Optimal Production Run Length of a Deteriorating Process with Investment in Setup and Quality Improvement

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Abstract

Process deterioration is a leading cause of mediocre production plan adherence and hence not satisfying customers' demand. Process setup and quality improvement are two common factors that return the production system to its designated status. The proposed research integrates production control, capital investment, and quality control aspects to enhance the reliability and productivity of the production-inventory system. To achieve this goal, a mathematical model is formulated to decide the optimal production run length (OPRL), cost of setup, and cost of process quality improvement. It is assumed that the production process starts in an in-control state with a fixed defective rate and may shift to an out-of-control state with a linear increase of defective rate. A numerical example is provided to demonstrate model practicability and to derive managerial insights. For instance, investment in setup cost reduction and quality improvement can achieve a 61.07% reduction in the total cost.

Keywords
Production-inventory management; Process deterioration; Production cycle; Capital investment.

1. Introduction
In today's highly competitive world markets, businesses need to employ a cost-effective production-inventory policy to meet customer demand. The optimal production run length (OPRL) model is an extension of the Economic Order Quantity (EOQ) and Economic Production Quantity (EPQ) models. The classical OPRL is one of the most popular and appealing issues of production-inventory system management. The run length of the production is determined economically to minimize the total inventory and production costs by balancing the inventory holding cost and the average fixed ordering cost. Many models with more complicated and/or practical assumptions, on the other hand, have been thoroughly investigated in recent years. Furthermore, the OPRL model has had limitations over the years and cannot be considered a universal model (Chiu et al., 2010; Taleizadeh et al., 2014), as many key assumptions of the classical OPRL model relax factors that are inevitable in many manufacturing systems.

The classical OPRL model assumes that the production process starts in an in-control state, and the status may change after an uncertain time to an out-of-control state. It is assumed that no defective parts are produced during the in-control state, and all produced items are of perfect quality, i.e., the system is failure-free. Although in the real production systems, quality is not at zero-level defective during the in-control period. Different policies are adopted in the industry, e.g., discard, repair or rework, to deal with the defective items based on the item type and degree of imperfectness. Moreover, quality has a considerable share of the total cost. Therefore, the inventory management plan proposed by the traditional model might be inappropriate (Hou, 2007).

Although several research studies have modeled the production-inventory system, they have ignored the effect of investment in quality improvement on the system's performance and reliability. This research contributes to the literature by incorporating various aspects, such as the deterioration of the production process, investment in quality improvement during the in- and out-of-control periods, and investment in setup-cost reduction. The process has a constant defective rate during the in-control period, and as the shift occurs, the process deterioration begins to increase to the end of the cycle.

The subsequent sections of this paper are organized as follows: section 2 reviews the literature. Next, section 3 presents the notations and assumptions and derives the proposed mathematical model. Then, the solution of the model is developed, and a numerical example is solved in section 4. Eventually, conclusions and recommended future research are provided in section 5.

2. Literature Review

Several studies are conducted to determine the OPRL for the production-inventory integrated system. However, so far, most of them have ignored the investment needed to improve the process's quality. The reviewed papers are presented chronologically to demonstrate the literature's state-of-the-art by discussing the models' main aspects, such as assumptions, objective function, decision variables, and solution methodology.

Rosenblatt and Lee (1986) investigated the consequences of an imperfect production process on the OPRL. They assumed that the process starts in an in-control state and shifts to an out-of-control state, then the process starts deteriorating. The time to shift is assumed to be a random variable following an exponential distribution. Later on, (Kim et al., 1999) generalized (Rosenblatt et al., 1986) work by assuming that the time to shift is arbitrarily distributed. They examined the effect of process deterioration under constant, linear, and exponential trends.

Yeh et al. (2000) formulated the production deterioration as a two-state continuous-time Markov chain. However, they could not achieve a closed-form expression for the total cost, i.e., cost of restoration and holding inventory, and were satisfied with an approximate solution. Kim et al. (2001) considered the OPRL and the optimal number of inspections together. Chung and Hou (2003) generalized (Kim et al., 1999; Rosenblatt et al., 1986) models by allowing shortage. While (Chen et al., 2006) assumed that shortage is allowed and completely backordered, and products sold with a free minimal repair warranty.

