Determining the Matchdays for the Turkish Super League using Non-Linear Binary Integer Programming

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Abstract

Sports scheduling has become an important and difficult research problem in recent years. Deciding which teams will play on which pitch and at what time is a challenge to sports administrators because it requires the achievement of fairness accounting for various factors that affect stakeholders. This paper proposes an integer programming model for the problem of determining the optimal matchday schedule for an entire season of Turkish Super League under two fairness criteria. The objective is to ensure that team-specific matchday distributions are as uniform as possible. We model the problem as a nonlinear binary integer program. Deviations from ideal matchday assignments obtained through the optimal schedule are compared with those of manual schedules constructed by the Turkish Football Federation. The results indicate that seasonal fixture can be significantly improved by implementing the optimal schedule.

Keywords
OR in sports, Sports scheduling, Matchday scheduling, Integer programming

1. Introduction

Sports have long been an essential part of our life. Often, we see different people from different backgrounds brought together by sport. Just as teams need fans to prosper, fans need teams to entertain them. Several organizations come together to form the basis of sports and each member contributes in one way or the other. There are certain objectives that are being targeted by different stakeholders. While fans want matches to be played on certain days to facilitate ease of attendance, TV broadcasters demand that certain matches are broadcasted on their channels. We observe such demands and objectives across all organizations and all related parties. This brings about the problem of fairness and balance in terms of scheduling sports tournaments.

Specifically, in soccer, certain restrictions need to be satisfied to achieve a fair schedule. Factors such as rest duration, breaks, carry-over effect, club's revenue, and TV broadcasting revenue should be targeted for achieving an optimal

schedule. The objective of obtaining a fair schedule may be dented by poor coordination of matches done by sports administrators. Inevitably, bad results, loss of revenue, dissatisfaction among fans, and even decline in attendance may be noticed over the course of a season. That makes scheduling an important topic of research whereby those losses are reduced. However, the fact that many criteria interrelate with one another makes it difficult to obtain a feasible schedule that satisfies all constraints.

This paper presents an optimal matchday schedule that significantly outperforms the manual schedule developed by the Turkish Football Federation (TFF). The TFF is the highest governing football body in Turkey. It manages, controls, and arranges all football related activities in the country. Among its duties are deciding which teams play, which venue hosts, what time to schedule a game, what rules are applied, etc. When finalizing the seasonal schedule for the Turkish Super League (TSL), the TFF must take into consideration the encounters in the European stage both at the club and the national level as well as the games played in the domestic Turkish Cup. TSL games are normally scheduled from Friday to Monday. Both the international games and the Turkish cup games are usually played weekdays. TFF must guarantee a minimum number of rest days for each team between consecutive matches. Furthermore, many clubs have been vocal about their matchday and time preferences in Turkish media over more than a decade citing economic and performance related concerns. Their criticisms publicly stated over the press suggest that an even distribution of matchdays not only throughout the season, but also up to any round of the fixture is a necessity to instill a perception of fairness among the clubs and the fans. What compounds the problem is the uncertainty that the single-elimination structure of the European and the Turkish Cup tournaments bring to the decision problem. Therefore, a dynamic approach is required to address multiple criteria as the results of each knockout stage in single-elimination tournaments unfold resolving the uncertainty on a gradual basis about future of the season.

We present the review of related studies in the next section. The third section is where we present a technical description of our problem. The fourth section includes the problem formulation followed by the results in the fifth section. Section 6 concludes the paper.

1.1 Objectives

The approach we take is to develop a dynamic model that addresses all the aforementioned issues germane to this decision problem. We solve a non-linear binary integer program round by round in the light of the most recent information about the progress of teams and their schedule of play in other tournaments which impose several rest duration constraints. The objective is to minimize deviations from an ideal schedule that achieves a preferable assignment of games to certain time slots in each round. We compare our results to that of the TFF for the purpose of examining the differences and possible improvements.

2. Literature Review

Sports scheduling has been a research theme in the last few decades studied by many academicians delving deep into the topic and shedding valuable insights. Such problems should be solved under the need for establishing equality among participants in competitions, meeting matchday preferences often in conflict, ensuring fans' satisfaction, increasing the profitability of TV broadcasters (see Kendall et al. 2010, Bilgesu 2016 and Ribeiro 2012). Inarguably, these are complex problems, and addressing all concerns and requirements simultaneously is a daunting task.

