\sigma^2\frac{\partial^2 V}{\partial I^2} = 0 \tag{8}$$

This is a partial differential equation with boundary conditions given by (4).

The solution to the above problem is obtained as follows:

$$\text{Let}\ \ \text{V(I,t)} = \text{Q(t)I}^2 + \text{R(t)I} + \text{M(t)} \tag{9}$$

Then

$$\left.\begin{aligned} \frac{\partial V}{\partial t} &= \dot{Q}\text{I}^2 + \dot{R}I + \dot{M} \\ \frac{\partial V}{\partial I} &= 2QI + R \\ \frac{\partial^2 V}{\partial I^2} &= 2Q \end{aligned}\right\} \tag{10}$$

Substituting from (10) in equation (8) we get

$$\text{I}^2\left(\dot{Q} + \frac{Q^2}{k} - h\right) + I\left(\dot{R} + \frac{QR}{k} - 2\text{Q}\sum_{i=1}^{n} D_{it}\right) + \left(\dot{M} - \text{R}\sum_{i=1}^{n} D_{it} + \frac{R^2}{4k} + \sigma^2 Q\right) = 0 \tag{11}$$

From equation (11), we get the following system of non-linear differential equations:

$$\dot{Q} + \frac{Q^2}{k} - h = 0, Q(T) = 0 \tag{12}$$

$$\dot{R} + \frac{QR}{k} - 2\mathrm{Q}\sum_{i=1}^{n} D_{it} = 0, R(T) = 0 \tag{13}$$

$$\dot{M} - \mathrm{R}\sum_{i=1}^{n} D_{it} + \frac{R^2}{4k} + \sigma^2 Q = 0, M(T) = 0 \tag{14}$$

The solution to the above system of equations is given by:

$$Q(t) = \frac{\sqrt{hk}\left(e^{-2\sqrt{h/k}(T-t)} - 1\right)}{1 + e^{-2\sqrt{h/k}(T-t)}} \tag{15}$$

$$R(t) = e^{-\int \frac{Q}{k}dt}\left(C_1 + \int 2\mathrm{Q}\sum_{i=1}^{n} D_{it}\, e^{\int \frac{Q}{k}dt}\, dt\right) \tag{16}$$

$$M(t) = -\int_t^T \left(\mathrm{R}\sum_{i=1}^{n} D_{it} - \frac{R^2}{4k} - \sigma^2 Q\right) d\tau \tag{17}$$

where C_1 is a constant which is determined from the terminal condition R(T) = 0.
Hence, the optimal production rate as a function of the state variable and time is as follows:

$$P_t = \frac{1}{2k}\left[2I_t \frac{\sqrt{hk}\left(e^{-2\sqrt{h/k}(T-t)} - 1\right)}{1 + e^{-2\sqrt{h/k}(T-t)}} + e^{-\int \frac{Q}{k}dt}\left(C_1 + \int 2\mathrm{Q}\sum_{i=1}^{n} D_{it}\, e^{\int \frac{Q}{k}dt}\, dt\right)\right] \tag{18}$$

The expected value of the Inventory level corresponding to the optimal production rate is given by:

$$E[I_t] = e^{\int \frac{Q}{k}dt}\left[I_0 + \int \left(\frac{R}{2k} - \sum_{i=1}^{n} D_{it}\right) e^{-\int \frac{Q}{k}dt}\, dt\right] \tag{19}$$

3.2 Multi-Segmented Production, Inventory and Demand

In this case we consider a situation, in which the production units as well as the warehouses are located in the different market segments according to the respective demands.