Rahim and Al-Hajailan (2006) studied an inventory-production system with a variable production rate of imperfect items during the out-of-control state. Chiu (2007) and Chiu et al. (2007) considered different actions for the imperfect items, such as rework and scrape, and stochastic machine breakdowns. They implemented the no-resumption policy and the renewal reward theorem to deal with the variable cycle length. They used the bisection method based on the intermediate value theorem to obtain the OPRL because they could not find a closed-form expression.

Hou (2007) proposed an algorithm to determine the OPRL, setup cost, and process quality. They investigated the advantages of extended capital investments in setup cost reduction and quality improvement by extending the proposed models (Rosenblatt et al., 1986; Kim et al., 1999). They found that investing in reducing setup costs decreases the OPRL and lot size. On the other hand, investment in improving quality increases production run length and lot size. Further relevant research on capital investment in setup cost reduction (Hofmann, 1998; Sarker et al., 1997).

Lin and Lin (2007) and Ma et al. (2010) treated the produced items during the out-of-control period as scrape with additional cost. The objective was to minimize the expected unit cost, e.g., setup, production, holding inventory, screening, and defective costs. In comparison, (Hu et al., 2009; Shih et al., 2016) assumed that all defective items could be reworked. Therefore, they implemented a policy of reworking all produced items without inspection after the shift occurs. Lee (2009) formulated the maintenance cost to restore the system to the in-control status, and defective items are sold with a free minimal repair warranty. They suggested that a shorter production run length is preferred to minimize warranty costs, consistent with the conclusion (Rahim et al., 2011). In addition, although it may lead to higher setup and restoration costs, it may result in fewer defective items.

Wang and Tang (2009) assumed that the time to shift is a fuzzy variable considering the rework cost of the defective items during the out-of-control state. Later on, (Hu et al., 2010) used the bisection method to find the OPRL after obtaining the lower and upper bounds.

Pearn et al. (2011) separated the production process into two stages. In stage 1, the raw material is transformed into a semi-finished product, and in stage 2, the semi-finished product is transformed into a finished product. They considered the OPRL and process quality as decision variables to reduce production accumulation between the two stages. Wee et al. (2013) and Hsu et al. (2016) considered that the inspection rate could be less than the production rate, so the backorder occurs. Salmasnia et al. (2017) minimized the total cost of holding, ordering, maintenance, sampling, and quality control to determine the OPRL. They considered a production system with multiple assignable causes and used a variable sampling interval policy to keep the failure rate constant across sampling intervals. Chen et al. (2017) integrated production and marketing characteristics and assumed that the warranty period for an imperfect production system is a function of the product's selling price. Therefore, the OPRL and the warranty period were the decision variables.

Fekri (2019) categorized defective items as repairable or non-repairable. Furthermore, after producing a certain number of defective items, the production system is shut down for maintenance. Improving the learning rate decreased the setup and production times and the number of defective items. Öztürk (2021) investigated the impact of two inspection policies, namely in- and after-production, on the expected profit. According to the numerical analysis, the highest profit can be obtained if the inspection cost is the same in both scenarios.

To summarize, the proposed research is an attempt to fill a gap identified in the literature review by integrating various aspects into a model, such as production process deterioration, investment in quality improvement during the in- and out-of-control periods and investment in setup-cost reduction.

3. Model Development

This section describes the steps used to formulate the proposed model.

3.1 Notation

Part of the following notation is mainly adapted from (Hou, 2007) to develop the proposed model.

TAC: Total Annual Cost
D: demand rate in units per unit time
P: production rate in units per unit time ($P > D$)
T: production cycle length
t: production run length in each cycle ($t < T$)
t_{opt}: optimal production run length in each cycle
K: setup cost for each production run without investment
K_{inv}: setup cost for each production run after the investment
K_{opt}: optimum setup cost for each production run
h: holding cost of unit per unit time
s: rework cost for an imperfect unit
$E(N)$: expected number of imperfect units during the production run
$E(N_0)$: expected number of imperfect units during an in-control period
$E(N_1)$: expected number of imperfect units during an out-of-control period
α: proportion of imperfect units during the in-control state without the investment
α_{inv}: proportion of imperfect units during the in-control state after the investment
α_{opt}: optimal proportion of imperfect units during the in-control state
β: proportion of imperfect units during the out-of-control state without the investment
β_{inv}: proportion of imperfect units during the out-of-control after the investment
β_{opt}: optimal proportion of imperfect units during the out-of-control
$Ø_k$: capital investment in setup cost reduction
$Ø_\alpha$: capital investment in process quality improvement in the in-control state
$Ø_\beta$: capital investment in process quality improvement in the out-of-control state
a: fraction of the reduction in K per dollar increase in investment
b: fraction of the reduction in α per dollar increase in investment
c: fraction of the reduction in β per dollar increase in investment
i: capital cost per dollar per year