TSL runs according to a round robin schedule. A round robin schedule is one in which all teams play each other a specified number of times. In the Turkish League, each team meets an opponent exactly twice, one on the home pitch, the other at the opponent's venue. This format is called double round robin. As a common format for similar league tournaments, round robin scheduling has been extensively analyzed, mostly from a seasonal fixture determination point of view. A good survey of round robin scheduling can be found in Rasmussen and Trick (2008). This study gives us a broad understanding of the types of schedules and terminologies in sports scheduling. It is customary to see that leagues are scheduled as double round robin. Nonetheless, single, triple, and quadruple round robin schedules also occur in some leagues. Examples to such designs are discussed in Goossens and Spieksma (2012), Durán et al. (2007) and Rasmussen (2008).

A widely studied component of research on round robin tournaments is fixture determination. One may classify this research based on how objective functions are formulated and which criteria are addressed in the solution. In addition to the rest balance and matchday distribution criteria explained in the introduction and addressed in this paper, there are other criteria that should be handled in the fixture phase. For example, in any league format, home and away games

should occur in sequence. However, this cannot be achieved for the entire season and consecutive home or away games that are called breaks are observed. Minimization of such breaks is an objective in Rasmussen and Trick (2007), van't Hof et al. (2010), Durán et al. (2017), Recalde et al. (2013), and Bulck and Goossens (2020).

In most of the work, integer programming is used to solve sports scheduling problems, but other methods and heuristics can also be employed when exact solution approaches fail. Briskorn and Drexl (2009) proposed a formulation using integer programming for round robin schedules by adapting constraints on fairness for breaks, allowance of forbidden games, and the carryover effect. Another application of integer programming on minimization of carryover effects is by Guedes and Ribeiro (2011). The authors used integer programming to minimize weighted carryover effects and brought forth a heuristic that offers an effective solution. Günneç and Demir (2019) and Recalde et al. (2013) follow a similar technical approach (albeit with varying objectives) for the Turkish Super League and the Ecuadorian football league, respectively. Focusing on another branch of sports, we see in Kim (2019) an analysis of the Korean baseball league schedules with an emphasis on minimizing travel distance and establishing match fairness through integer programming. The author also proposes a heuristic algorithm that supports large-sized problems.

This paper is an extension to an earlier work by Göçgün and Bakır (2021). We improve on this prior work in two significant respects. First, the precision of the schedules we generate is augmented by the inclusion of daily time slots. Second, through the addition of time slots, we are able to factor in constraints for the assignment of popular games to specific time slots and for prevention of simultaneous scheduling of certain matches to ensure operational feasibility. In the next section, we elaborate more on the specifics of our model as well as the differences with Göçgün and Bakır (2021).

3. Methods

3.1. Problem Definition

Turkish Super League (TSL) is the top professional soccer league in Turkey. Scheduled as a double round robin, the league starts in August and ends in May. Teams play each other twice over the course of a season with one game played at home venue and another at opponent's venue in the reversed fixture. At the end of the tournament, the team with the highest points is crowned champion of the league while the three teams at the bottom get relegated to the second league. Based on the end-of-season rankings and Turkish Cup performances, teams qualify to participate various tournaments organized by UEFA, Europe's highest football authority. The number of teams that achieve this status may change from one year to another based on the past levels of success attained by the Turkish teams in European tournaments and by the national team in various competitions.

Since they compete in multiple tournaments over the course of the season, fixture loads of the league participants are not the same. This adds another level of complexity to the matchday scheduling problem and is usually another source of the criticisms raised frequently by the stakeholders. As indicated earlier, each round is scheduled from Friday to Monday. Let #(*fri*), #(*sat*), #(*sun*), and #(*mon*) symbolize variables for the number of games scheduled in each round on Friday, Saturday, Sunday, and Monday, respectively. Like in Göçgün and Bakır (2021), we assume for the most rounds in baseline case analyses that #(*fri*) = 1, #(*sat*) = 3, #(*sun*) = 4, and #(*mon*) = 1. While there is no written rule or a publicly disclosed preference by the league administrators to follow this format, this is in practice the most common daily distribution of games in any round. We further specify the rough time slots of the games assigned for Saturdays and Sundays, which is an important detail not available in Göçgün and Bakır (2021).

