Hybrid Model for Locating Fire Station In A City

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Abstract

Fire outbreak, traffic accidents, rescue operations etc. are fairly common in a human settlement all over the globe. The government funded fire stations are responsible to control such situations at the earliest possible. The response time and road route must be minimized to ensure that the adversity is taken under control quickly. To set up such a fire station at an optimal location we also need to consider the budget involved and the value of the locality which it would serve. This paper presents three models i.e. establishment cost, optimal service level and hybrid model of locating a fire station in a city by extending the mixed linear programming formulation using binary genetic algorithm. The proposed models are tested for problem sizes of n = 30 and 60 using GAMS 23.5 and comparisons of optimum results are reported. Hybrid model performs best that help to decision-makers to enhance their decision capabilities.

Keyword
Fire Station Location, Genetic Algorithm, Facility Location Problem, Hybrid Model

1. Introduction

We all have seen bright red coloured fire trucks rushing on the roads. Fire-fighters are summoned whenever there is a fire outbreak, traffic accidents, rescue operation or any kind of disaster and emergency situations. Time is the biggest challenge for them as each second could be fatal for human lives and could also bring huge loss to property as well. Presently, most of the fire department operations are concentrated mostly in the urban areas. Fire trucks take the shortest distance to the place of incident along with ambulance to control the adversity and provide medical assistance to the victims. To improve the rescue operations, the most important factor that comes into the play is locating the fire stations at the optimal places in a city. Since, fire department is run by the government and public funds are used for the setup, the fire department need to work according to the budget provided by the government. As this is an emergency requirement for the society, fire department can not compromise on their quality of service and the requirements which are stated in the regulations. Thus, numerous constraints and variables are taken into account before deciding the optimal location of a fire station. For example, if a locality has a high value; meaning the locality comprises of industries, large human settlements, expensive properties etc. then a fire station should be set up at a near distance from it.

This type of facility location problem comes under the umbrella of hard to solve problems. Hence, computer is required to solve these kinds of problems for the time being. The advancement of computer technology has brought convenience to the solution approaches. General Algebraic Modelling Systems (GAMS) is a mathematical modelling programming environment which is useful for solving complex mathematical problems easily. The problem is fed and coded in the GAMS IDE. Various linear models are made and ran in the software by importing data tables from MS Excel. The outputs are recorded and put up for tests and analysis. The best model is then selected on the basis of the results produced.

2. Literature Review

The first studies about fire station layout started with Plane and Hendrick (1974). They developed a mathematical model that meets demand based on projected time and distances of the previous studies of emergency service station locations by Toregas et al (1971). The importance of this study is that it shows the solution of both the binary integer model and the linear model for the first time. Authors of this study also proposed a modified median model. Median models are frequently observed in the studies next to the cluster coverage model. In the literature it is observed that FLP implacable to locate fire station in a city attempt to optimize the distance, to meet the demand and to minimize the response time, in general. Hakimi (1983), Infante and Taborga (2001), Lorenna and Senne (2004), Choi and Lee (2007), Osman and Amadi (2007), Resende and Werneck (2007), Dominguez and Munoz (2007) and Bischoff and Dachert (2009) can all be counted as the studies that developed models that meet demands at the minimum amount of time. Galveo et al (2000), Hong et al(2004) and ReVelle et al. (2008) made the studies about locating the facility to meet out the service/customers demands. Another study area about fire station location is economic approach including budget evaluation processes.

Badri et al. (1998) suggested a model for the fire station location problem using multi-target programming. During this study authors used constraints that will minimize service operation costs aside from the fixed costs. While Church et al. (2009) pointed out that the yearly repair and maintenance costs of fire stations are very high. In this study, personnel expenses also create high costs aside from installation and fixed costs. These studies showed that determining fire station location is very important both strategically and economically which is the primary concerns of present the study (Sharma 2019).