3.2 Assumptions

The following assumptions are proposed to formulate the suggested mathematical model has, and figure 1 depicts the assumptions:

1. The production process begins in an in-control state and may shift to an out-of-control state.
2. The demand is assumed to be deterministic, and the setup time is negligible, i.e., the setup time is zero.
3. The elapsed time to the process shift, X, is assumed to be exponentially distributed with a mean $1/\lambda$.
4. The shift cannot be detected until the end of the production cycle.
5. The proportion of imperfect parts produced during the in-control state is fixed at α level.
6. The process begins a linear deterioration as the shift occurs, and the proportion of the imperfect parts is $\alpha + \beta\tau$.
7. The imperfect parts cannot be detected until the end of the production cycle.
8. All imperfect parts can be reworked under a specified cost.
9. The process is restored to the in-control state with each setup.
10. The effect of capital investment on setup cost reduction and quality improvement can be a logarithmic investment cost function (Hofmann, 1998).

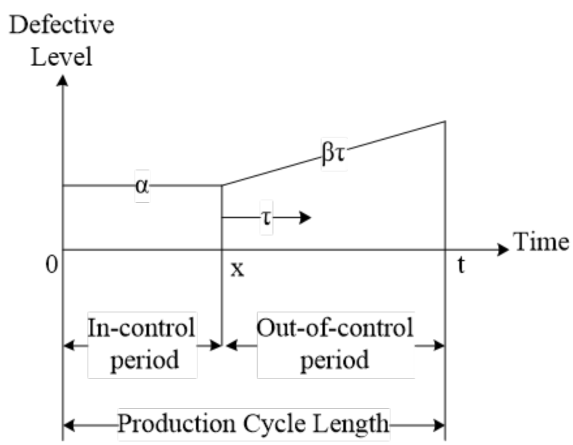


Figure 1. Production cycle representation

3.3 Mathematical Model

Based on the mentioned assumptions, the total annual cost, TAC, of the production-inventory system is composed of the following components:

1. Cost of setup: is the setup cost for each production cycle, K, multiplied by the number of cycles per year, $1/T$.
2. Cost of holding inventory: this cost is calculated by multiplying the holding cost per unit, h, by the average inventory level $(P-D)/2$, by the production run length, t.
3. Cost of rework: this cost is obtained by multiplying the rework cost of an imperfect unit, s, by the expected number of imperfect units, $E(N)$, by the number of orders per year, $1/T$.
4. Cost of capital investment: the summation of the capital investment in setup cost reduction and process quality improvement in the in- and out-of-control states.

$$TAC(t) = \frac{K}{T} + \frac{h(P-D)t}{2} + \frac{s}{T}\,E(N) + i\,[\emptyset_{\mathrm{k}} + \emptyset_{\alpha} + \emptyset_{\beta}] \tag{1}$$

Substituting by $TD = Pt \Longrightarrow T = \frac{Pt}{D}$

$$TAC(t) = \frac{KD}{Pt} + \frac{h(P-D)t}{2} + \frac{sD}{Pt}\,E(N) + i\,[\emptyset_{\mathrm{k}} + \emptyset_{\alpha} + \emptyset_{\beta}] \tag{2}$$