One remark should be made right here about one critical aspect of our problem formulation. While the relevance of the approach described in this paper is not limited to a specific round robin league format, our focal point is the football enterprise in Turkey and hence we demonstrate its utility in TSL. In the most recent 2020-21 season, there were 21 teams in TSL, which was unusual because relegation was canceled at the end of the 2019-20 season citing the negative influence of the global pandemic. As COVID-19 pandemic reduces its spread around the globe with vaccination efforts, TFF plans a gradual transition to the format in the pre-pandemic period with 18 teams. In fact, as a step into that direction, the 2021-22 season will be organized with 20 teams. While for various reasons changes in the number of teams are not historically unheard of, TSL was stabilized in this sense with 18 teams from 1994 until the breakup of the pandemic. Against this backdrop, it is our plan in this research to continue under the assumption of an 18-team organization in TSL ignoring the temporary health-related modifications to the format and illustrate the improvements using data obtained from seasons up to 2019.

Table 1. Total seasonal day distribution for top teams 2014-2019

Seasonal Day Distributions 2014-2019				
	Friday	Saturday	Sunday	Monday
Galatasaray	20	72	55	19
Başakşehir	17	60	68	22
Beşiktaş	18	48	67	33
Trabzonspor	16	61	61	28
Fenerbahçe	13	49	76	29

Table 1 displays the game day distributions of the top teams in TSL. All the teams are indisputably the most popular for historical reasons except Başakşehir FK. Başakşehir FK was included in the list for its gradually rising seasonal rankings in the last decade which eventually culminated in their first ever championship in 2020. This table gives us an idea about the matchday distribution imbalance in TSL. For instance, we see that Beşiktaş played 33 times on Mondays during that period whereas Galatasaray played 19 games. The picture is similar for the remaining participants of the league as well.

TFF schedules Friday and Monday games in the evenings for not risking attendance during a workday. Ideally, one game apiece should be scheduled on those days. Three time slots are modeled for Saturdays and Sundays. These time slots refer to three periods of the day: noon, afternoon, and evening. Based on historical data, we set the ideal in greater part of the season to have one game at each period on Saturdays whereas Sundays should have one game in the noon, another in the afternoon, and two scheduled in the evening. In early parts of the season, the picture is different, though. In examining the scheduling pattern for early months of the season in four consecutive seasons from 2015 to 2019, we obtain the evening match percentages. Statistics indicate that games are not frequently scheduled to the noon and afternoon slots in the beginning of the season. In fact, most fixtures occur in the evening. This can be largely attributed to the hot weather conditions. Therefore, the ideal time slot distribution is allowed to be different for the earlier months of the season.

Based on the observations made on games played each day, we set the time slot distribution in each round as shown in Table 2. In other words, Table 2 is a portrayal of how the ideal time slot distributions are in our analyses that we later provide in this paper for the 2018 - 19 season. One more note should be added to clarify the numbers in the table. The occasional zeros for Mondays have been entered to account for the national team meetings in the European stage. TFF never schedules a match on Mondays if the national team is scheduled to play several qualification games for the UEFA or FIFA (which is highest football authority in the world) organizations after a specific TSL round.

Table 2. Ideal time slot distributions used in analyzing the 2018 - 19 season.

Round	**Friday**	**Saturday noon**	**Saturday afternoon**	**Saturday evening**	**Sunday noon**	**Sunday afternoon**	**Sunday evening**	**Monday**
1 to 3	1	0	0	3	0	0	4	1
4	1	0	1	3	0	1	3	0
5 to 7	1	0	1	2	0	1	3	1
8	1	1	1	2	1	1	2	0
9 to 11	1	1	1	1	1	1	2	1
12	1	1	1	2	1	1	2	0
13 to 25	1	1	1	1	1	1	2	1
26	1	1	1	2	1	1	2	0
27 to 33	1	1	1	1	1	1	2	1
34	1	1	1	2	1	1	2	0

The scheduling problem we address in this work is to determine matchdays as well as rough time slots (for certain days) of matches for each round of the season. The details of our mathematical formulation used for solution are given next.