3. Problem and Formulation

For the purpose of the study an urban metropolitan city is assumed where population density, public utility services and locality cots is high. The road connectivity is uniform throughout the city. The formulation of locating fire stations is stated below:

3.1. Constants and Variables

Hypothetical cities are taken into consideration and three models are formulated for analysis. The notations, parameters and indices which are used in the models are given below:

k	:	representative point of localities
i	:	representative point of fire stations
V(k)	:	value of the locality (measured in ₹ 1,00,000)
AV(k)	:	minimum number of fire stations a locality should be attached with
F(i)	:	cost of setting up the fire station (measured in ₹ 1,00,000)
AF(i)	:	maximum number of localities a fires station can serve
D(i,k)	:	distance between i^{th} fire station and k^{th} locality (measured in km)
BUD	:	total budget for setting of fire stations across the city (measured in ₹ 1,00,000)
SERVL	:	minimum level of service required (lesser the value, better the service)

$Y(i) = \{1, \ \ if\ we\ locate\ a\ fire\ station\ at\ 'i'\ 0, \ \ otherwise$

$Z(i,k) = \{1, \ \ if\ \ i^{th}\ fire\ station\ is\ assigned\ to\ k^{th}\ locality\ 0, \ \ otherwise$

3.2. Model # 01 : Budget constraint

Maximizing the service within a budget given.

Minimize: $\sum_i \quad \sum_k \quad Y(i) * D(i,k) * V(k)$ [1]

Constraints:

1. $\sum_k \quad Z(i,k) \leq Y(i) * AF(i) \qquad \forall\ i = 1 \dots \dots n$
2. $\sum_i \quad Z(i,k) \geq AV(k) \qquad \forall\ k = 1 \dots \dots n$
3. $\sum_i \quad F(i) * Y(i) \leq BUDGET$
4. $Z(i,k) = \{0,1\} \qquad \forall\ i,k = 1 \dots \dots n$
5. $Y(i) = \{0,1\} \qquad \forall\ i = 1 \dots \dots n$

3.3. Model # 02 : Service constraint

Minimizing the cost within a service level given.

Minimize: $\sum_i F(i) * Y(i)$ [2]

Constraints:

1. $\sum_k Z(i,k) \leq Y(i) * AF(i) \quad \forall\, i$=1......n
2. $\sum_i Z(i,k) \geq AV(k) \quad \forall\, k = 1 \ldots\ldots n$
3. $\sum_i \sum_k Y(i) * D(i,k) * V(k) \leq SERVL$
4. $Z(i,k) = \{0,1\} \quad \forall\, i,k = 1 \ldots.n$
5. $Y(i) = \{0,1\} \quad \forall\, i = 1 \ldots..n$

3.4. Model # 03: Hybrid

Minimizing cost and maximizing service as well within a given budget and service level.

Minimize: $\sum_i \sum_k Y(i) * D(i,k) * V(k) + \sum_i F(i) * Y(i)$ [3]

Constraints:

1. $\boldsymbol{\sum_k Z(i,k) \leq Y(i) * AF(i) \quad \forall\, i = 1 \ldots\ldots n}$
2. $\sum_i Z(i,k) \geq AV(k) \quad \forall\, k = 1 \ldots..n$
3. $\sum_i F(i) * Y(i) \leq BUDGET$
4. $\sum_i \sum_k Y(i) * D(i,k) * V(k) \leq SERVL$
5. $Z(i,k) = \{0,1\} \quad \forall\, i,k = 1 \ldots\ldots.n$
6. $Y(i) = \{0,1\} \quad \forall\, i = 1 \ldots\ldots.n$

4. Data Generation and Computation of Results

All the three models are coded in the GAMS 28.2.0 software. Two inputs are taken for each formulation comprising random data for 30 cities. The first input and second input are a set of thirty cities with 30x30 and 60x60 matrices of distance between the fire stations and localities respectively. The cities are divided into small, medium and big cities in a cluster of 10 in one. V(k), AV(k), AF(i), F(i) and D(i,k) are uniformly distributed according to the values listed in table 1.