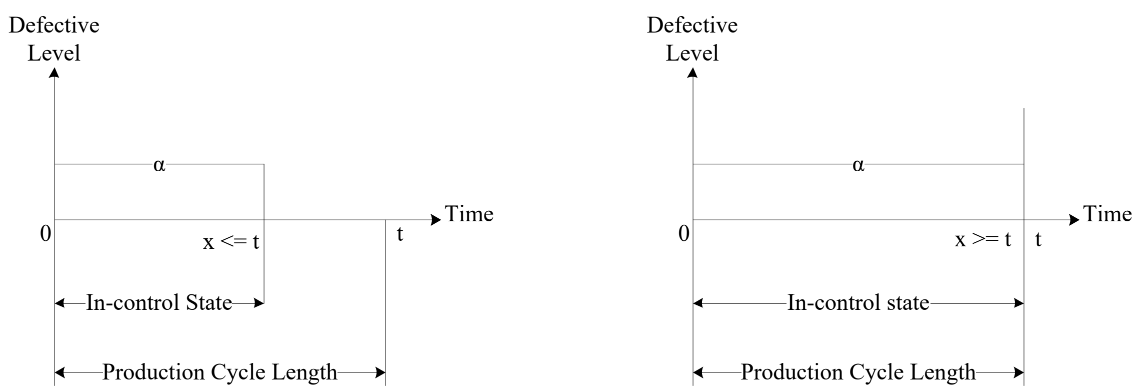
3.3.1 Linear Deterioration

Hou (2007) considered that the percentage of defectives is fixed during the out-of-control state and that the items produced during the in-control period are of ideal quality. However, the process quality is not ideal before the shift occurrence, during the in-control stage. Therefore, the total number of defective items before and after the shift is the expected quantity of defective items during production.

- During the in-control state: Two situations could occur by considering a constant defective rate α in an in-control period, as shown in Figure 2.

$$E(N_0) = E[\alpha P \min(X,t)] = \alpha P \int_{x=0}^{t} x f(x) dx + \alpha P t \int_{x=t}^{\infty} f(x) dx$$

$$= \frac{\alpha P}{\lambda}[1 - e^{-\lambda t}] \tag{3}$$



(a) Situation 1: $x \leq t$ (b) Situation 2: $x \geq t$

Figure 2. The process status during the in-control state

- During the out-of-control state: the process deterioration increases linearly

$$E(N_1) = \int_{x=0}^{t} \int_{\tau=0}^{t-x} (\alpha + \beta\tau) P \, d\tau \; f(x) dx$$

$$E(N_1) = \frac{P\alpha}{\lambda}\left[e^{-\lambda t} + \lambda t - 1\right] + \frac{P\beta}{\lambda^3}[1 + \frac{\lambda^2 t^2}{2} - \lambda t - e^{-\lambda t}] \tag{4}$$

- During the production run: total defectives during the production process.

$$E(N) = E(N_0) + E(N_1) = \alpha P t + \frac{P\beta}{\lambda^3}[1 + \frac{\lambda^2 t^2}{2} - \lambda t - e^{-\lambda t}] \tag{5}$$

3.3.2 Capital Investment

Hofmann (1998) investigated the influence of capital investment on the setup and the production process, assuming that the investments under examination have a single effect. Furthermore, as illustrated in Figure 3, the reduction of setup cost, K, is a convex function and strictly decreases as the investment, ϕ, increases. Eq. (6) expresses the relationship between capital investment and setup cost, where the cost of setup can theoretically reach zero if capital investment approaches infinity (Hofmann, 1998).

$$K_{inv} = K\, e^{-\frac{\phi_k}{a}} \implies \phi_k(K) = a \ln \frac{K}{K_{inv}} \quad for\ 0 < K_{inv} \le K \tag{6}$$

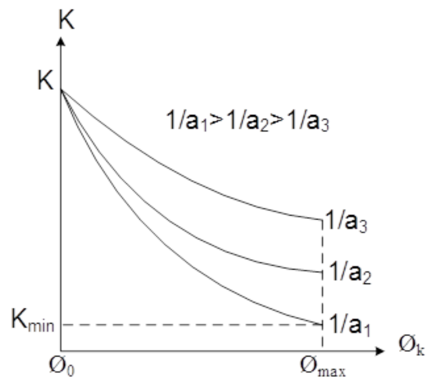


Figure 3. Relation between investment and setup cost

The parameter a represents the responsiveness of the setup cost to changes in capital investment. Whereas the more investments made, the less setup cost reduction obtained through additional investments (Hofmann, 1998).