The stock-flow equation for the problem in the i^{th} segment is given by the $It\hat{o}$ stochastic differential equation as follows:

$$dI_{it} = (P_{it} - D_{it})dt + \sigma_i dZ_{it} \tag{20}$$

with I_{i0} denoting the initial inventory level in the i^{th} segment. Z_{it} denotes the standard Weiner process in the i^{th} segment, whose derivative represents the white noise denoting some random fluctuations in the inventory level. Since the manufacturer aims to find the optimal production rate that minimizes the total expected costs, the objective function for the problem is given as:

$$\underset{P_t}{Min}\ E\left(\int_0^T \sum_{i=1}^{n} [K_i(P_{it}) + H_i(I_{it})]\, dt\right)$$

Taking $K_i(P_{it}) = k_i {P_{it}}^2$ and $H_i(I_{it}) = h_i {I_{it}}^2$, we can rewrite the objective function as:

$$\underset{P_{it}}{Max}\ E\left(-\int_0^T \sum_{i=1}^{n} \left[\, k_i {P_{it}}^2 + h_i\, {I_{it}}^2\right] dt\right) \tag{21}$$

The objective function can be written as $J = \sum_{i=1}^{n} J_i$, where

$$J_i = \underset{P_{it}}{\mathrm{Max}}\ \mathrm{E}\left(-\int_0^T \left[\, k_i {P_{it}}^2 + h_i\, {I_{it}}^2\right] dt\right) \tag{22}$$

Here J_i denotes the expected value of profit in the i^{th} segment. By the monotonicity of integral, the segment dependent inventory problem is equivalent to the problem determining a production rate and associated inventory rate satisfying equation (20) for each segment.

Let $V_i(I_i,t)$ which is the value function be the expected value of the objective function in the i^{th} segment from time t to T. By the Hamilton-Jacobi-Bellman equation:

$$\underset{P_t}{\text{Max}}\left(-k_i{P_{it}}^2 - h_i\, {I_{it}}^2 + \frac{\partial V_i}{\partial t} + \frac{\partial V_i}{\partial I_i}(P_{it} - D_{it}) + \frac{1}{2}{\sigma_i}^2\frac{\partial^2 V_i}{\partial {I_i}^2}\right) = 0 \tag{23}$$

with boundary conditions $V_i(I_i,T) = 0$

We now differentiate the expression in equation (23) with respect to P_{it} and equate it to zero, to get the optimal value of the production rate which minimizes the cost as follows

$$P_{it} = \frac{1}{2k_i}\frac{\partial V_i}{\partial I_i} \tag{24}$$

In order that the production rate is non-negative, the optimal production rate is given by

$$P_{it} = Max\left\{0, \frac{1}{2k_i}\frac{\partial V_i}{\partial I_i}\right\} \tag{25}$$

On substituting the value of production rate P_t from equation (24) in equation (23) we get:

$$\frac{1}{4k_i}\left(\frac{\partial V_i}{\partial I_i}\right)^2 - h_i{I_{it}}^2 + \frac{\partial V_i}{\partial t} - \frac{\partial V_i}{\partial I_i}D_{it} + \frac{1}{2}{\sigma_i}^2\frac{\partial^2 V_i}{\partial {I_i}^2} = 0 \tag{26}$$

This is a partial differential equation with boundary conditions $V_i(I_i,T) = 0$.

The solution to the above problem is obtained as follows:

$$\text{Let } V_i(I_i,t) = Q_i(t)I_i^2 + R_i(t)I_i + M_i(t) \tag{27}$$

Then equation (26) can be written as

$$ {I_{it}}^2\left(\dot{Q}_i + \frac{Q_i^2}{k_i} - h_i\right) + I_{it}\left(\dot{R}_i + \frac{Q_iR_i}{k_i} - 2Q_iD_{it}\right) + \left(\dot{M}_i - R_iD_{it} + \frac{R_i^2}{4k_i} + {\sigma_i}^2Q_i\right) = 0 \tag{28}$$

with boundary conditions $V_i(I_i,T) = 0$.

On solving we get

$$Q_i(t) = \frac{\sqrt{h_ik_i}\left(e^{-2\sqrt{h_i/k_i}(T-t)} - 1\right)}{\left(e^{-2\sqrt{h_i/k_i}(T-t)} + 1\right)} \tag{29}$$

$$R_i(t) = e^{-\int\frac{Q_i}{k_i}dt}\left(C_2 + \int 2Q_iD_{it}e^{\int\frac{Q_i}{k_i}dt}\,dt\right) \tag{30}$$

$$M_i(t) = -\int_t^T\left(R_iD_{it} - \frac{R_i^2}{4k_i} - {\sigma_i}^2Q_i\right)d\tau \tag{31}$$

where C_2 is a constant which is determined from the terminal condition $R_i(T) = 0$.