3.2. Mathematical Formulation

We formulate the matchday scheduling problem as a nonlinear binary integer program for each round where the objective function consists of a weighted sum of two matchday distribution metrics. The formulation that we propose in this paper should be solved for each round as up-to-date information from other tournaments is received. In an 18-team format, there are 34 rounds. Therefore, our proposed procedure requires an iterative solution of 34 nonlinear binary integer programs. For brevity, we present the components of the mathematical model only for the 10th round of the 2018-19 season. This choice is not arbitrary; there are a significant number of games in the Turkish Cup and in various UEFA organizations either right before or after this round that should be dealt with in the model. In the next section though, we present a comparative analysis for the entire season. The fixture of the 10th round is given in Table 3.

Table 3. The round 10 fixture of the 2018-19 season.

Home Team	Home Team Index	Away Team Index	Away Team
Bursa	16	9	Alanya
Erzurum	17	14	Kasımpaşa
Konya	8	2	Başakşehir
Antalya	7	4	Trabzon
Kayseri	10	12	Sivas
Malatya	5	1	Galatasaray
Fenerbahçe	6	13	Ankaragücü
Akhisar	18	15	Göztepe
Beşiktaş	3	11	Çaykur Rize

We first explain the rationale behind the matchday distribution metrics that are part of the objective function. The objective function is expressed as:

$$\text{Min } c_1 \text{ x } \sum_{\text{d2} \in \text{DS}} w_{d2} \text{ x } D_{d2} + c_2 \text{ x V} \quad (1)$$

where
D_{d2}: seasonal matchday deviation metric where d2 ∈ DS = {Fri, Sat, Sun, Mon},
V: round-specific matchday deviation metric,
c_1: weight assigned to the summation of the weighted seasonal matchday deviations,
c_2: weight assigned to the round-specific matchday deviation,
w_{d2} : weight assigned to deviation D_{d2} in the objective function.

It is worth noting that seasonal and round-specific deviation metrics were first introduced in Göçgün and Bakır (2021). We adapt these metrics into our formulation for solving the underlying scheduling problem, the unique aspect of which is the level of detail in match assignments. In particular, assignments are made to rough time slots on matchdays, which is much more realistic compared to that in Göçgün and Bakır (2021).

As mentioned earlier, two issues arise in arranging the matchdays fairly. First and arguably the more major of the two is regarding the equitable assignment of days to teams from Friday to Monday throughout the season. Main challenge here is to maintain a fair assignment not only by the end of the season, but also up to any round in the season as well. The deviation metric in the objective function (1) has the term D_{d2} to judge the extent to which we achieve the first fairness criterion. It takes a different functional form for each day of the round and has to be calculated separately for each team. The second issue is about the assignment of 9 games in each round from Friday to Monday according to a reasonable pattern that satisfies several constraints on rest time as well as other organizational requirements. The metric in (1) used for this purpose is V. V calculates a penalty term for deviating from the scheduling pattern.

Matchday distribution metrics D_{d2} to V are explicitly stated below. The seasonal deviation metric D_{d2} is inspired by the goal chasing metric in the Monden heuristic which is a method used for production sequencing on assembly lines in accordance with just in time principles. This heuristic keeps the actual usage rate of items remain as close as possible to their average usage rates on an assembly line (see Vollmann et al. 2004 p. 662 as well as Göçgün and Bakır 2021 for details). The underlying logic of keeping a constant rate production for each component on a continuous basis is similar to that of maintaining a uniform distribution of matchdays across the teams in each round of the season.
We are now in a position to state our matchday distribution metrics:

$$D_{d2} = \sum_{i=1}^{I}[x_{i,r,d2} + \sum_{k=1}^{r-1} x_{i,k,d2} - (r \text{ x } a_{d2})]^2, \text{ if d2 = \{Fri, Mon\}}, \quad (2)$$

$$D_{d2} = \sum_{i=1}^{I}[x_{i,r.Sat1} + x_{i,r,Sat2} + x_{i,r,Sat3} + (\sum_{k=1}^{r-1} x_{i,k,Sat1} + x_{i,k,Sat2} + x_{i,k,Sat3}) - (r \text{ x } a_{d2})]^2, \text{ if d2 = \{Sat\}}, \quad (3)$$

$$D_{d2} = \sum_{i=1}^{I}[x_{i,r,Sun1} + x_{i,r,Sun2} + x_{i,r,Sun3} + (\sum_{k=1}^{r-1} x_{i,k,Sun1} + x_{i,k,Sun2} + x_{i,k,Sun3}) - (r \text{ x } a_{d2})]^2, \text{ if d2 = \{Sun\}}, \quad (4)$$