Table 1. Dataset Distribution of cities (all ranges are inclusive)

	Small	Medium	Large
V(k)	500 to 5,000	1,000 to 10,000	5,000 to 50,000
AV(k)	0 to 2	1 to 5	2 to 6
AF(i)	1 to 3	2 to 6	3 to 8
F(i)	50 to 500	300 to 1,500	100 to2,500
D(i,k)	5 to 15	10 to 30	15 to 80

All the values are generated using random module of python. These inputs are imported to the GAMS from Excel sheet and the program is run to get the results. The outputs of our interest are CPU time, number of iterations and the optimized value of the objective functions for all the three models. All the proposed models were able to produce optimal results. These outputs are noted for their comparative analysis also. The Z-score and ANOVA analysis results are reported in table 2 for the no of iterations and computation time of each of the models to report the effective and efficient one out of three developed models. Three z-tests on the CPU time between model1 & model2, model2 & model3 and model3 & model1. Similarly, three more z-tests are performed on the number of iterations. Null hypothesis is when the mean differences of the two models is equal to '0'$(\mu_1 = \mu_2)$; both for mean time and no. of iteration differences. The results of the tests are shown below.

Table 2. Models Comparisons on CPU Time (efficiency) and Number of Iteration (Effectiveness)

		Model1	Model2	Model3	Z_{12}	Z_{23}	Z_{13}
Run Time	$Mean_{30*30}$ $Variance_{30*30}$	1.48366 0.01764	1.39433 0.00321	1.39196 0.00091	Z=3.389 **p(0.001)<.05**	Z=0.202 P(0.840)>.05	Z=3.688 **P(0.000)<.05**
	$Mean_{60*60}$ $Variance_{60*60}$	1.8487 0.00897	1.7156 0.0054	1.76303 0.02338	Z=6.082 **p(.000)<.05**	Z=1.531 p(0.1257)>.05	Z=2.609 **p(0.009)<.05**
No of Itrns.	$Mean_{30*30}$ $Variance_{30*30}$	263.5 11895.58	216.46 5026.64	210.53 2403.11	Z=1.980 **p(0.048)<.05**	Z=0.377 P(0.706)>.05	Z=2.426 **p(0.015)<.05**
	$Mean_{60*60}$ $Variance_{60*60}$	543.43 53498.71	459.1 8830.95	543.43 53948.71	Z=1.850 p(0.064)>.05	Z=2.181 **p(0.029)<.05**	Z=0.233 p(0.815)>.05

From the z-tests between model 1 and model 2, it is quite evident that model 2 performs better than the model 1. In the case for model 3 and model 1, model 3 performs better. The result of model 2 and model 3 is pretty close as 3 out 4 tests show no significant differences but in one test, model 2 comes out to be significantly better than the model 3. Hence, the best suitable model comes out to be model 2. To ensure the robustness of the findings of z-test ANOVA (Analysis of Variance) is performed to test the significant differences among the model with respect to CPU time and no of iterations to solve the problems under different input sets of 30*30 and 60*60. ANOVA tests indicate that 3 out 4 times all models are significantly different. Hence, we can safely conclude that all three models are different significantly and it also verifies the result we achieved from the z-tests.

Table 3. ANOVA for multiple comparisons of the Models

Problem Set	***Source of Variation***	***SS***	***Df***	***MS***	***F***	***P-value***	***F crit***
no. of iteration (30x30)	Between Groups	0.16395	2	0.08197	10.92095	5.87E-05	3.10130
	Within Groups	0.65304	87	0.00751			
	Total	0.81699	89				
no. of iteration (60x60)	Between Groups	169476.36	2	84738.18	**2.14140**	0.12364	3.10130
	Within Groups	3442714.37	87	39571.43			
	Total	3612190.72	89				
CPU time (30x30)	Between Groups	0.27304	2	0.13652	10.48260	8.34E-05	3.10130
	Within Groups	1.13306	87	0.01302			
	Total	1.40610	89				
CPU time (60x60)	Between Groups	0.27304	2	0.13652	10.48260	8.34E-05	3.10130
	Within Groups	1.13306	87	0.01302			
	Total	1.40610	89				

