New Heuristic for Solving Capacitated Lot Sizing Problem with Setups, Production, Inventory and Back Orders (CLSPWSIPB)

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Abstract

Here we give standard 0-1 mixed integer linear program of CLSPSPIB. In this formulation we eliminate shortage variables and setup (0-1 integer) variables to pose the problem as 'all real' linear program in inventory and production variables. Based on its solution, we keep select variables as zeros or ones in the original CLSPSPIB. We get very good solutions in much reduced CPU time. Detailed computational experience is given.

Keywords
Capacitated Lot Sizing Problem (CLSP), CLSPWSPIB, Heuristic.

1. Introduction
For a thorough literature review on capacitated lot sizing problem reader is referred to Mayank Verma (2012). Recently Sharma and Sinha (2018) gave a new formulation of multi item capacitated lot sizing problem that has several advantages. This line was further towed by Parvathy (2018) who suggested a novel heuristic for getting good solutions to single item capacitated lot sizing problem. Md Amir (2020) showed (empirically) that the new formulation due to Sharma and Sinha (2018) has merit.

We give a novel heuristic approach to CLSPWSIPB that requires solving a LP. The solution of LP is used to selectively set 0-1 setup variables at zero or one. This leads to good solutions to original CLSPWSIPB at much less computational burden. Lot sizing problem with backorder, inventory, setup and production cost is posed as problem P below.

2. Contribution
Below we outline our new approach.

Problem P (Original Problem):

Min sum(t), [XP(t)*CXP(t) + XINV(t)*CINV(t) + CSETUP(t)*y(t)+
XSHT(t)*CSHT(t)] (1)

s.t.

XP(t) <= CAP(t)*y(t) for all t (2)

XINV(0) + sum(t=1 to t1), XP(t) + XSHT(t1) = sum(1 to t1), DEM(t) + XINV(t1) (3)

XSHT(T) = 0 (4)

XINV(t), XP(t), XSHT(t) >= 0 for all t (5)

Y(t) = (0,1) binary for all t (6)

Method 1: Solve problem P by feeding it to GAMS (for T = 50: 20 problems). We record the following: CPU time taken, Objective Function Value for each of the problems.

Eliminate XSHT(t1) by using equation (3). Now the constraint in problem RP is (7):

sum(1 to t1), DEM(t) + XINV(t1) - sum(t=1 to t1), XP(t) - XINV(0) >= 0 (7)

This approach was first given by SM Ali and Sharma (2007) and in several papers due to RRK Sharma (2019, 2019, 2020, 2020, 2021, and 2022), see references. Sharma, Sinha and Verma (2018) have also deployed a similar approach.

next we put y(t) = XP(t)/CAP(t) for all t (8)

thus we have eliminated binary variable y(t) and real variable XSHT(t). Thus the reduced problem RP has less number of variables and has no binary variable y(t). It is expected to yield computational advantages.

Reduced Problem RP:

Min sum(t1 = 1 to T), [XP(t1)*CXP(t1) + XINV(t1)*CINV(t1) +
CSETUP(t1)*XP(t1)/CAP(t1) + {sum(t=1 to t1), DEM(t) + XINV(t1) –
sum(t=1 to t1), XP(t) - XINV(0)}*CSHT(t1)] (9)
XP(t) <= CAP(t) for all t (10)
sum(1 to t1), DEM(t) + XINV(t1) - sum(t=1 to t1), XP(t) - XINV(0) >= 0 (7)
XINV(t), XP(t) >= 0 for all t (11)

Method 2:

Feed RP to GAMS. Note down CPU time for each problem. Now compute XP(t)/CAP(t) for each ‘t’ and if XP(t)/CAP(t) >= 0.75 (an arbitrary number) assign that y(t) = 1. Let set of y(t) = 1 be SET_y(t)_ONES and its cardinality be N_ONES_Y (a positive integer).

