Behavioural Finance (BF) And Portfolio Optimization Problem (POP)

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Abstract

Return and risk are two important attribute of any portfolio. On likes to max return by keeping at a given level or min risk by keeping return at a min level. Or one takes a composite of return and take negative of risk aversion factor multiplied by risk of a portfolio and max it subject to wealth constraint (this is the traditional mean-variance approach: see Subramanyam and Patel () and Shama and Pankaj ()). In this model the risk aversion factor remains the same whatever be the wealth available with the decision maker. It is well known that as wealth of an individual increases his risk proneness also increases. This aspect is considered in the model that is presented in this paper by assuming that as wealth band changes, then also changes the risk aversion factor. Now we give two models of such situation that considers the mean-variance approach to portfolio selection, and conduct numerical investigation to see which of the two models do better.

Key Words
Behavioural Finance, Portfolio Optimization Problem and Mean Variance Approach

1. Introduction and Literature Review

Sum(i), x(i)*R(i) + x(0)*R – a*(sum(i), Sum(j), x(i)*x(j)*s(i)*s(j)*p(i,j)) – t*sum(y(i) (1)
Model: Maximise (1):

Such that
Sum(i), x(i) +x(0) <= W (2)
X(i) , X0 >= 0 (3)
X(i) <= M*Y(i) for all i (4)
Y(i) = (0,1) for all i (5)

Model P:

Max (1), s.t. (2) to (5)

Where,
- x(i) is the amount invested in security I; i=1,2,…..n
- x(0) is the amount invested in riskless security
- Y(i) =1 if security is part of portfolio else 0

- Ri = 1+expected rate of return on security i: i= 1,2,…..n
- R0 = 1+expected rate of return on riskless security
- a = risk aversion factor of investor (a>0)
- s(i) = standard deviation of the rate of return on security i: i= 1,2,…..n
- p(i,j) = correlation coefficient between the returns on securities I and j: i,j= 1,2,…..n
- t = transaction cost incurred for each risky security included in the portfolio
- W = Total money available to be invested
- M is big positive number.

Model P has been studied by Patel and Subrahmanyam (1982), Pankaj (2006) and Sharma and Pankaj (2007).

2. New Problem Formulation

We define following more variables and parameters.

- 'k' is the index for wealth range.
- YW(k) = 1 if W is in range 'k' otherwise 0.
- a(k) is risk aversion factor when wealth lies in 'k' th band.

Now if LB(k) <= W <= UB(k), then a = a(k) (12)
✓ a + M*(1 – YW(k)) >= a(k) above for all 'k' (6)
✓ a – M*(1 – YW(k)) <= a(k) above for all 'k' (7)

and sum(k), YW(k) = 1 (8)

YW(k) = (0,1) for all 'k' (9)

LB(k) <= W + M*(1 – YW(k)) above for all 'k' (10)
UB(k) >= W – M*(1 – YW(k)) above for all 'k' (11)

Model I: MONOLITHIC MODEL

Max (1), s.t. (2) to (11)

Model II (k): (when wealth belongs to 'k' th range)
Max (1), s.t. (2) to (5) and (12) for a given 'k'

It can be seen that Model I and Max (k), Model II(k) (the fragmented approach) are equivalent problems.
We compare optimal objective values of two approaches (Model I and Model II) and respective CPU times.

3. Computational Experience

Reviewing computational results in appendix A and there compiled score as presented in Table 1, it can be easily seen that monolithic model takes less CPU time and hence is computationally more attractive. Based on the computational observation pertaining to 12 problems, return on portfolio investment and computation CPU time is compared and reported in Table 1. The t-test score on the returns from portfolio computed from model 1 and model 2 has no statistical difference while it can be easily seen that monolithic model takes less CPU time and hence is computationally more attractive.

Table 1: Comparison of Computational Results obtained from Model1 and Model 2

Problem No	Model 1		Model 2		%age Difference in returns
	Return	CPU Time	Return	CPU Time	
1	137800	0.25	31799	0.719	-76.9238
2	158999	0.312	212002	1.719	33.33543
3	107060	0.281	159004	1.813	48.51859
4	53009	0.328	52999	1.234	-0.01886
5	107070	0.234	52999	1.596	-50.5006
6	107060	0.343	106039	2.374	-0.95367
7	106000	0.266	52999	1.609	-50.0009
8	159000	0.266	212002	1.516	33.33459
9	63620	0.219	63601	1.594	-0.02986
10	42400	0.156	42400	1.843	0
11	127205	0.25	190804	2.547	49.99725