Similarly, the relation between α and β and capital investment in-process quality improvement:

$$\alpha_{inv} = \alpha\, e^{-\frac{\phi_\alpha}{b}} \implies \phi_\alpha(\alpha) = b \ln \frac{\alpha}{\alpha_{inv}} \quad for\ 0 < \alpha_{inv} \le \alpha \tag{7}$$

$$\beta_{inv} = \beta\, e^{-\frac{\phi_\beta}{c}} \implies \phi_\beta(\beta) = c \ln \frac{\beta}{\beta_{inv}} \quad for\ 0 < \beta_{inv} \le \beta \tag{8}$$

Substituting Eqs. (5), (6), (7), and (8) into Eq. (2) result in the following expression of the TAC:

$$TAC(t, K, \alpha, \beta) = \frac{KD}{Pt} + \frac{h(P-D)t}{2} + s\alpha D + \frac{sD}{t\lambda^3}\beta[1 + \frac{\lambda^2 t^2}{2} - \lambda t - e^{-\lambda t}]$$
$$+ i\left[a \ln\left(\frac{K}{K_{inv}}\right) + b \ln\left(\frac{\alpha}{\alpha_{inv}}\right) + c \ln\left(\frac{\beta}{\beta_{inv}}\right)\right] \tag{9}$$

4. Solution Procedure and Example

The total annual cost, TAC, expressed in Eq. (9), is convex in terms of setup cost and process quality since the Hessian matrix of Eq. (9) is positive with respect to (K_{inv}, α_{inv}, and β_{inv}). Taking the first partial derivative of Eq. (9) with respect to (K_{inv}, α_{inv}, and β_{inv}) and equating it to zero, the following equations are obtained for the optimum values as a function of (t).

$$\frac{\partial TAC}{\partial K_{inv}} = \frac{D}{Pt} - \frac{ia}{K_{inv}} = 0 \implies K_{inv} = \frac{iaPt}{D} \tag{10}$$

$$\frac{\partial TAC}{\partial \alpha_{inv}} = sD - \frac{ib}{\alpha_{inv}} = 0 \implies \alpha_{inv} = \frac{ib}{sD} \tag{11}$$

$$\frac{\partial TAC}{\partial \beta_{inv}} = \frac{sD}{t\lambda^3}\left[1 + \frac{\lambda^2 t^2}{2} - \lambda t - e^{-\lambda t}\right] - \frac{ic}{\beta_{inv}} \quad \Rightarrow \quad \beta_{inv} = \frac{ic}{\frac{sD}{t\lambda^3}\left[1 + \frac{\lambda^2 t^2}{2} - \lambda t - e^{-\lambda t}\right]} \tag{12}$$

Suppose the setup cost after investment, K_{inv}, is higher than the original setup cost, K. In that case, there is no need for investment, and use $K_{opt} = K$. The same is true for the process quality if the original quality is better than after the investment, so there is no need to improve quality. Then, the optimal production run length, t_{opt}, is determined by applying a systematic search method under different values of the production run length.

The flowchart of the systematic search procedure is shown in Figure 4. By Substituting the values of (K_{inv}, α_{inv}, and β_{inv}) into Eq. (9), TAC(t) is convex in terms of t since its second derivative is positive. OPRL can be obtained by applying a systematic search over t.