If XP(t)/CAP(t) <= 0.15 (an arbitrary number) assign that y(t) = 0. Let set of y(t) = 0 be SET_y(t)_ZEROS and its cardinality be N_ZEROS_Y (a positive integer). Now solve following problem P1.

Problem P1:

Min (1)
s.t.

(2) to (6) and:

Sum(t1 belongs to SET_y(t)_ONES, y(t1) = N_ONES_Y (12)
Sum(t1 belongs to SET_y(t)_ZEROS, y(t1) = 0 (13)

Feed problem P1 to GAMS and record the following: CPU time, Objective function value. Please note method 2 is a heuristic and does not give ‘OPTIMAL’ solution but gives a good solution in attractive CPU time.

3. Computational Experience

The Computation results of the CPU Time and outputs corresponding to Objective functions are presented in Table 1. The sample statistics of the computational results are presented in table 2. The statistical results of comparing both the models using t-statistic and their significance is presented in table3. Looking at these results it is evident that model 2 is taking lesser CPU time. While there is no claim on the optimized output of the model 2 still the objective function's output obtained from model 2 is statistically inferior to model 1 (refer Table 2 and table 3).

Table 1. Computation Results Obtained from Model 1 and Model 2

DatasetNo.	Model 1 CPU time	Model 1 Obj. Function	Model 2 CPU Time	Model 2 Obj. function
1	0.02	730129403.1	0	944598003
2	0.02	677208477.6	0	908177370.2
3	0.01	540538990.5	0	795268355.7
4	0.02	678835215.1	0	855108334.4
5	0.02	653503643	0	908167358.1
6	0.02	647750293	0.03	865558955.2
7	0.01	654242378	0	883558707.9
8	0.02	616692118	0.02	838845251.8
9	0.02	544383067	0	776367255.9
10	0.02	616190829	0.08	829178608.5
11	0.02	607833588	0	852146462
12	0.02	607693828.7	0	852127534.7
13	0.02	607874254	0	852152894.4
14	0.02	607722676.3	0.03	853977487.2
15	0.03	607752556.1	0.02	847914557
16	0.03	607717307.9	0.02	847865969.4
17	0.03	607731251.7	0.02	852151907.4
18	0.03	607710306.4	0.08	853966495.8
19	0.03	607720783.3	0.02	852150890.8
20	0.02	607712703.7	0.02	852152836.3
21	0.02	607718055.5	0.02	847861928.2
22	0.03	607713886.8	0.03	854003514.6
23	0.02	561502334.9	0.01	766859563.8
24	0.03	608609017.4	0	824753508
25	0.02	600793173.1	0.02	778277003.9
26	0.02	696788810.4	0.01	915920340.5
27	0.02	698883744.5	0	917240142.2
28	0.02	579498332.2	0	800199469.9
29	0.02	602485315	0	803314694
30	0.02	709552896.4	0	906256590.7

Table 2. Computational results Statistics

	Statistic	Mean	N	Std. Deviation	Std. Error Mean
Pair 1	M1_CPU_Time	.0217	30	.00531	.00097
	M2_CPU_Time	.0143	30	.02096	.00383
Pare 2	M1_Obj_Fun	6.2368E8	30	4.60091E7	8.40008E6
	M2_Obj_Fun	8.5120E8	30	4.37535E7	7.98826E6

Table 3. Comparison of Model 1 and 2 against their execution CPU Time and Objective Functions

		t	df	Sig. (2-tailed)
Pair 1	M1_CPU_Time V/S M2_CPU_Time	2.045	29	.050
Pair 1	M1_Obj_Fun V/S M2_Obj_Fun	-59.155	29	.000

4. Results and Conclusions:

In this paper we present a new method for solving capacitated lot sizing problem with setup, backorders and shortages. Using a different flow balance constraint, we eliminate the shortage variables and eliminate the setup variables. Thus leading to computational advantages. Then we use the heuristic method given by SP Singh and RRK Sharma to solve mixed 0-1 integer programming problems by solving an associated all real problem and set 0-1 variable as 1 if its value in associated all real problem is greater than 0.85 (an arbitrary number) and 0 if its value in all real problem is less than 0.15 (again an arbitrary number). This has been known to give good solutions in competitive CPU times.