and

$$V = \sum_{d \in DS2}\left(\frac{\sum_{i=1}^{I} x_{ird}}{2} - b_d\right)^2, \quad (5)$$

where

$$x_{ird} = \begin{cases} 1, & \text{if team i plays in round r on day/period d} \\ 0, & \text{otherwise} \end{cases}$$

are the decision variables of our mathematical model. Here i = 1, 2, . . ., I where I = 18; r = 1, 2, . . ., R where R = 34; d ∈ CD = {Wed, Thur, Fri, Sat1, Sat2, Sat3, Sun1, Sun2, Sun3, Mon, Tue}. I and R symbolize the number of teams and the number of rounds, respectively. The parameters in expressions (2) to (5) are introduced below:

w_{d2} : weight assigned to deviation D_{d2} in the objective function where d2 ∈ DS = {Fri, Sat, Sun, Mon},
c_1: weight assigned to the summation of the weighted deviations,
c_2: weight assigned to deviation V,
a_d: the number of matches that should be scheduled on day *d* in a single round, *d* in DS, under ideal conditions,
$b_d(r)$: the ideal number of games during a time slot/period *d* in DS2 in round *r*, where DS2 = {Fri, Sat1, Sat2, Sat3, Sun1, Sun2, Sun3, Mon}

In set DS2, Sat1 denotes Saturday noon time slot whereas Sat2 and Sat 3 denote Saturday afternoon and evening time slots, respectively. Similar definitions are made for Sun1, Sun2, and Sun3. As mentioned earlier, both deviation metrics are like the ones used in Göçgün and Bakır (2021) with slight differences due to the added precision in scheduling coming with the introduction of time slots. A similar non-linear structure in the objective function is also available in Yavuz et al. (2008) where the aim is to accomplish a fair assignment of referees by keeping a relatively balanced officiating load in the entire season and assigning each referee to an equal number of games played by each participating team.

Some explanations of the deviation metrics presented in Equations (2) to (5) are in order. The seasonal deviation metric in Equation (2) has the same structural form for games played on Fridays and Mondays. Up to any round *r*, the number of games that should be played on a Friday or a Monday is $r \text{ x } a_{Fri}$ or $r \text{ x } a_{Mon}$, respectively. Terms $r \text{ x } a_{Fri}$ and $r \text{ x } a_{Mon}$ do not give us integer quantities for most rounds much like an ideal number of components that should be processed on an assembly line for any number in production sequence in Monden heuristic is not an integer. The expression $\sum_{k=1}^{r-1} x_{i,k,d2}$ in (2) consists of binary decision variables that store information about day assignments up to round *r*. Using the optimal values of decision variables in mathematical programs developed for all the rounds prior to *r*, $\sum_{k=1}^{r-1} x_{i,k,d2}$ yields the sum of past matchday assignments for a specific team on either Friday or Monday. The only term including a binary decision variable of the program is $x_{i,r,d2}$; accordingly the summation $x_{i,r,d2} + \sum_{k=1}^{r-1} x_{i,k,d2}$ d2 ∈ {Fri, Mon} gives us the number of Friday or Monday games that are scheduled by the end of round

r. Since *r* x a_{d2} d2 ∈ {Fri, Mon} is the ideal number, the difference gives us the deviation. Square is taken to avoid potential negatives.

Metrics in (3) and (4) represent seasonal deviations for Saturdays and Sundays, respectively. Those metrics are developed under the same idea. For all the metrics in (2) to (4), the value of a_d is crucial. It is given by

$$a_d = \begin{cases} \frac{s^*}{34} & \text{if d} = \text{Fri}, \\ \frac{t^*}{34} & \text{if d} = \text{Sat}, \\ \frac{u^*}{34} & \text{if d} = \text{Sun}, \\ \frac{v^*}{34} & \text{if d} = \text{Mon}, \end{cases} \tag{6}$$

where (s^*, t^*, u^*, v^*) symbolize the fair split of matchdays over the season for any team. Here s^*, t^*, u^*, and $v^*$are the number of matches that should be assigned to any team on Fridays, Saturdays, Sundays and Mondays, respectively. Since we perform our numerical analysis for an 18-team seasonal format, $s^* + t^* + u^* + v^* = 34$ (Göçgün and Bakır 2021).

