Model for Locating Hospitals in a City

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Abstract

This paper develops and analyzes a model for hospital location and capacity allocation.Thefocus is on an urban/developing area. The first model is the convectional model and the secondis an improved model with better constraints, previous work in the planning of hospitals and/orhealth centers in a region has been done with less constraints hence is easily implementableand has less administrative load but costs may be higher.This work focuses on developing andanalyzing a model for hospital location and capacity allocation while optimizing a number ofobjectives, e.g. minimizing cost of establishment and maximizing coverage of patients.The model 1 is easily implementable and has less administrative load hence costs may behigher, prescribed Model has more variables (hence more CPU time) and requires more administrative load for implementation but costs are probably lesser.Hence cost can be reducedconsiderably if we can trade with speed of solution.

Keywords
Facility location, Location of hospitals and service level constraints.

1.Introduction

Hospitals or health care centers play a pivotal role in mitigating serious injuries that occur in a region and are a critical resource because of limited space and large construction investment and operating costs. They are shared among different types of patients with different access targets determined by their location, previous work in the planning of hospitals and/or health centers in a region hasbeen done with less constraints hence is easily implementable and has less administrative load but costs may be higher. Determining the precise location of healthcare services is essential because ofthe vital importance of these centers. In most cities, lack of proper space allocation and optimal location of the city's services with facilities, especially health services (hospitals and clinics), have led to an ascending increase in urban and citizen problems. This work focuses on developing and analyzing a model for hospital location and capacity allocation while optimizing a number of objectives, e.g. minimizing cost of establishment and maximizing coverage of patients. The remainder of this paper is organized as follows. Section 2 presents the problem and Model 2, and its formulation. Section 3 demonstrates solution strategies used by the model. Section 4 focuses on computational results of the model. Finally, the last section contains our conclusions, implications and suggests some directions for future work.

2. Problem and Formulation

Health centers are centers that directly contribute to the health of the individuals and communities. Quick, timely and inexpensive access, to these centers is very important for any community, especially in urban communities. Determining the precise location of healthcare services is essential because of the vital importance of these centers. In most cities, lack of proper space allocation and optimal location of the city's services with facilities, especially health services (hospitals and clinics),have led to an ascending increase in urban and citizen problems. What distinguish Model 2fromModel 1 is , as model 1 is easily implementable and has less administrative load hence costs may be higher, but Model 2 has more variables (hence more CPU time) and requires more administrative load for implementation but costs are probably lesser. Choices of capacities of the service facilities are as important as are the choice of their locations. In maximal covering problems the capacities are allocated to sites based on the size of demand at the node, which in case of a region is the number of patients. Averbakh et al. [3] studied plant location with demand dependent capacity allocation. Previous work has dealt with capacity allocation based on demand, however in case of regional planning for hospitals; there exists constraints with respect to the available budget which influences the total capacity available to be allocated. A higher capacity hospital has a greater chance of having more number of operating rooms and beds hence serving more patients. The output of our model is a set of hospital locations and capacity allocations. These are determined with the objective of providing services to the patients in a manner that minimizes the normalize cost of building hospital while meeting hospital capacity constraints. Since hospitals have a limited capacity it is possible that not all patients receive service. We present the formulation for a specific damage scenario, i.e., when the needed constraints are assumed to be known. This research is aimed to optimize the hospital location by Integer programming optimization techniques.

Locating hospitals and allocating hospital capacity in a region is a classic example of decision making. One way to approach these types of problems is through the technique of robust optimization, wherein an effort is made to optimize the worst-case performance of the system. Similarly a risk based approach could also be used to find a solution that works well over all the scenarios. To determine the best place for healthcare centers, these important principles and indicators should be considered:

- Cost Principle: Cost of locating hospital considering its life.
- Distance principle: Distance between patient area `i' and hospital `k'.
- Capacity: Maximum serving capacity of hospital `i'
- Service Level: Service level ordered
- Location: Denotes where centers should be located

Before we present the model we elaborate on the issue of hospital capacity. The least capacity thatany real life hospital should have is equal to the Minimum base volume (smallest volume that thesmallest sized hospital should be able to take). At the same time there is an upper limit on the maximum capacity that any real life hospital could have, which is equal to Maximum critical volume(maximum volume that the biggest sized hospital would be able to take). These maximum valuesare used when we decide hospital capacities in our model.We introduce the following notation:

I	: Areas a region is divided into
K	: Denotes the points that are potential points where hospitals can be located
NV_i	: Measure of persons in that area that needs medical attention (it is positive integer)
Y_k	: Denotes the points where hospitals are located
CAP_k	: Number of patients hospital 'k' can handle
D_{ik}	: Distance between area 'i' and hospital 'k'
$SERVLH_k$	: Service level of hospital 'k'
$SERVLA_i$	: Service level of area 'i'
F_k:	:Normalized cost of locating a hospital considering its life

$$Z(i,k) \quad : \begin{cases} 1, & \text{if area 'i' is assigned to hospital 'k'} \\ 0, & \text{otherwise} \end{cases}$$

2.1 Model 1with conventional administrative constraints
Based on this assumption, objective function becomes:

Minimize $$\sum_{k\in K} F_k * Y_k + \sum_{k\in K}\sum_{i\in I} X_{ik} * D_{ik} \quad (1)$$

Subject to:

1. $X_{i,k} <= NV_i * Y_k \quad \forall k \in K \quad \forall i \in I$ (2)
2. $\sum_{i\in I} X_{ik} \le Y_k * CAP_k \quad \forall k \in K$ (3)
3. $\sum_{k\in K} X_{ik} \equiv NV_i \quad \forall i \in I$ (4)
4. $X_{i,k} \ge 0 \quad \forall k \in K \quad \forall i \in I$ (5)

2.2 Model 2: Advancement of conventional approach with additional administrative constraints
Minimize the objective function (1) by advancing the Model 1with following additional administrative constraints:

1. $Z_{ik} \le Y_k \quad \forall k \in K \forall i \in I$ (6)
2. $\sum_{i\in I} Z_{ik} \le SERVLH_k \quad \forall k \in K$ (7)
3. $\sum_{k\in K} Z_{ik} \le SERVLA_i \quad \forall i \in I$ (8)
4. $\sum_{k\in K} CAP_k * Y_k \ge \sum_{i\in I} NV_i$ (9)
5. $\sum_{i\in I} Z_{ik} \le Y_k * SERVLH_k \quad \forall k \in K$ (10)

3. Findings

In this section, we present our computational results. We use two sets of problem input generated randomly for the to distinguish model performance on different complexity. The choice of model parameters is based onthe random data and some global data available for computation. For each category we solvedthirty instances for both models. All problems were run on a Intel i5 4th-gen, 2.90 GHz processorand 8 GB RAM. The numbers reported in the results table are the objective function values (cost)obtained after solving the instances of (P1) using MIP.

Table1. Average Score of Computational Results of Models (Using output of 30 dataset Appendix Table 3/4)

	Minimized Average Cost		Average CPU Time		Average Iterations	
Data-set	**Model 2**	**Model 1**	**Model 2**	**Model 1**	**Model 2**	**Model 1**
30*30 (N=30)	6831.064	7059.001	2.153	1.947	72	79
60*60 (N=30)	100443.95	101990.58	2.39	2.16	231	245

The table 1 compares the average percentage decrease in cost and iterations of Model 2 compared to Model 1 with two data-set, medium data-set consists of data-sets where a region is divided into 30 zones, and large data-set consists of data-sets where a region is divided into 60 zones. Each test is performed 30 times. Based on the computational results obtained corresponding to two sets of data inputs and the generated results as mentioned in appendixes table A and B, the t-test score of model comparison is mentioned in table 2 on the basis of cost, time complexity (CPU Time) and speed of obtaining solution (number of iterations). The t-score suggest that the Model 2 is performing significantly better that Model 1 thus significantly good for solving more complex problem with such configuration of computational facility without compromising the cost computation to locate the hospital under the given constraints and assumptions as stated in para 3of the paper. While looking at the solution convergence speed (number of iteration), it is evident that model 2 is more robust and useful for larger problem size again without compromising cost computation of locating new hospital in a city.

