Optimization Approach for Quai Crane Scheduling Problem Under a Combined Constraints

Imane Torbi
Student researcher, Mechanical department
National high School of Electricity and mechanics ENSEM /Hassan II University
Casablanca, MOROCCO
torbi.imane@gmail.com

Sanae Aidi
Student researcher, Mechanical department
National high School of Electricity and mechanics ENSEM /Hassan II University
Casablanca, MOROCCO
sanaa.aidi@gmail.com

Mohamed Mazouzi
Professor, Mechanical department
National high School of Electricity and mechanics ENSEM /Hassan II University
Casablanca, MOROCCO
m.mazouzi@ensem.ac.ma

Abstract

Most researchers have demonstrated the importance and the value of maritime terminal scheduling and optimization. The following work is focusing first, on the problem of unloading operations of outbound containers. Thus, the sequencing of both Quay crane and truck is studied at the same time. Moreover, the possibility of potential interferences between quay cranes and the problem of shifting which significantly influence the performance of quay cranes is considered. In the literature, there is no work that combines interferences between quay cranes and shifting problem at the same time which is the innovation point of our work.

The scheduling problem of the operations unloading containers is formulated as a mixed linear program model. The objective function is to minimize the makespan of handling operations by the quay cranes. The mathematical model is based on various assumptions and it includes the potential interferences and the shifting problem. After modeling the mixed linear program, we used AMPL programming software to validate the mathematical formulation and to derive the optimization results.

The solution obtained by AMPL allows us to determine a completion time (483 seconds) for the processing of the 10 containers presented in the case study as well as the operation of each equipment and the working sequence of each (quay crane and truck).

Keywords
Optimization , quay crane, truck, container, synchronization ,constraint, interference ,shifting.

1. Introduction

In the field of intermodal transport, the sea route is widely used to transport thousands of tons of goods. The products are loaded into containers of standardized dimensions, which are then embarked on ships ensuring the connection between the ports. Land transport, making it possible to link these ports to the origins and/or destination of the goods, is then carried out by trains or trucks. Commercial ports and terminals therefore constitute the land-ship interface. They form large logistics platforms where the goods will transit and where the containers will undergo a set of handling and storage operations.

In general, many decision/planning problems need to be resolved, among which we distinguish the problems related to the arrival of ships in a port, the loading/unloading of these ships, the transport of containers to a storage location (and vice versa) (Vis et al. 2003) , ( Steeken et al., 2004).
At the strategic level: the dimensioning of the platforms (number of spaces available); the sizing and location of container storage;
- At the tactical level: the assignment of quay locations to ships arriving in port (berth allocation), the determination of the number of handling hoists assigned simultaneously to a ship (to minimize the time spent by the ship at the terminal), and the number of vehicles needed to move the containers within the time allowed; the calculation of the number of handling resources at the stock level (deposit and withdrawal) (yard cranes or ASCs for automated cranes)
- At the operational level: the loading and unloading of containers by a quay (quay crane scheduling), taking into account the location of the containers on the ship, the risks of collision of the quays which run on a single rail; the assignment and routing of vehicles transporting containers between the quay and the stock (to minimize the distance travelled, empty distances, transport time..) Figure 1; the scheduling of handling resources at the level of storage/removal of goods.

![Figure 1. Quay crane and truck operations](image)

The objective of this work is to solve a Quay Crane Scheduling Problem (QCSP) and specially optimize the operations of unloading containers by the quay cranes and their synchronization with the trucks operations by taking into account the interference constraints between quay cranes and no-productive movements of containers that is called a shifting problem. There is no work that combines between these constraints at the same time which is the innovation point of our work.

For this, we will start with the literature review of Quay Crane Scheduling Problems (QCSP), we will then present the assumptions of the model taking into account the framework of the modeling. Thereafter, we will approach the mathematical formulation of the problem of optimization of the movements of containers and we will present its various components: parameters, variables of decision, function objective and the constraints. Then, to fully understand the problem, we will present a situation based on fictitious data. Then, for the resolution of this problem we will use the AMPL Studio programming software. Finally, we will represent the results in the form of graphs containing the Route and the sequences of the different handling machines will be presented.