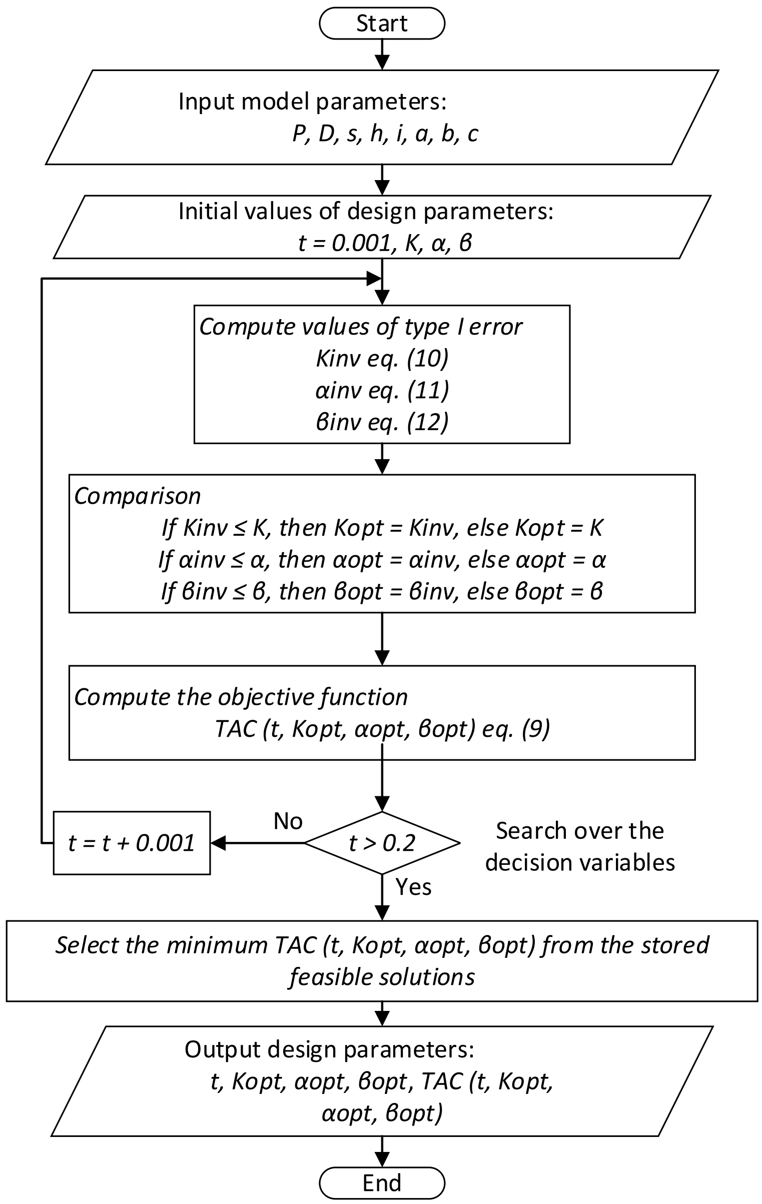


Figure 4. Flowchart of the systematic search procedure

4.1. A Numerical Example

This section illustrates the practicability of the proposed model. The numerical example is adopted from (Hou, 2007): P = 1500 units per year, D = 1000 units per year, K = \$50 per production run, $\alpha = 0.02$, $\beta = 0.02$, s = \$40 per unit, h = \$80 per unit, $i = 0.12$, $a = 1450$, $b = 30$ and $c = 100$. Figure 5 depicts the behavior of the total cost, *TAC*, in relation to the production run length, t. The *TAC* is convex over time; as expected, as t became longer, the holding cost, h, and rework cost became higher, s. The relation between setup cost, K, and investment in setup cost reduction, $\O_k$, with t is depicted in Figure 6. Figure 7 outlines the distribution of different cost elements in the case of considering investment and without investment. It is clear that the rework cost contributes significantly to the *TAC* without investment, but after investment and implementing the setup and quality improvement, the rework cost reduces.

Table 1 summarizes the results using the algorithm proposed by (Hou, 2007). Applying the proposed model by considering the defective items during the in-control state and the effect of process deterioration during the out-of-control state, the optimal values of the decision variables are summarized in table 1. Both models are solved under different values of λ from 0.05 to 4.0 by step 0.05. It has been found that both models (Hou, 2007) and the proposed model are insensitive to the change in λ.