Table 2. t-test Score of Models Comparison (Using output of 30 dataset Appendix Table 3/4)

	Cost Model2 v/s Model 1		CPU Time Model2 v/s Model 1		Iterations Model2 v/s Model 1	
	t-value	Sig (2-tailed)	t-value	Sig (2-tailed)	t-value	Sig (2-tailed)
data-set 30*30 (N=30)	-1.922	.065	12.111	0.000	-1.373	0.18
data-set 60*60 (N=30)	0.749	0.460	11.648	0.000	-3.5	0.002

5. Implications

In this section we present a set of implications based on our case study runs.For the case when the total capacity is more than the number of patients, reallocation ofcapacities is not desirable since the current solution is usually acceptable.For the dual case,i.e. when the total capacity is less than the number of casualties, real-location is desirable andthe extent of it depends on the cost of reallocating a unit of capacity.For the case when the total capacity is greater than the number of casualties, the optimal number of facilities could be less than N, the pre-specified number of hospitals to be built. Howeverfor the case when total capacity is less than or equal to the number of casualties, the optimalnumber of facilities could be greater than N, the pre-specified number of hospitals to be built.This observation is of considerable significance to planners in deciding the number of hospitalsto be built. This is because in low population regions there could be some facilities whichactually do not serve any casualties. The capacities of these hospitals could be reallocated toother facilities and the cost can be made saved. Similarly in a high population region, it ispossible that results would have been better if there were a larger number of hospitals.

In model 1 an area is partially assigned to a hospital, where as in model 2 entire area is assigned to a hospital (which is administratively easier to implement). It is expected that computational results (cost and CPU time) are data dependent. Here we give selected computational result & more computational experience is required.

5.Conclusions and Future Scope

The hospital locations and capacity allocations obtained using Model 2is much better than theoriginal convectional model in terms of cost and iterations. This model could be used to enhancehospital planning efforts for a urban/developing region if we can trade with speed of solution.In the current research, patient density in a region is considered to have the same severity. However, the routing of the casualties needs to take into account this severity so as to maximize thenumber of patients served. Capacity allocation too needs to take into account this factor.Similarlywe also have to consider that the patient density changes with season, Monsoon season may havemore patients as compared to Summer/Fall hence this needs to be considered. This study has taken into account only average number of patients over all seasons, however similar models can also bedeveloped for season specific time-frame and designers can act accordingly.

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Biographies

DrSheela Sharma is medical practitioner and Professor and Head (Department of Obstetrics and Gynaecology) at Rama Medical College Kanpur (India). She has 10 publications to her credit.

Prof. RRK Sharma: He is B.E. (mechanical engineering) from VNIT Nagpur India, and PhD in management from I.I.M., Ahmedabad, INDIA. He has nearly three years of experience in automotive companies in India (Tata Motors and TVS-Suzuki). He has 32 years of teaching and research experience at the Department of Industrial and Management Engineering, I.I.T., Kanpur, 208016 INDIA. To date he has written 1192 papers (peer-reviewed (389) /under review (22) / working papers 781 (not referred)). He has developed over ten software products. To date, he has guided 64 M TECH and 21 Ph D theses at I.I.T. Kanpur. He has been Sanjay Mittal Chair Professor at IIT KANPUR (15.09.2015 to 14.09.2018) and is currently a H.A.G. scale professor at I.I.T. Kanpur. In 2015, he received "Membership Award" given by IABE USA (International Academy of Business and Economics). In 2016 he received the "Distinguished Educator Award" from IEOM (Industrial Engineering and Operations Management) Society, U.S.A. In 2021, he received IEOM Distinguished Service Award. In 2019 and 2020, he was invited by the Ministry of Human Resources Department, India, to participate in the NIRF rankings survey for management schools in India. In 2019, he was invited to participate in the Q.S. ranking exercise for ranking management schools in South Asia.

Dr. Vinay Singh: He has earned his Bachelor Degree in engineering (Computer Science and Engineering) from RBS College Agra, Masters in Human Resource Development and Management from IIT Kharagpur and PhD in Management from IIT Kanpur. Currently he is working as Assistant Professor in the department of Management at ABV-Indian Institute of Information Technology and Management Gwalior, India since Nov 2012. So far he has 32 publications in peer review journals to his credit. He has supervised 92 Masters Students and guided 02 PhD theses. He has also earned two national patents in embedded products design and has developed three software packages. He has received 03 research project grants from prestigious agencies of India.

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Prof. KK Lai: He is currently President of CYUT Taiwan. He has numerous publications to his credit.