We thus see that method 2 gives good solutions at competitive CPU times. This is a useful contribution of our paper. We give GAMS codes of our programs in the appendix.

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Biographies

Prof. RRK Sharma: He is B.E. (mechanical engineering) from VNIT Nagpur India, and PhD in management from I.I.M., Ahmedabad, INDIA. He has nearly three years of experience in automotive companies in India (Tata Motors and TVS-Suzuki). He has 32 years of teaching and research experience at the Department of Industrial and Management Engineering, I.I.T., Kanpur, 208016 INDIA. To date he has written 1195 papers (peer-reviewed (395) / under review (18) / working papers 782 (not referred)). He has developed over ten software products. To date, he has guided 64 M TECH and 21 Ph D theses at I.I.T. Kanpur. He has been Sanjay Mittal Chair Professor at IIT KANPUR (15.09.2015 to 14.09.2018) and is currently a H.A.G. scale professor at I.I.T. Kanpur. In 2015, he received "Membership Award" given by IABE USA (International Academy of Business and Economics). In 2016 he received the "Distinguished Educator Award" from IEOM (Industrial Engineering and Operations Management) Society, U.S.A. In 2021, he received IEOM Distinguished Service Award. In 2019 and 2020, he was invited by the Ministry of Human Resources Department, India, to participate in the NIRF rankings survey for management schools in India. In 2019, he was invited to participate in the Q.S. ranking exercise for ranking management schools in South Asia.

Dr. Vinay Singh: He has earned his Bachelor Degree in engineering (Computer Science and Engineering) from RBS College Agra, Masters in Human Resource Development and Management from IIT Kharagpur and PhD in Management from IIT Kanpur. Currently he is working as Assistant Professor in the department of Management at ABV-Indian Institute of Information Technology and Management Gwalior, India since Nov 2012. So far he has 26 publications in peer review journals to his credit. He has supervised 92 Masters Students and guided 02 PhD theses. He has also earned two national patents in embedded products design and has developed three software packages. He has received 03 research project grants from prestigious agencies of India.

Mr. Saurabh Sontakke: She is undergraduate student at the Dept of Mechanical Engineering at IIT Kanpur 208016 India.

Prof. KK Lai: He is currently President of CYUT Taiwan. He has numerous publications to his credit.

APPENDIX

Here we give the GAMS codes developed by us in this research. This is likely to be useful to future researchers.

Method 1:

```
*0-1 production model.
set t 'time'/1*50/;
set tt(t)/1*50/;

parameter CXP(t) Cost of production
$call GDXXRW CXP.xlsx par = CXP rdim=1 trace=3 rng = Sheet1!A1:A50
$GDXIN CXP.gdx
$LOAD CXP
$GDXIN

Display CXP;

parameter CXINV(t) Inventory carrying cost
$call GDXXRW CXINV.xlsx par = CXINV rdim=1 trace=3 rng = Sheet1!A1:A50
$GDXIN CXINV.gdx
$LOAD CXINV
```

```
$GDXIN

parameter CBO(t) Back Order Cost
$call GDXXRW CBO.xlsx par = CBO rdim=1 trace=3 rng = Sheet1!A1:A50
$GDXIN CBO.gdx
$LOAD CBO
$GDXIN

parameter f(t) fixed cost
$call GDXXRW f.xlsx par = f rdim=1 trace=3 rng = Sheet1!A1:A50
$GDXIN f.gdx
$LOAD f
$GDXIN

parameter Dem(t) demand at time t
$call GDXXRW Dem.xlsx par = Dem rdim=1 trace=3 rng = Sheet1!A1:A50
$GDXIN Dem.gdx
$LOAD Dem
$GDXIN

parameter Cap(t) Capacity at time t
$call GDXXRW Cap.xlsx par = Cap rdim=1 trace=3 rng = Sheet1!A1:A50
$GDXIN Cap.gdx
$LOAD Cap
$GDXIN

*parameter zzz(t)
Variables
 z objective function for total Optimal_Cost
 y(t) binary variable to denote prodn.
 XP(t) production at time t
 XINV(t) inventory at time t
 XSHT(t) shortage cost at time t;

Positive Variable XP,XINV,XINV,XSHT;
Binary Variable y ;
Equations
cost total optimal cost
prodcons(t) production constraint
demcon(t) demand balance constraint
*end_inv ending inventory is zero
end_sht ending shortage quantity is zero;
* objective function
cost.. z=e=sum((t),CXP(t)*XP(t) + CXINV(t)*XINV(t) + CBO(t)*XSHT(t) + f(t)*y(t));
*constraints
prodcons(t).. XP(t) =l= Cap(t)*y(t);
end_sht.. XSHT('50') =e= 0;
demcon(t).. sum(tt$(ord(tt)<=ord(t)),XP(tt)) + XSHT(t) =e= sum(tt$(ord(tt)<=ord(t)),DEM(tt)) + XINV(t) ;

Model transport /all/;
solve transport using mip minimizing z;
display z.l;
display y.l;
```