Hence, the optimal production rate as a function of the state variable and time is as follows:

$$P_{it} = \frac{1}{2k_i}\left[2I_{it}\frac{\sqrt{h_ik_i}\left(e^{-2\sqrt{h_i/k_i}(T-t)} - 1\right)}{\left(e^{-2\sqrt{h_i/k_i}(T-t)} + 1\right)} + e^{-\int\frac{Q_i}{k_i}dt}\left(C_1 + \int 2Q_iD_{it}e^{\int\frac{Q_i}{k_i}dt}\,dt\right)\right] \forall i = 1, \dots, n \tag{32}$$

The expected value of the Inventory level in the i^{th} segment corresponding to the optimal production rate is given by:

$$E[I_{it}] = e^{\int\frac{Q_i}{k_i}dt}\left[I_{i0} + \int\left(\frac{R_i}{2k_i} - D_{it}\right)e^{-\int\frac{Q_i}{k_i}dt}\,dt\right] \tag{33}$$

4. Special Cases

We now consider some special cases for the demand rate. For this we will consider the Single Source Inventory and Multi-Segmented Demand problem discussed in section 3.1.

4.1 Constant demand rate

We assume that the demand rate at time t for each segment is a constant, i.e. $D_{it} = d \quad \forall\, i = 1, \dots, n$. Hence $\sum_{i=1}^n D_{it} = nd = D(say)$.

Hence, the optimal production rate is given by:

$$P_t = D + \left[\frac{\sqrt{h/k}\, I_t\left(e^{-2\sqrt{h/k}(T-t)} - 1\right) - 2De^{-\sqrt{h/k}(T-t)}}{1 + e^{-2\sqrt{h/k}(T-t)}}\right] \tag{34}$$

Thus, the optimal production rate is equal to the Demand rate added to a correction factor.

The expected value of the Inventory level corresponding to the optimal production rate is given by:

$$E[I_t] = \frac{e^{-\sqrt{h/k}(T-t)}\left(I_0\sqrt{h}\left(1+e^{2\sqrt{h/k}(T-t)}\right)+D\sqrt{k}\left(-1+e^{2\sqrt{h/k}(T-t)}\right)\right)}{2\sqrt{h}} \tag{35}$$

4.2 Exponential demand rate

We assume that the demand rate at time t for each segment is an exponential expression given by
$D_{it} = e^{-2\sqrt{h/k}(T-t)} \quad \forall\, i = 1, \ldots, n.$ Hence $\sum_{i=1}^{n} D_{it} = ne^{-2\sqrt{h/k}(T-t)}$
Hence, the optimal production rate is given by:

$$P_t = \frac{n\left(-3+e^{-2\sqrt{h/k}(T-t)}+2e^{\sqrt{h/k}(T-t)}\right)-3\sqrt{h/k}\,I_t\left(-1+e^{2\sqrt{h/k}(T-t)}\right)}{3\left(1+e^{2\sqrt{h/k}(T-t)}\right)} \tag{36}$$

The expected value of the Inventory level corresponding to the optimal production rate is given by:

$$E[I_t] = \frac{\left(3e^{4T\sqrt{h/k}}+3e^{2\sqrt{h/k}(T+t)}\right) I_0\sqrt{h} + n\sqrt{k}\left(2e^{2T\sqrt{h/k}} - e^{3T\sqrt{h/k}} + 2e^{2t\sqrt{h/k}} - 2e^{3t\sqrt{h/k}} + e^{(3T+2t)\sqrt{h/k}} - 2e^{(2T+3t)\sqrt{h/k}}\right)}{3\sqrt{h}e^{(2T+t)\sqrt{h/k}}\left(1+e^{2T\sqrt{h/k}}\right)} \tag{37}$$