The round-specific metric is given in (5). It measures the squared deviation from the ideal matchday time slot distribution. The value of $b_{d(r)}$ for each round *r* is obtained from Table 2. For instance, $b_{Sat2(5)} = 1$, which means that in round 5, the ideal case decision is scheduling 1 match on a Saturday afternoon.

3.2.1. The binary integer program for a specific round

We now exemplify how the non-linear binary integer formulation is constructed for each round. We select the 10th round of the season, the fixture of which is given in Table 3 (i.e., *r* = 10 in the below model).

The objective function:

Min c_1 x $\sum_{\text{d2} \in \text{DS}} w_{d2}$ x D_{d2} + c_2 x V

Feasibility constraints:

$$x_{i,r,Fri} + x_{i,r,Sat1} + x_{i,r,Sat2} + x_{i,r,Sat3} + x_{i,r,Sun1} + x_{i,r,Sun2} + x_{i,r,Sun3} + x_{i,r,Mon} = 1, \quad i = 1,2,..,18 \tag{7}$$

$$x_{i,r,Thur} + x_{i,r,Fri} + x_{i,r,Sat1} + x_{i,r,Sat2} + x_{i,r,Sat3} \leq 1, \quad i = 1,2,..,18 \tag{8}$$

$$x_{i,r,Sun1} + x_{i,r,Sun2} + x_{i,r,Sun3} + x_{i,r,Mon} + x_{i,r,Tue} \leq 1, i = 1,2,..,18 \tag{9}$$

$$x_{i,r,Wed} + x_{i,r,Thur} + x_{i,r,Fri} \leq 1, \quad i = 1,2,..,18 \tag{10}$$

$$x_{i,r,Mon} + x_{i,r,Tue} + x_{i,r,Wed} \leq 1, \quad i = 1,2,..,18 \tag{11}$$

$$x_{i,r,Wed} + x_{i,r,Sat1} + x_{i,r,Sat2} \leq 1, \quad i = 1,2,..,18 \tag{12}$$

$$x_{i,r,Thur} + x_{i,r,Sun1} + x_{i,r,Sun2} \leq 1, \quad i = 1,2,..,18 \tag{13}$$

$$x_{i,r,Sat1} = 0, \text{i} \in \{\text{GS, FB, BJK, BSK, TS}\} \tag{14}$$

$$x_{i,r,Sun1} = 0, \text{i} \in \{\text{GS, FB, BJK, BSK, TS}\} \tag{15}$$

$$\sum_{\text{i} \in T_{ist}} x_{i,r,d} \leq 2, \quad \text{d} \in \{\text{Fri, Mon}\} \tag{16}$$

$$\sum_{\text{i} \in T_{ist}} x_{i,r,Sat1} + x_{i,r,Sat2} + x_{i,r,Sat3} \leq 2 \tag{17}$$

$$\sum_{i \in T_{ist}} x_{i,r,Sun1} + x_{i,r,Sun2} + x_{i,r,Sun3} \leq 2 \quad (18)$$

$$\sum_{i \in T_{ist}} x_{i,r,k} \leq 1, \qquad k \in \{Sat2, Sat3, Sun2, Sun3\} \quad (19)$$

$$\sum_{i \in T_{tophome}} x_{i,r,k} \leq 1, \qquad k \in \{Sat2, Sat3, Sun2, Sun3\} \quad (20)$$

where GS, FB, BJK, BSK and TS are acronyms for Galatasaray, Fenerbahçe, Beşiktaş, Başakşehir, and Trabzon, respectively and

T_{ist}: Set of Istanbul-based teams having home games in round 10 (or any round for which the formulation is made),

$T_{tophome}$: Set of top teams in the league playing at home pitch, where top teams are Galatasaray, Fenerbahçe, Beşiktaş, Başakşehir, Trabzon.

Assignment constraints:

$$x_{16,r,d} - x_{9,r,d} = 0, \qquad d \in DS2 \quad (21)$$

$$x_{17,r,d} - x_{14,r,d} = 0, \qquad d \in DS2 \quad (22)$$

$$x_{8,r,d} - x_{2,r,d} = 0, \qquad d \in DS2 \quad (23)$$

$$x_{7,r,d} - x_{4,r,d} = 0, \qquad d \in DS2 \quad (24)$$

$$x_{10,r,d} - x_{12,r,d} = 0, \qquad d \in DS2 \quad (25)$$

$$x_{5,r,d} - x_{1,r,d} = 0, \qquad d \in DS2 \quad (26)$$

$$x_{6,r,d} - x_{13,r,d} = 0, \qquad d \in DS2 \quad (27)$$

$$x_{18,r,d} - x_{15,r,d} = 0, \qquad d \in DS2 \quad (28)$$

$$x_{3,r,d} - x_{11,r,d} = 0, \qquad d \in DS2 \quad (29)$$

Additional constraints:

$$x_{1,r,Wed} = 1 \quad (30)$$

$$x_{3,r,Thur} = 1 \quad (31)$$

$$x_{18,r,Thur} = 1 \quad (32)$$

$$x_{7,r,Tue} = 1 \quad (33)$$

$$x_{14,r,Tue} = 1 \quad (34)$$

$$x_{16,r,Tue} = 1 \quad (35)$$

$$x_{8,r,Wed} = 1 \quad (36)$$

$$x_{10,r,Wed} = 1 \quad (37)$$

$$x_{5,r,Wed} = 1 \quad (38)$$

$$x_{9,r,Wed} = 1 \quad (39)$$

$$x_{4,r,Wed} = 1 \quad (40)$$

$$\sum_{d \in CD} x_{i,r,d} = 1, \quad i = 1, \ldots, I \;\&\; i \neq \{1,3,4,5,7,8,9,10,14,16,18\} \quad (41)$$

A brief explanation of the purpose for imposing the constraints in expressions (7) to (41) is provided here. Constraint (7) permits only one slot for each team in every round. Constraints (8) to (13) are required because of the minimum rest restrictions and enforces that each team plays only one game on the respective days. To restate more succinctly, each team must have at least two full days of rest between the matchdays. Constraints (14) and (15) prevent a noon time schedule for the top five teams. Since Istanbul is the only city that has more than 2 teams participating in the league, constraints (16) through (18) enforce that at most two teams based in the same city (which is Istanbul for the underlying season) can be scheduled to the same day. Constraint (19) states that although two teams from the same city can be scheduled on the same day, they must not be scheduled at the same time. Constraint (20) ensures that the schedules of the top five teams do not overlap.

Feasibility constraints are followed by the fixture-specific assignment constraints. Those constraints (21 through 29) ensure that the match of two teams that play against each other in the respective round is assigned to the same time slot. They are followed by constraints (30) to (40) that are imposed according to fixtures in other tournaments that affect TSL. Finally, constraint (41) ensures that teams that do not have extra matches are scheduled to only one day (or one time slot).

Since our model is dynamic in nature, we modify constraints when solving the binary integer program for a given round, according to the games preceding/succeeding that round. We solve the integer program for each round using AMPL, CPLEX. In the next section, we discuss the results of our comparative analysis for the 2018-19 season.

4. Results and Discussion

First, it is worth pointing out the baseline assumptions made in this work. In line with Göçgün and Bakır (2021), our model assumes that under ideal conditions $s^* = v^* = 5$ and $t^* = u^* = 12$. Round-specific ideal matchday distributions are as given in Table 2. The weights for the objective function terms are $c_1 = 1$; $c_2 = 1$; $w_{Fri} = 0.11$, $w_{Sat} = 0.33$, $w_{Sun} = 0.44$, $w_{Mon} = 0.11$.

Table 4 shows the seasonal matchday distribution rendered by the manual TFF schedule. For each team, the total games played on all the available days are recorded. To illustrate, we observe that Galatasaray played 7 games on Friday, 11 on Saturday, 13 on Sunday and finally 3 games on Monday throughout the season. This is in sharp contrast with Akhisar's matchday distribution where only 1 game took place on Friday, 11 on Saturday, 15 on Sunday and 7 on Monday. It is this inequity that we aim to avoid through the optimal schedule. In comparison, Table 5 displays the distribution given by the optimal schedule obtained by our model. A significant improvement that one can even spot visually is there. In spite of the added constraints that may have forced the system to deviate from ideal, the results are pretty aligned with our ultimate objectives.