Method 2:

Program 1:

```
set t 'time'/1*50/;
set tt(t)/1*50/;


parameter CXP(t) Cost of production
$call GDXXRW CXP.xlsx par = CXP rdim=1 trace=3 rng = Sheet1!A1:A50
$GDXIN CXP.gdx
$LOAD CXP
$GDXIN
parameter CXINV(t) Inventory carrying cost
$call GDXXRW CXINV.xlsx par = CXINV rdim=1 trace=3 rng = Sheet1!A1:A50
$GDXIN CXINV.gdx
$LOAD CXINV
$GDXIN

parameter CBO(t) Back Order Cost
$call GDXXRW CBO.xlsx par = CBO rdim=1 trace=3 rng = Sheet1!A1:A50
$GDXIN CBO.gdx
$LOAD CBO
$GDXIN

parameter f(t) fixed cost
$call GDXXRW f.xlsx par = f rdim=1 trace=3 rng = Sheet1!A1:A50
$GDXIN f.gdx
$LOAD f
$GDXIN

parameter Dem(t) demand at time t
$call GDXXRW Dem.xlsx par = Dem rdim=1 trace=3 rng = Sheet1!A1:A50
$GDXIN Dem.gdx
$LOAD Dem
$GDXIN

parameter Cap(t) Capacity at time t
$call GDXXRW Cap.xlsx par = Cap rdim=1 trace=3 rng = Sheet1!A1:A50
$GDXIN Cap.gdx
$LOAD Cap
$GDXIN

*parameter zzz(t)

variable
XP(t) Quantity produced at time t
XINV(t) Inventory at time t
XBO(t) Back order quantity at time t
y_zeros(t) Ys that are 0
y_ones(t) Ys that are 1;

positive variable  XP,XINV,XBO;

Free variable
z objective function;

Equations
```

```
objfun objective function
ccons(t) capacity constraint
demcon(t) modified demand balance constraint
bocons Back order at T=50 is 0
;

objfun.. z =e= sum((t),CXP(t)*XP(t)+CXINV(t)*XINV(t)
            +CBO(t)*(sum(tt$(ord(tt)<=ord(t)),Dem(tt)-XP(tt)) + XINV(t)) +
                 f(t)*XP(t)/Cap(t));
demcon(t).. sum(tt$(ord(tt)<=ord(t)),Dem(tt)-XP(tt)) + XINV(t) =g= 0;
ccons(t).. XP(t) =l= Cap(t);
*at t=last,XBO=0
bocons.. sum((t),Dem(t)-XP(t))+ XINV("50") =e= 0;

Model SingleItem1 /objfun,demcon, ccons, bocons/;

Solve SingleItem1 using lp minimizing z;

parameter zz(t), y(t);

zz(t) = XP.l(t);

y(t) = zz(t)/Cap(t);
display y;
loop(t,
if (y(t) <= 0.15,
  y_zeros.l(t)=1;
elseif (y(t)>=0.85),
  y_ones.l(t)=1;
else
  continue;
);
);
display y_zeros.l,y_ones.l;
execute_unload "y_zeros.gdx" y_zeros.l
execute 'gdxxrw.exe y_zeros.gdx o=y_zeros.xls var=y_zeros.l'

execute_unload "y_ones.gdx" y_ones.l
execute 'gdxxrw.exe y_ones.gdx o=y_ones.xls var=y_ones.l'
```