5. Numerical Results

For the case considered in section 4.1, a set of following values for the parameters have been used to illustrate the model :
h = 1;
k = 1.5;
T = 4;
I_0= 305;
D = 505;
The graphs for the Expected Optimal Inventory level and the Expected Optimal Production rate are given below:

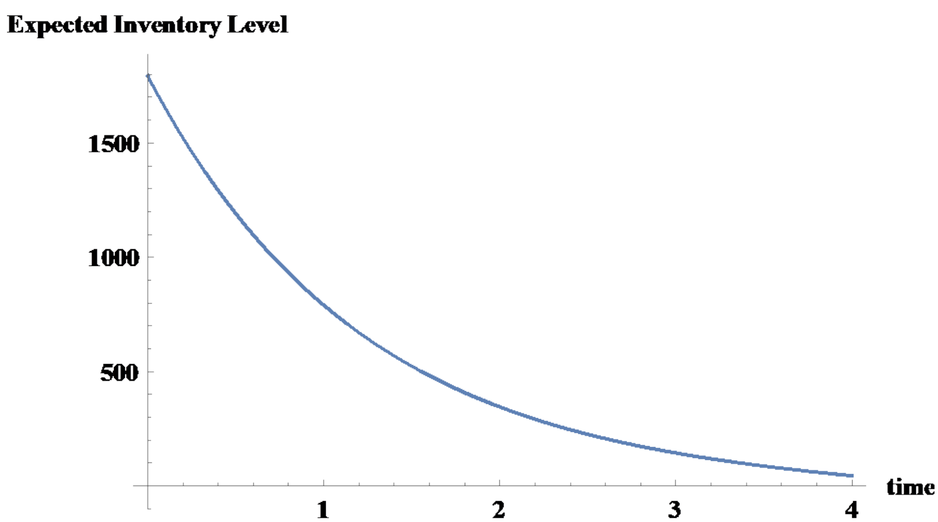

Figure 1: Expected Optimal Inventory level

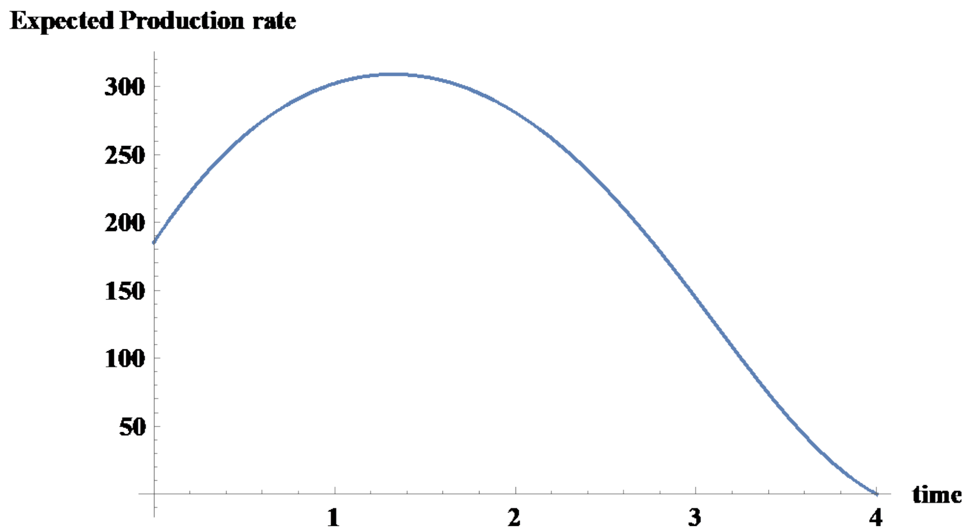

Figure 2: Expected Optimal Production Rate

6. Conclusion

In the paper a production and inventory planning problem considering the concept of market segmentation, has been formulated. The inventory levels are governed by some random fluctuations. The production units, warehouses and demands are considered in different ways. Firstly, the process of production and storing occurs at a single source and then the products are distributed in the various segments. Secondly, the processes are carried in the respective segments. Two different cases for the demand rate function are taken and a numerical example is taken to illustrate a special case. The stochastic versions of these models increase their applicability as they can reflect the real-life problems in a more effective manner.

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