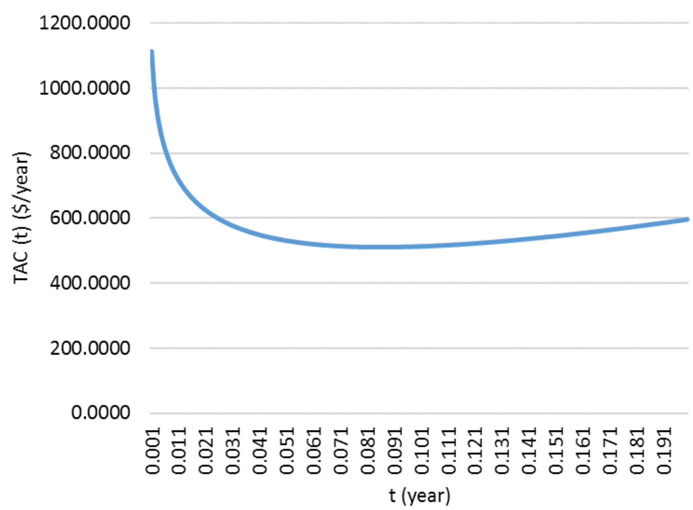


Figure 5. Total annual cost over time

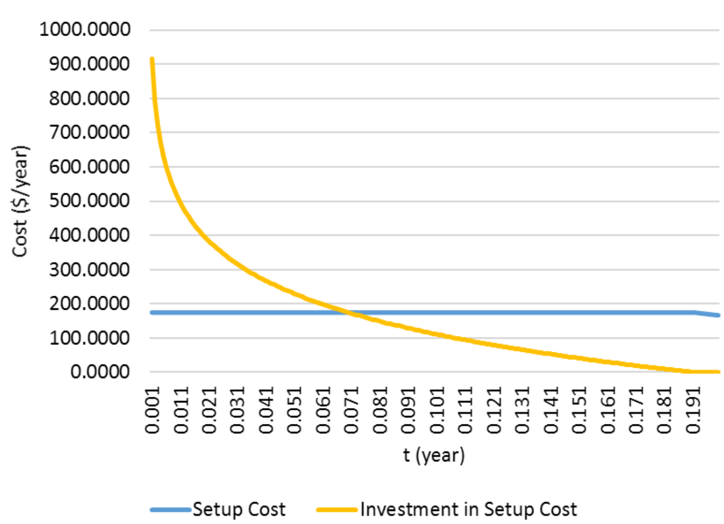
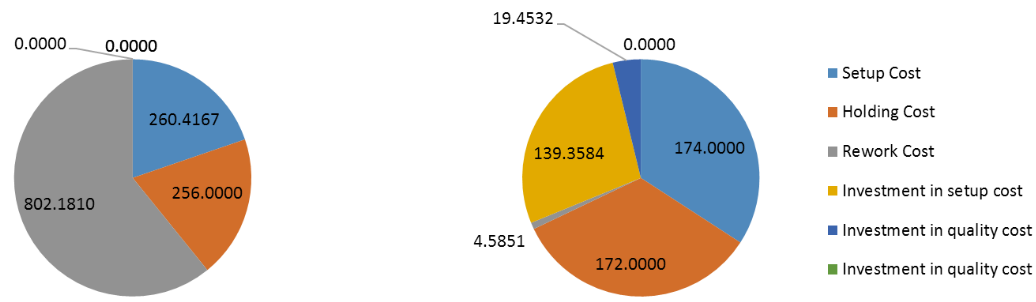


Figure 6. Setup cost and investment in setup cost reduction over time



(a) Before investment (b) After investment

Figure 7. Total cost distribution

Table 1. Summary of the optimal solutions

	λ	K^*	α^*	β^*	t^*	$\varnothing_k$	$\varnothing_\alpha$	$\varnothing_\beta$	$TC(t^*)$
(Hou, 2007)	0.05:0.40	26.10	0.0200	0.02	0.100	113.12	0.00	0.00	496.12
Our[1]	0.05:0.40	22.45	0.0001	0.02	0.086	139.36	19.45	0.00	513.36
Our[2]	0.05:0.40	50.00	0.0200	0.02	0.128	0.00	0.00	0.00	1,318.60

[1] case with investment and [2] case without investment

The proposed model considers the defective rate during both in- and out-of-control states. As expected, the production run length is longer in the proposed model than in the (Hou, 2007) model. Longer t^* to compensate for the defective parts and saving in *TAC* is realized by reducing the required investments in setup cost reduction and quality improvement.

5. Conclusions

The proposed model computes the OPRL, setup cost, and process quality. The production process starts in an in-control state and may shift to an out-of-control state after an elapsed time. The time to the shift is assumed to be exponentially distributed. After the shift, the process became out-of-control, and the deterioration increases linearly to the end of the production cycle. Defective items are produced at a constant rate during in-control; this rate increases linearly after the shift due to process deterioration. The results from the proposed model show a direct relationship between investment in setup cost reduction and OPRL, whereas an inverse relationship exists between investments in-process quality and OPRL. The OPRL depends on *a*, *b*, and *c*, representing how costly it is to reduce setup cost and how costly it is to make process quality improvement. Since an increase or a reduction in OPRL affects the produced quantity and hence the inventory level and shortage, it is important to investigate the optimal allocation of investments among different options. As future work considers the deterioration during the out-of-control state increases exponentially and integrates maintenance planning to restore the process to its original condition, i.e., in-control state.

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