6. Conclusions

Present study addresses locating fire station in city which is believed to be NP hard in nature. Under assumption of fixing certain constraints. Models are proposed and tested for three objectives with two sets of inputs data with 30 experimental outcomes corresponding to each model solutions. From the various z-tests performed above, it can be concluded that model 3 is more realistic while model 2 outperformed the other models on the experimental data. Model 3 is comparably way better than model 1 but stayed a little short of the model 2. In the first input (30x30), model 3 and model 2 were at par but as the dataset is increased to the second input (60x60), model 2 gave better results. The mean of CPU time taken by model 2 is 1.3943 and 1.7156 seconds with variance of 0.0032 and 0.0054 for input1 and input2 respectively. The mean number of iterations it took is 216.46 and 459.1 for input1 and input2 with a data of 30 cities each. Thus, minimizing the cost within a given service level seems to be the best possible way to locate a fire station according to the dataset we took for the study. Through this working model it is ensured that there is no compromise in the service, which is indeed the topmost priority of a fire station and also the cost is minimized that will help in setting the budget for the fire department accordingly. This is a result we obtained under ideal conditions; thus, it is subject to change when applied practically in the real-life depending on a lot of other quantitative and qualitative aspects which are not taken into consideration in our analysis.

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Biographies

Prof. RRK Sharma: He is B.E. (mechanical engineering) from VNIT Nagpur India, and PhD in management from I.I.M., Ahmedabad, INDIA. He has nearly three years of experience in automotive companies in India (Tata Motors and TVS-Suzuki). He has 32 years of teaching and research experience at the Department of Industrial and Management Engineering, I.I.T., Kanpur, 208016 INDIA. To date he has written 1192 papers (peer-reviewed (389) /under review (22) / working papers 781 (not referred)). He has developed over ten software products. To date, he has guided 64 M TECH and 21 Ph D theses at I.I.T. Kanpur. He has been Sanjay Mittal Chair Professor at IIT KANPUR (15.09.2015 to 14.09.2018) and is currently a H.A.G. scale professor at I.I.T. Kanpur. In 2015, he received “Membership Award” given by IABE USA (International Academy of Business and Economics). In 2016 he received the “Distinguished Educator Award” from IEOM (Industrial Engineering and Operations Management) Society, U.S.A. In 2021, he received IEOM Distinguished Service Award. In 2019 and 2020, he was invited by the Ministry of Human Resources Department, India, to participate in the NIRF rankings survey for management schools in India. In 2019, he was invited to participate in the Q.S. ranking exercise for ranking management schools in South Asia.

Dr. Vinay Singh: He has earned his Bachelor Degree in engineering (Computer Science and Engineering) from RBS College Agra, Masters in Human Resource Development and Management from IIT Kharagpur and PhD in Management from IIT Kanpur. Currently he is working as Assistant Professor in the department of Management at ABV-Indian Institute of Information Technology and Management Gwalior, India since Nov 2012. So far he has 26 publications in peer review journals to his credit. He has supervised 92 Masters Students and guided 02 PhD theses. He has also earned two national patents in embedded products design and has developed three software packages. He has received 03 research project grants from prestigious agencies of India.

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Appendix

Table 4. Optimized Solution for 30 Problem (Size i=30 & k=30)