Appendix

Table 3. Computational Results (30 Outputs for Data Set of problem Size i=30 & k-30)

	Model 2			**Model 1**		
Sr.No	**Minimum Cost**	**CPU Time**	**Iterations**	**Minimum Cost**	**CPU Time**	**Iterations**
1	7280.8447370	2.247	66	8349.7152320	2.09	87.00
2	5530.9340360	2.272	80	5723.2252800	2.07	59.00
3	7565.2254110	2.302	67	7726.2579250	2.14	61.00
4	4928.2983410	2.238	64	5210.3053020	1.864	58.00
5	7584.2300410	2.317	73	7370.9911760	2.08	76.00
6	6130.2623600	2.218	55	6609.9013850	2.13	81.00
7	5560.5409630	2.273	87	5480.3780510	2.13	88.00
8	6920.5037830	2.301	53	8113.2474300	2.152	82.00
9	8339.2668940	2.215	41	9745.4627060	2.018	115.00
10	8228.1168560	2.236	129	8186.8338820	1.933	83.00
11	4647.5916070	2.161	78	4909.4116900	1.908	77.00
12	7228.7071430	2.021	60	8547.6722250	1.742	79.00
13	6577.3641740	2.016	85	7029.7808020	1.992	96.00

14	6656.0000000	2.136	101	5614.7548460	1.847	87.00
15	6770.2543580	2.162	66	6787.1532980	1.988	77.00
16	6274.4689350	2.166	59	6745.9065300	1.97	79.00
17	6109.0359290	2.178	51	6378.7824280	1.816	76.00
18	6578.9688560	1.939	116	5715.0090310	1.769	76.00
19	7109.2102380	2.096	60	7036.1571990	1.879	78.00
20	8220.2885340	2.01	80	7286.6240480	1.893	79.00
21	6669.7578390	2.127	59	8015.5979380	1.896	85.00
22	6486.2102020	2.123	66	6862.7050480	1.929	90.00
23	6332.0483470	2.075	92	5824.7705010	1.836	53.00
24	7037.5243060	2.156	66	7751.0390010	1.89	86.00
25	8078.5910890	2.1	51	8476.9096240	1.863	83.00
26	6806.9062160	2.095	46	7630.3888890	1.992	94.00
27	9104.7171190	2.029	65	8677.5002650	1.957	69.00
28	6092.7896000	2.133	106	5942.9966560	1.677	82.00
29	7095.1573350	2.1	84	7222.4694970	1.981	67.00
30	6988.1083820	2.148	64	6798.0833370	1.97	64.00

Table 4. Computational Results (30 Output for data Set of problem size i=60 & k=60)

	Model 2			**Model 1**		
Sr. No	**Minimum Cost**	**CPU Time**	**Iterations**	**Minimum Cost**	**CPU Time**	**Iterations**
1	118036.31	2.473	262	106004.30	2.213	226
2	83043.56	2.502	187	109217.61	2.27	329
3	110818.00	2.501	250	102856.38	2.253	290
4	102914.02	2.477	267	85111.97	2.199	211
5	98470.00	2.579	182	115200.00	2.317	251
6	111513.57	2.484	265	115627.62	2.317	332
7	74577.52	2.501	225	82729.00	2.139	288
8	104455.53	2.544	267	109203.70	2.24	356
9	99868.36	2.168	267	103634.00	2.009	291
10	79969.57	2.27	224	75423.58	2.144	241
11	107493.50	2.247	268	100776.03	1.985	280
12	118110.02	2.381	232	107363.99	2.144	209
13	71672.25	2.236	181	92366.23	2.172	269
14	118098.84	2.344	196	117356.19	2.105	221
15	115309.94	2.264	274	114284.09	2.077	294
16	110684.66	2.259	243	104043.96	2.037	217
17	73253.19	2.372	206	76737.07	2.143	289
18	90157.80	2.264	235	87730.78	2.004	265
19	99207.68	2.348	244	103745.26	2.165	303
20	113281.00	2.369	227	110810.03	1.919	271

21	88570.20	2.22	202	96907.67	2.074	257
22	97877.03	2.411	281	109332.60	2.26	343
23	106404.47	2.388	218	101396.62	2.096	225
24	78997.13	2.499	224.0	80047.72	2.319	226
25	106918.46	2.629	180.0	119509.00	2.244	238
26	131840.05	2.356	288.0	124339.74	2.365	270
27	110684.66	2.259	243	104043.96	2.037	217
28	73253.19	2.372	206	76737.07	2.143	289
29	90157.80	2.264	235	87730.78	2.004	265
30	99207.68	2.348	244	103745.26	2.165	303