Table 4. The seasonal matchday distribution resulting from the manual TFF schedule

Team	Team index	Friday	Saturday	Sunday	Monday
Galatasaray	1	7	11	13	3
Başakşehir	2	4	10	13	7
Beşiktaş	3	5	10	13	6
Trabzon	4	6	13	11	4
Malatya	5	4	10	16	4
Fenerbahçe	6	4	11	11	8
Antalya	7	3	13	13	5
Konya	8	4	11	14	5
Alanya	9	4	11	12	7
Kayseri	10	1	17	13	3
Çaykur Rize	11	3	14	12	5
Sivas	12	5	13	13	3
Ankaragücü	13	4	12	14	4
Kasımpaşa	14	5	11	10	8
Göztepe	15	5	14	12	3
Bursa	16	7	10	15	2
Erzurum	17	4	11	15	4
Akhisar	18	1	11	15	7

Table 5. The seasonal matchday distribution resulting from the optimal schedule.

Team	Team index	Friday	Saturday	Sunday	Monday
Galatasaray	1	5	12	13	4
Başakşehir	2	4	12	14	4
Beşiktaş	3	4	12	13	5
Trabzon	4	5	12	13	4
Malatya	5	4	12	13	5
Fenerbahçe	6	4	12	13	5
Antalya	7	4	13	13	4
Konya	8	4	12	13	5
Alanya	9	5	12	13	4
Kayseri	10	5	12	13	4
Çaykur Rize	11	4	12	14	4
Sivas	12	4	12	14	4
Ankaragücü	13	5	12	13	4
Kasımpaşa	14	4	12	13	5
Göztepe	15	5	13	12	4
Bursa	16	5	12	12	5
Erzurum	17	4	12	13	5
Akhisar	18	5	12	12	5

Table 6. The seasonal day distribution data summary for the manual TFF schedule.

	Friday	Saturday	Sunday	Monday
Standard deviation	1.63	1.86	1.59	1.88
Maximum value	7	17	16	8
Minimum value	1	10	10	2
Range	6	7	6	6

Table 7. The seasonal day distribution data summary for the optimal schedule.

	Friday	Saturday	Sunday	Monday
Standard deviation	0.51	0.32	0.59	0.51
Maximum value	5	13	14	5
Minimum value	4	12	12	4
Range	1	1	2	1

Tables 6 and 7 provide some basic statistics that demonstrate the practical utility of our model. Using data in Tables 4 and 5, we compute the sample standard deviations for each potential matchday and report the range in data. Sample standard deviations are good measures of fairness in seasonal matchday distribution: the smaller the better. We observe that standard deviations are reduced by 60 to 85% for each matchday. Change in ranges is even more striking: the minimum value for the range is 6 with the manual TFF schedule whereas the optimal schedule reduces the range statistic to a maximum of 2. Such a drastic change in the range of each potential matchday's distribution is a clear indication of the improvement in terms of the seasonal fairness criterion that we seek to achieve in this study. Our model reduces the seasonal imbalance by almost 80%, which should address the publicly stated concerns in this regard.

6. Conclusion

In this paper, we investigate whether a matchday schedule that strikes a good balance among the participant teams in the Turkish Super League (TSL) can be achieved. The problem of interest is motivated by the widespread criticisms pronounced by various club administrators in Turkey on unfair matchday distribution among the football teams. Since

football still retains its position as one of the most popular sport branches, there is a constant need to make it better and eliminate any perception of unfairness. Accordingly, we aim to find an optimal schedule for TSL whereby matches are assigned to days and time slots consistently in a fair manner.

Two types of deviation from an ideal are mainly discussed here. One measures the degree to which teams have equal number of encounters scheduled on particular days of each of the 34 rounds throughout the season. The second is about the extent the games are assigned to selected time slots in alignment with an ideal that seeks to increase spectator interest. Since the criticisms on the current decisions made the Turkish Football Federation (TFF) suggest a dynamic approach, we propose a method in which weighted sum of these two deviations are minimized on a round basis subject to various types of constraints. The non-linear objective function which is a generalized version of a well-known squared sum of deviations is without units; hence, comparative analyses are made based on practical implications of the solutions.

The results are promising. Our solutions ensure major improvements to the current schedule used by the TFF. Results show that the seasonal deviations can be substantially decreased and imbalance in the schedule be reduced to the minimal. Further, even in the worst case of improvement, the range of seasonal matchday distributions among teams is reduced to one-third (on Sunday).

Opportunities for extension into future research also exist. Rest time between consecutive meetings was not actively treated in this paper. Instead, we impose several constraints that guarantee minimum number of days between consecutive meetings. One may explore the effect of bringing rest time actively into the decision-making process. Another line of extension may be about reducing the negative impact of the carryover effect.

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