Program 2:

```
set t 'time'/1*50/;
set tt(t)/1*50/;

parameter CXP(t) Cost of production
$call GDXXRW CXP.xlsx par = CXP rdim=1 trace=3 rng = Sheet1!A1:A50
$GDXIN CXP.gdx
$LOAD CXP
$GDXIN

Display CXP;

parameter CXINV(t) Inventory carrying cost
$call GDXXRW CXINV.xlsx par = CXINV rdim=1 trace=3 rng = Sheet1!A1:A50
$GDXIN CXINV.gdx
$LOAD CXINV
```

```
$GDXIN

parameter CBO(t) Back Order Cost
$call GDXXRW CBO.xlsx par = CBO rdim=1 trace=3 rng = Sheet1!A1:A50
$GDXIN CBO.gdx
$LOAD CBO
$GDXIN

parameter f(t) fixed cost
$call GDXXRW f.xlsx par = f rdim=1 trace=3 rng = Sheet1!A1:A50
$GDXIN f.gdx
$LOAD f
$GDXIN

parameter Dem(t) demand at time t
$call GDXXRW Dem.xlsx par = Dem rdim=1 trace=3 rng = Sheet1!A1:A50
$GDXIN Dem.gdx
$LOAD Dem
$GDXIN

parameter Cap(t) Capacity at time t
$call GDXXRW Cap.xlsx par = Cap rdim=1 trace=3 rng = Sheet1!A1:A50
$GDXIN Cap.gdx
$LOAD Cap
$GDXIN

parameter y_ones(t) Ys that are 1
$call GDXXRW y_ones.xls par = y_ones cdim=1 trace=3 rng = Sheet1!A1:AZ1
$GDXIN y_ones.gdx
$LOAD y_ones
$GDXIN
Display y_ones

parameter y_zeros(t) Ys that are 0
$call GDXXRW y_zeros.xls par = y_zeros cdim=1 trace=3 rng = Sheet1!A1:AZ1
$GDXIN y_zeros.gdx
$LOAD y_zeros
$GDXIN
Display y_zeros
*parameter zzz(t)

variable
XINV(t) Inventory at time t
XP(t) Production at time t
y(t) set up variable
z objective function;


positive variable XINV;
binary variable y;
Free variable z;

Equations
objfun objective function
demcon(t) modified demand balance constraint
bocons Back order at T=50 is 0
```

```
y_o(t) Ys that will be 1 for sure
y_z(t) Ys that will be 0 for sure
;

objfun.. z =e= sum((t),CXP(t)*Cap(t)*y(t)+CXINV(t)*XINV(t)
                    +CBO(t)*(sum(tt$(ord(tt)<=ord(t)),Dem(tt) - Cap(tt)*y(tt)) + XINV(t)) +
                             f(t)*y(t));
y_o(t)$ y_ones(t).. y(t) =e= 1;
y_z(t)$ y_zeros(t).. y(t) =e= 0;
demcon(t).. sum(tt$(ord(tt)<=ord(t)),Dem(tt)-Cap(tt)*y(tt)) + XINV(t) =g= 0;
*at t=last,XBO=0
bocons.. sum((t),Dem(t)-Cap(t)*y(t))+ XINV("50") =e= 0;

Model SingleItem1 /all/;

Solve SingleItem1 using mip minimizing z;
display z.l;
display y.l;
```