Table 2: Sample Statistics obtained from Problem Computation

Sets	Computational Parameters	Mean	N	Std. Deviation	Std. Error Mean
Pair 1	Profit_Max1	1.0629E5	11	39661.44761	11958.37640
	Profit_Max2	1.0697E5	11	72276.84712	21792.28936
	M1_vs_M2_Return(%age of change in return)	-1.2038	11	42.46042	12.80230
Pair 2	CPU_Time1	.2641	11	.05284	.01593
	CPU_Time2	1.6876	11	.49470	.14916

Table 3: t-test score of computational comparison of the results obtained from Model 1 and Model 2

Sets	Computational Parameters	t	df	Sig. (2-tailed)
Pair 1	Profit_Max1 - Profit_Max2	-.041	10	.968
	M1_vs_M2_Return(comparison of %age return)	-.094	10	.927
Pair 2	CPU_Time1 - CPU_Time2	-1.42355	10	.000

4. Conclusions

Fragmented model (model 2) existed in literature. We give here a monolithic model (model 1) for the first time. We also find that monolithic model and fragmented model (model 2) give objective function values that are comparable; and monolithic model (model 1) took significantly less CPU time. This is a very useful contribution we make.

References

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Appendix: Computational Results obtained from Model 1 and Model 2

Problem 1: Number of securities = 50

W < 300000				
	W obtained	**Return**	**CPU Time**	**A**
Model 1	129999	137800	0.250	0.04
Model 2				
B1 100-29999	29999	31799	0.219	0.06
B2 30000-74999	68580	1.00000E+10	0.157	0.05
B3 75000-149999	75000	1.00000E+10	0.140	0.04
B4 150000-299999	158492	1.00000E+10	0.203	0.03

Problem 2: Number of securities = 50

W < 250000				
	W obtained	**Return**	**CPU Time**	**A**
Model 1	149999	158999	0.312	0.05
Model 2				
B1 100-49999	49999	53000	0.453	0.07
B2 50000-99999	99999	106000	0.422	0.06
B3 100000-149999	149999	159000	0.312	0.05
B4 150000-199999	199999	212002	0.250	0.04
B5 200000-250000	250000	1.527762E+8	0.282	0.03

Problem 3: Number of securities = 50

W < 150000				
	W obtained	**Return**	**CPU Time**	**A**
Model 1	100999	107060	0.281	0.05
Model 2				
B1 100-999	999	13173	0.328	0.07
B2 1000-9999	9999	10600	0.297	0.065
B3 10000-19999	19999	21200	0.313	0.06
B4 20000-39999	39999	42399	0.203	0.055
B5 40000-74999	74999	79499	0.297	0.05
B6 75000-99999	99999	105999	0.203	0.045
B7 100000-150000	150000	159004	0.172	0.04

Problem 4: Number of securities = 50

W < 150000				
	W obtained	**Return**	**CPU Time**	**A**
Model 1	100999	107060	0.281	0.05
Model 2				
B1 100-999	999	13173	0.328	0.07
B2 1000-9999	9999	10600	0.297	0.065
B3 10000-19999	19999	21200	0.313	0.06
B4 20000-39999	39999	42399	0.203	0.055
B5 40000-74999	74999	79499	0.297	0.05
B6 75000-99999	99999	105999	0.203	0.045
B7 100000-150000	150000	159004	0.172	0.04

Problem 5: Number of securities = 40

W < 50000				
	W obtained	**Return**	**CPU Time**	**A**
Model 1	49999	53009	0.328	0.02
Model 2				
B1 100-999	999	1059	0.328	0.06
B2 1000-4999	4999	5299	0.250	0.05
B3 5000-9999	9999	10599	0.156	0.04
B4 10000-29999	29999	31799	0.266	0.03
B5 30000-50000	49999	52999	0.234	0.02

Problem 6: Number of securities = 40

W < 500000				
	W obtained	**Return**	**CPU Time**	**A**
Model 1	100999	107070	0.234	0.04
Model 2				
B1 100-999	999	1059	0.328	0.06
B2 1000-4999	4999	5299	0.250	0.057
B3 5000-9999	9999	10599	0.203	0.055
B4 10000-29999	29999	31799	0.187	0.052
B5 30000-49999	49999	52999	0.266	0.05
B6 50000-99999	69453	1.00000E+10	0.204	0.45
B7 100000-499999	100000	1.00000E+10	0.156	0.04

Problem 7: Number of securities = 40

W < 150000				
	W obtained	**Return**	**CPU Time**	**A**
Model 1	100999	107060	0.343	0.04
Model 2				
B1 100-999	999	1059	0.422	0.075
B2 1000-9999	9999	10599	0.406	0.07
B3 10000-19999	19999	21200	0.312	0.065
B4 20000-29999	29999	31800	0.297	0.06
B5 30000-49999	49999	53000	0.187	0.055
B6 50000-69999	69999	74199	0.203	0.05
B7 70000-99999	99999	106039	0.297	0.045
B8 100000-150000	150000	6.308998E+8	0.250	0.04