### 2. Literature Review

Several researchers addressed the QCSP in the literature. (Moghaddam et al. 2008) presented a mixed integer programming (MIP) model for the quay crane scheduling and assignment problem, after that (Wang et al. 2009) proposed the sequence of unloading operation, with the objective to minimize the weighted operation time of jobs and travel time. (Chung et al. 2012) created a genetic algorithm to deal with the problem and to test the optimization reliability of the proposed algorithm., meanwhile (Yi et al. 2012) aimed to minimize the total handle time of all tasks, using a polynomial time algorithm to solve the problem. (Azza et al. 2014) founded the handling sequence of tasks at ship; their objective is to minimize the time spent by the vessel at berth bay using a Mixed integer linear programming and ant colony algorithm. (Liu et al. 2015) used the branch-and-price framework to create a models and algorithm for the general double-cycling problem where reshuffle containers are allowed to move directly from one stock to another (Liu et al. 2016) studied a scheduling problem with two uniform quay cranes; the objective was to minimize the turn-around time of a vessel, then (Alnaqbi et al. 2016) used Mixed-integer linear programming and hybrid algorithm to minimize the latest starting time for the vessel and its handling time. (Zhen and al. 2016) also studied the QCSP and proposed a program in mixed numbers (MIP) and a genetic algorithm to solve a model of quay crane and truck scheduling based on the number of containers independently at bays, so the interference constraint does not exist, (Haoyuan et al. 2017) proposed a simulation model of container terminal, their objective to optimize the shortest total delay time for all vessels, (Salhi et al. 2017) developed a genetic algorithm to solve a model that combines three distinct problems, berth allocation, quay crane assignment, and quay crane scheduling that arises in container ports, (Xiazhong et al. 2017) aimed to find the optimal storage location of container groups while minimizing both the maximum completion time and the traveling time of trucks. They used mixed-integer linear programming and tabu search to solve the problem, (Skaf et al. 2019) are developed a MIP and a genetic problem based on the work of (Zhen et al. 2016), but they considered a only quay crane and added a forklift in the storage area.
As part of the planning of transport vehicles and handling equipment, it is easier if each piece of equipment is studied separately. However, the analysis becomes more complicated by grouping two or more elements together. Subsequently, the work decreases by studying the operations of the different equipment simultaneously. This is why we have chosen to combine between the quay crane and the transport truck taking into account the problem of interference between the quays and the shifting problem.

3. Mathematical Formulation

3.1 The assumptions of the model
To develop our mathematical model, a number of assumptions will be taken in consideration and will be integrated:

- We limit our work to the unloading operations of containers (outbound).
- Location of containers is given.
- Quay cranes moves between bays are possible.
- All containers in a bay are destined for the same storage.
- The possibility of interference from quay cranes in the quay is taken into account.
- The possibility of unloading a same bay by 2 quay cranes but not at the same time.
- Non-productive movements of containers (shifting) are taken into account.
- We do not consider the unloading time of the trucks, it is an instantaneous unloading.
- We are only interested in 40 feet containers.

3.2 The mathematical model

3.1.1 Model parameters
The indices used in the model are as follows:
- \(i, j\): Indices relative to the containers; \(i, j \in C\).
- \(i = 0\): fictitious container.
- \(t\): Index for trucks; \(t \in T\).
- \(q\): Index of the quay Crane; \(q \in Q\).
- \(b\): Index of the bay; \(b \in B\).

Sets and parameters used in the model:
- \(C\): Set of containers to be processed in the quay.
- \(T\): Set of trucks available in the quay.
- \(B\): Set of bay in the Vessel.
- \(Q\): Set of Quay Crane (QC) available on the quay.
- \(Q_0\): Initial number of quay Crane in a bay \(b\), \(b \in B\).
- \(L\): Set of locations for the storage of export containers (location address formed by the number of the block, bay, row and floor).
- \(l_i\): Location of a container \(i\) in the vessel.
- \(b_i\): Bay number in a bay for a container \(i\) in a location \(l_i\).
- \(f_i\): Floor number for a container \(i\) in a location \(l_i\).
- \(r_i\): Row number for a container \(i\) in a location \(l_i\).
- \(h_1\): Handling time of a container.
- \(h_2\): Handling time of a container to pick it after remove the container above (shifting); it is assumed that \(h_2 = 2 \times h_1\).
- \(U\): Set of container pairs that cannot be handled at the same time due to interference. As an example, two containers in the same bay cannot be handled at the same time by two quay cranes because they are too close to each other.
- \(R\): Large positive constant.
- \(k_{ij}\): Travel time of a quay crane from container \(i\) to container \(j\) regardless of the location of \(i\) and \(j\) (\(i \neq j\)).
- \(T_{Ei}\): Time for a truck to transport container \(i\), from the quay to the destination on the storage area; \((i \in C)\).
- \(tv_{qa}\): Empty return time of a truck from the storage area to the quay.