Model 1 (Budget Constraint)			**Model 2 (Service Constraint)**			**Hybrid Model**		
Optimised Value	Time elapsed	Iterations	Optimised Value	Time elapsed	Iterations	Optimised Value	Time elapsed	Iterations
12299810.0	1.553	270	3768	1.375	227	12304370.0	1.404	226
9116093.0	2.125	123	1422	1.594	105	9118275.0	1.512	133
6717921.0	1.461	141	1135	1.493	113	6720228.0	1.346	156
8236155.0	1.601	131	1851	1.475	187	8238463.0	1.374	105
8463909.0	1.465	226	2006	1.418	235	8467090.0	1.414	232
9281936.0	1.455	144	2530	1.483	194	9284838.0	1.375	190
7933020.0	1.455	202	1689	1.362	180	7935302.0	1.371	196
8051945.0	1.478	184	1662	1.291	214	8054511.0	1.377	121
11692530.0	1.429	165	3107	1.343	296	11696120.0	1.385	220
7696319.0	1.428	166	2385	1.351	229	7699784.0	1.389	238
63761640.0	1.52	279	19267	1.42	215	60739760.0	1.374	178
69461520.0	1.387	226	18056	1.402	209	69481360.0	1.424	196
54132940.0	1.486	358	13329	1.357	180	54148950.0	1.39	282
76251460.0	1.59	536	18732	1.315	180	76270820.0	1.401	285
48523780.0	1.426	173	12877	1.364	168	48537820.0	1.427	164
52624380.0	1.428	193	15499	1.395	203	52640330.0	1.394	182
70670580.0	1.427	388	16566	1.401	171	70688580.0	1.423	163
50614660.0	1.42	189	18197	1.385	187	50633390.0	1.354	177
63807900.0	1.553	230	18925	1.389	295	63828610.0	1.396	193
54368750.0	1.43	191	11282	1.357	171	54381380.0	1.396	307
615040900.0	1.484	232	18182	1.402	223	615065000.0	1.382	232
811537700.0	1.407	462	33688	1.36	213	811574200.0	1.352	216
777747000.0	1.427	411	17736	1.356	217	777767900.0	1.366	232
695680600.0	1.459	237	23022	1.388	236	695706000.0	1.359	249
686723400.0	1.425	241	24965	1.408	532	686750900.0	1.411	210
715035700.0	1.463	376	16283	1.395	231	715054900.0	1.392	236
559668600.0	1.418	258	19373	1.405	198	559693600.0	1.392	209
673298500.0	1.372	361	17249	1.385	244	673320800.0	1.398	222
646563100.0	1.378	469	19337	1.401	209	646584700.0	1.388	274
638978000.0	1.56	343	18575	1.36	232	639001500.0	1.393	292

Table 5. Optimized Solution for 30 Problem (Size i=60 & k=60**)**

Model 1 (Budget Constraint)			**Model 2 (Service Constraint)**			**Hybrid Model**		
Optimised Value	Time elapsed	Iterations	Optimised Value	Time elapsed	Iterations	Optimised Value	Time elapsed	Iterations
32080322	1.825	332	4544	1.746	464	32085550	1.579	371
38641950	1.816	494	5421	1.738	474	38648790	1.628	492
33631380	1.793	463	5252	1.63	524	33638220	1.748	410
41962200	1.73	542	4704	1.771	717	41968770	1.669	575
33881900	1.796	380	3510	1.736	373	33887090	1.641	362
25727160	1.788	360	2545	1.737	387	25731130	1.658	335
37830420	1.816	400	4978	1.764	689	37836760	1.723	420
37862420	1.892	421	4887	1.752	588	37868860	1.71	421
34203570	1.884	369	3618	1.709	378	34208740	1.631	446
42969680	1.726	422	4969	1.629	438	42975760	1.651	429
202944200	1.835	415	26336	1.993	402	202972500	1.826	387
246931600	1.936	651	33893	1.648	368	246747600	1.897	817
247235600	1.834	372	32987	1.733	419	247270400	1.876	388
252729900	2.118	897	34890	1.745	411	252491200	2.415	785
240849400	1.809	542	37502	1.665	409	240888100	1.844	558
284407500	2.015	469	38919	1.758	442	284447500	1.867	639
203559400	1.816	892	27742	1.74	372	203590900	1.894	1016
277472300	1.813	606	31135	1.761	684	277505800	1.749	607
250608600	1.8	344	28818	1.733	362	250639900	1.632	336
252532900	2.015	949	32848	1.715	360	252567900	1.852	1141
2651499000	1.742	477	31104	1.605	465	2651532000	1.661	493
3407573000	2.079	1478	43583	1.72	417	3403478000	1.922	1280
2897853000	1.827	515	37763	1.58	434	2897900000	1.62	480
2661523000	1.765	528	37317	1.667	472	2661570000	1.741	481
2891858000	1.902	594	45084	1.661	456	2891908000	1.733	540
2720605000	1.825	458	35951	1.726	516	2720648000	1.756	467
2627396000	1.793	512	43909	1.727	437	2627445000	1.762	500
2503041000	1.836	506	38338	1.738	407	2503084000	1.715	516
2853158000	1.786	467	35167	1.732	459	2853202000	1.752	476
3016727000	1.849	448	40811	1.609	449	3016776000	1.739	551