Problem 8: Number of securities = 35

W < 100000				
	W obtained	**Return**	**CPU Time**	**A**
Model 1	99996	106000	0.266	0.04
Model 2				
B1 100-999	999	1059	0.328	0.07
B2 1000-9999	9999	10599	0.187	0.065
B3 10000-19999	19999	21199	0.219	0.06
B4 20000-29999	29999	31799	0.250	0.055
B5 30000-39999	39999	42399	0.203	0.05
B6 40000-49999	49999	52999	0.219	0.045
B7 50000-100000	90635	1.00000E+10	0.203	0.04

Problem 9: Number of securities = 45

W < 250000				
	W obtained	**Return**	**CPU Time**	**A**
Model 1	149999	159000	0.266	0.05
Model 2				
B1 100-49999	49999	52999	0.204	0.07
B2 50000-99999	99999	105999	0.297	0.06
B3 100000-149999	149999	159000	0.234	0.05
B4 150000-199999	199999	212002	0.344	0.04
B5 200000-250000	250000	1.527762E+8	0.437	0.03

Problem 10: Number of securities = 45

W < 60000				
	W obtained	**Return**	**CPU Time**	**A**
Model 1	60000	63620	0.219	0.035
Model 2				
B1 100-999	999	1059	0.203	0.05
B2 1000-9999	9999	10599	0.218	0.0475
B3 10000-19999	19999	21199	0.235	0.045
B4 20000-29999	29999	31799	0.266	0.0425
B5 30000-39999	39999	44903	0.250	0.04
B6 40000-49999	49999	55765	0.172	0.0375
B7 50000-60000	60000	63601	0.250	0.035

Problem 11: Number of securities = 35

W < 40000				
	W obtained	**Return**	**CPU Time**	**A**
Model 1	40000	42400	0.156	0.03
Model 2				
B1 100-999	999	1059	0.125	0.07
B2 1000-4999	4999	5299	0.250	0.065
B3 5000-9999	9999	10599	0.281	0.06
B4 10000-14999	14999	15899	0.203	0.055
B5 15000-19999	19999	21199	0.172	0.05
B6 20000-24999	24999	26499	0.218	0.045
B7 25000-29999	29999	31799	0.203	0.04
B8 30000-34999	34999	37099	0.172	0.035
B9 35000-40000	40000	42400	0.219	0.03

Problem 12: Number of securities = 35

W < 200000				
	W obtained	**Return**	**CPU Time**	**A**
Model 1	119999	127205	0.250	0.045
Model 2				
B1 100-19999	19999	21199	0.125	0.07
B2 20000-39999	39999	42399	0.203	0.065
B3 40000-59999	59999	63600	0.235	0.06
B4 60000-79999	79999	88504	0.469	0.055
B5 80000-99999	99999	106001	0.281	0.05
B6 100000-119999	119999	127202	0.250	0.045
B7 120000-139999	139999	148404	0.297	0.04
B8 140000-159999	159999	169600	0.203	0.035

B9 160000-179999	179999	190804	0.312	0.03
B10 180000-200000	200000	1.00000E+10	0.172	0.025

Biographies:

Prof. RRK Sharma: He is B.E. (mechanical engineering) from VNIT Nagpur India, and PhD in management from I.I.M., Ahmedabad, INDIA. He has nearly three years of experience in automotive companies in India (Tata Motors and TVS-Suzuki). He has 32 years of teaching and research experience at the Department of Industrial and Management Engineering, I.I.T., Kanpur, 208016 INDIA. To date he has written 1192 papers (peer-reviewed (389) /under review (22) / working papers 781 (not referred)). He has developed over ten software products. To date, he has guided 64 M TECH and 21 Ph D theses at I.I.T. Kanpur. He has been Sanjay Mittal Chair Professor at IIT KANPUR (15.09.2015 to 14.09.2018) and is currently a H.A.G. scale professor at I.I.T. Kanpur. In 2015, he received "Membership Award" given by IABE USA (International Academy of Business and Economics). In 2016 he received the "Distinguished Educator Award" from IEOM (Industrial Engineering and Operations Management) Society, U.S.A. In 2021, he received IEOM Distinguished Service Award. In 2019 and 2020, he was invited by the Ministry of Human Resources Department, India, to participate in the NIRF rankings survey for management schools in India. In 2019, he was invited to participate in the Q.S. ranking exercise for ranking management schools in South Asia.

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