3.1.2 The decision variables of the model
- \(S_q^i\): Start time of picking container \(i\) by a quay crane; \((i \in C)\).
- \(S_t^i\): Start departure time of a truck transporting the container \(i\); \((i \in C)\).
- \(X_{jq} = 1\), if quay crane \(q\) processes container \(j\) just after container \(i\); 0 otherwise.
- \(W_{jt} = 1\), if truck \(t\) loads container \(j\) just after loading container \(i\); 0 otherwise.
- \(Z_{bbq} = 1\), if the quay crane \(q\) moves from bay \(b\) to bay \(b\); 0 otherwise (\(b \neq b\)).
\[V_{ij} = 1, \text{ if the processing of container } j \text{ begins after the end of the processing of container } i \text{ by a quay crane.}\]

\[p_t = h_1 + \sum_{r_i \leq r_j} h_2 \cdot V_{ij}\]

Where \(\sum_{r_i \leq r_j} h_2 \cdot V_{ij}\) is the non-productive movements of containers that is called a shifting.

### 3.1.3 The mathematical model

Minimize \(\{\text{Max } S_i^T\}\) \hspace{1cm} (1)

The constraints:

1. \[\sum_{i=1}^{n} \sum_{j=1}^{m} X_{ij} = 1; \forall j \in C\] \hspace{1cm} (2)
2. \[\sum_{i=1}^{n} \sum_{t=1}^{T} \sum_{q=1}^{Q} W_{ijt} = 1; \forall j \in C\] \hspace{1cm} (3)
3. \[\sum_{j=1}^{m} \sum_{q=1}^{Q} X_{0jq} = 1; \forall q \in Q\] \hspace{1cm} (4)
4. \[\sum_{j=1}^{m} X_{0jt} = 1; \forall t \in T\] \hspace{1cm} (5)
5. \[\sum_{i=1}^{n} X_{ijq} = \sum_{j=1}^{m} X_{qij}; \forall i, q \in Q\] \hspace{1cm} (6)
6. \[\sum_{i=1}^{n} \sum_{t=1}^{T} \sum_{q=1}^{Q} W_{ijt} = \sum_{j=1}^{m} \sum_{q=1}^{Q} W_{jlt}; \forall i, t \in T\] \hspace{1cm} (7)
7. \[p_t^1 = h_1 + \sum_{r_i \leq r_j} h_2 \cdot V_{ij} ; \forall i \in C\] \hspace{1cm} (8)
8. \[p_t^2 = \sum_{r_i \leq r_j} h_2 \cdot V_{ij} ; \forall i \in C\] \hspace{1cm} (9)
9. \[S_j^Q \geq S_i^Q + p_t^1 + p_t^2 + k_{ij} + R(X_{ijq} - 1); \forall i \in C, j \in C, q \in Q\] \hspace{1cm} (10)
10. \[S_j^Q \geq S_j^Q + k_{ij} + R(X_{ijy} - 1); \forall i \in C, j \in C, q \in Q\] \hspace{1cm} (11)
11. \[S_j^Q \geq S_j^Q + (p_t^1 + p_t^2) + R(V_{ij} - 1); \forall i \in C, j \in C, (i, j) \in U\] \hspace{1cm} (12)
12. \[S_j^Q \geq S_i^Q + T; \forall i \in C, j \in C, t \in T\] \hspace{1cm} (13)
13. \[S_i^T \geq S_i^Q + p_t^1; \forall i \in C\] \hspace{1cm} (14)
14. \[Z_{bbq} \geq X_{ijq}; \forall i \in C, j \in C, q \in Q, b \neq b^\prime\] \hspace{1cm} (15)
15. \[S_{bb}^Q + \sum_{b \neq b^\prime} \sum_{b \in B} \sum_{b \in B} \sum_{q=1}^{Q} Z_{bbq} \leq 1, b^\prime \in B, q \in Q\] \hspace{1cm} (16)
16. \[S_j^Q - S_i^Q - (p_t^1 + p_t^2) \leq V_{ij}; \forall i \in C, j \in C\] \hspace{1cm} (17)
17. \[S_j^Q + (p_t^1 + p_t^2) - S_i^Q \leq (1 - V_{ij}); \forall i \in C, j \in C\] \hspace{1cm} (18)
18. \[V_{ij} + V_{ji} = 1; \forall (i, j) \in U\] \hspace{1cm} (19)
19. \[\sum_{q=1}^{Q} X_{ijq} \leq V_{ij} + V_{ji}; \forall i, j, b_i < b_j, j = i + 1\] \hspace{1cm} (20)
20. \[\sum_{q=1}^{Q} X_{jj^\prime q} \leq V_{ij} + V_{ji}; \forall i, j, j^\prime, b_j < b_i < b\] \hspace{1cm} (21)
21. \[\sum_{q=1}^{Q} X_{i'iq} \leq V_{ij} + R(1 - u_{ij'j}); \forall i, j, j^\prime, i' | b_i = b_{i'}, b_j = b_j, r_i < r_j, r_i < r_{i'} < r_j\] \hspace{1cm} (22)
\[ \sum_{q=1}^{Q} X_{ij}'_q \leq V_{ij} + V_{ij} + Ru_{ij}'j', \forall i, j, i', j' | b_i = b_{i'}, b_j = b_j' \land r_i < r_{i'} < r_{j'} < r_j \]  
\[ 0 < S^i_q, S^j_q, p^i_q, p^j_q \geq 0; \]  
\[ X_{ij}, W_{ij}, Z_{bb'q}, V_{ij}, u_{ij}'j' \in \{0,1\}. \]

In the objective function (1), we seek to minimize the makespan (Start time of picking container by a quay crane). Constraints (2) and (3) ensure that each container is assigned to a single quay crane and a single truck. Constraints (4) and (5) guarantee that the quay cranes and the trucks leave the starting point only once. The constraints (6) and (7) ensure the conservation of flows for each equipment. Constraint (8) and (9) are for a problem of shifting, the first defines the handling time of container \( i \) and its loading on the truck. While, the second defines the time of reinstallation of the containers already removed. Constraint (10) ensures the order in which containers are processed by each quay crane. If a quay \( q \) processes container \( j \) just after container \( i \) then the quay \( q \) will take travel time \( k_{ij} \) to search the next container \( j \). Constraint (11) permits to present the order in which containers are processed by each quay according to the departure time of the trucks transporting the containers. Constraint (12) defines the container processing sequence for each quay in case \( V_{ij} = 1 \) (the processing of container \( j \) is done after the processing of container \( i \)). Constraint (13) guarantees the sequence order of the containers per truck. Constraint (14) determines the container processing sequence per truck. In constraint (15), the variable \( Z_{bb'q} \) takes the value 1 if the quay crane \( q \) processes container \( j \) just after container \( i \) such that the two containers \( i \) and \( j \) are stored in two different bays. Constraint (16) makes it possible to verify the hypothesis that in the same bay, there cannot be more than 1 quay crane. Constraint (17) ensures that at most one only movement of a quay crane \( q \) can occur to another bay (another destination). Constraints (18) and (19) also define the variable \( V_{ij} \). \( V_{ij} = 1 \) if processing of container \( j \) begins after the processing of container \( i \) is complete. Constraint (20) ensures the safety distance between the bays which is a bay so if crane \( q1 \) works in bay \( i \), crane \( q2 \) must not work in crane \( i+1 \), constraint (21) describe the first interference situation. The quay crane \( q \) is blocked by the quay crane \( q' \) and it cannot pick up the containers located to the right of the container \( j \). The same thing for the quay \( q' \) which is blocked by \( q \) and it cannot pick up the containers to the left of container \( i \). The situation where the two quay cranes may unload the containers in the same bay but not in the same time is what constraints (22) and (23) prevent. These two constraints introduce a new variable \( u_{ij}'j' \) to the model. The variable \( u_{ij}'j' = 1 \), if the quay crane \( q \) and \( q' \) operate in the same time (in other words, \( q \) will move from \( i \) to \( i' \) and \( q' \) from \( j \) to \( j' \)). The variable \( u_{ij}'j' = 0 \) otherwise; with \( i, i', j \) and \( j' \) belonging to the same bay and \( r_i < r_{i'} < r_{j'} < r_j \).

### 4. The Case Study

As part of a presentation of a preliminary solution of the problem treated, we try to propose a case study with fictitious data in order to describe the sequences of equipment in a container terminal:

- 4 bays: b1,b3,b4,b6 each with 3 rows, 3 floors
- 2 quay cranes: Q1 and Q2
- 4 trucks: T1, T2, T3 and T4
- 10 containers distributed in the 4 bays

The location of each container is as follows:

- C1: { bay 1, row 1, floor 1}
- C2: { bay 3, row 3, floor 1}
- C3: { bay 3, row 2, floor 1}
- C4: { bay 3, row 1, floor 1}
- C5: { bay 4, row 2, floor 1}
- C6: { bay 4, row 1, floor 1}
- C7: { bay 6, row 1, floor 1}
- C8: { bay 6, row 2, floor 1}
- C9: { bay 6, row 3, floor 1}
- C10: { bay 6, row 3, floor 2}

© IEOM Society International 1937
The locations of the containers are shown in Figure 2, this is the container storage at vessel:

![Figure 2. Locations of containers in the bays of vessel](image)

In this example we have 4 bays, b1, b3, b4, b6, the 2 cranes cannot unload the containers from b3 and b4 at the same time, because there is the interference constraint since we don’t have a safety distance. To integrate the shifting constraint in the case study, we added a container called CST in bay 3 above container 4, this container is not intended for unloading, in this case the crane has to unload it first and put it on the quay area and then it unloads container 4, so if the unloading time of a container is $h_1$, the unloading time of C4 with the shifting problem is $2*h_1$.

The other data relating to the handling time of the quay cranes $h_1$ that equal 70 sec according to (skaf et al., 2019). Travel time of a quay crane from container $i$ to container $j$ regardless of the location of $i$ and $j$ ($k_{ij}$), the crane travel speed is considered 45m/min and the bay length is 16m, so the time to go from bay $i$ to bay $i+1$ is 21sec. The transport time, and the empty return time of the trucks ($tvqa$) are presented in Table 1.

<table>
<thead>
<tr>
<th>Handling time of QC (sec)</th>
<th>Traveling time $K_{ij}$ (sec)</th>
<th>Time of transport (sec)</th>
<th>Time tvqa (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>70</td>
<td>$(j-i) \times 21$</td>
<td>120</td>
<td>60</td>
</tr>
</tbody>
</table>

5. Results and Discussion

After the presentation of the different categories of data, we use the AMPL Studio software in order to code the mathematical model and present a solution to the problem. In a first step, we wrote the mathematical model. Subsequently, CPLEX was used to solve this program, Figure 3.

AMPL integrates a modeling language for describing optimization data, variables, objectives, and constraints; a command language for browsing models and analyzing results; and scripting language for gathering and manipulating data and for implementing iterative optimization schemes. CPLEX is an integrated solver in AMPL that used in a large-scale for over three decades. Its efficiency and robustness have been demonstrated through varied applications.
The solution obtained in the AMPL programming software allowed us to have several information on the problem in question such as the value of the (makespan), Figure 4. The processing sequence of each quay crane and each truck. The completion time to process 10 containers is 483 seconds.

The solution allows us to determine the operation of each piece of equipment. The work sequence of each quay crane and truck is presented graphically in Figure 5 and Figure 6.

The quay crane QC1 will unload respectively C1, C4, C3, C2 and C5, QC2 will unload respectively C6, C10, C9, C8, C7, C5 and C4, these unloading sequences are generated following the data entered and the interference and shifting and availability of trucks.

In synchronization with the quay cranes, truck T1 will transport C1, C3 and C4 respectively; truck T2 will take C6, C9 and C7, truck number 3 T3 will transport C10, C8 and finally truck T4 will take C4 and C2.
So the most optimized cycle of dockside cranes, for Q2, it unloads C6, after it moves to unload C10, which is in bay 6, after it unloads C9 and C8, and finally it unloads C7 after a time of waiting due to unavailability of truck, all this cycle is done in 432 seconds. In parallel with Q2, Q1 unloads C1, moves to unload the CSF which is the container to test the shifting, C4, C3, C2 respectively, and finally C5 after a waiting time, it ends the cycle in 483 seconds which is the makespan.

For the truck cycle, T1 Load C1, it moves it to the storage area in a time of 120 sec, and it returns to look for the C3 in a time of 60 sec (tvqa), then it loads the C5 after a waiting time. the same operation for T2 which takes C6, C9, C7 and T3 which load C10, C8, and finally T4 which load C4 and C2.

6. Conclusion

In this article, we presented a mathematical formulation of the problem of planning the operations of qard cranes and the movements of transport trucks. This formulation allowed us to integrate the hypothesis of the problem as well as the objectives assigned. Also, and with fictitious data, we solved the problem with the AMPL software. We obtained a solution with a completion time (483 seconds) for the processing of the 10 containers presented in the case study as well as the processing sequences of the various handling equipment set up in the maritime terminal. Solving with AMPL is the first approach to solving the problem.

Then and in the context of the next step, we will try to solve the problem with a second resolution approach which is the genetic algorithm.

In contrast to the approach used in this work the heuristic methods have the ability and efficiency to solve optimization problems regardless of the size of the problem: 10, 20 and 100 containers. The data that will be used to test the approach will be fictitious and multiple instances can be generated by varying the number of containers and/or handling equipment available.

References


