Stochastic Programming Model for Perishable Material Inventory with Uncertain Demand and Multiple Suppliers

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Abstract

This paper considers a perishable material inventory problem from a multi-national company. The company purchases multiple perishable materials from different suppliers. Each material has a fixed expiry date issued by the suppliers. The suppliers are supplying the materials to the company based on the orders placed by the company. There is a supply lead-time for a supplier to deliver the ordered materials to the company. Different suppliers have different prices and different supply lead-times. The values of materials are constant before their expiry dates, and drop to zero upon their expiry dates are reached. While the issued expiry date is fixed, the material shelf-life is affected by the supply lead time that is a random variable. The materials are consumed on a first-expired-first-used basis. We formulate the perishable material inventory problem as a stochastic programming. The numerical experiments are conducted to demonstrate the proposed inventory model can help the company to reduce the inventory cost significantly.

Keywords
Perishable Materials, Multiple Suppliers, Supply Lead-times, Stochastic Programming

1. Introduction
The management of perishable materials is a prevalent problem in the chemical, pharmaceutical, and Food and Beverage (F&B) industries. The value of a perishable product after its expiry date drops to zero, and products which have expired may have no salvage value. Over-stocking of products will lead to high wastage and holding costs while under-stocking will incur shortage costs and reduction in business opportunities. The key to managing perishable products is to keep the right inventory at right time with the right length of stay in the warehouse.

Motivated by the practical problem from a multi-national company which is a global leading eye care device manufacturer. The company purchases multiple perishable materials from different suppliers. Each material has a fixed expiry date issued by the suppliers. The suppliers are supplying the materials to the company based on the orders placed by the company. There is a supply lead-time for a supplier to deliver the ordered materials to the company. Different suppliers have different prices and different supply lead-times. The values of materials are constant before their expiry dates, and drop to zero upon their expiry dates are reached. While the issued expiry date is fixed, the material shelf life is affected by the supply lead time that is a random variable. As such, the shelf-life of the same material from different suppliers may be different when the company receives the material. Thus, non-zero random supply lead times and random shelf-life are the two distinguishable features that we have to take into consideration in our material inventory model in this paper.

The organization of the paper is as follows. Section 2 reviews the available literature related to the inventory problem we consider in the paper. Section 3 presents the stochastic programming to mathematically model the material inventory problem. Section 4 conducts some numerical experiments based on the dataset from the eye care device manufacturer. Section 5 concludes the paper with some remarks.

1. Literature Review

There are two main streams of the literature in inventory management and optimization, which are related to this paper, perishable inventory management and inventory management with stochastic supply lead-times. This section reviews the most relevant literature in these two streams.

Perishable inventory management is to manage inventory of perishable products. The perishability is a distinguishable attribute of perishable products that have a fixed or variable shelf life or lifetime. The perishable products expire after a certain time, and the expired products cannot be used anymore. For instance, chemicals, pharmaceuticals and food products are typical perishable products. Perishable inventory problems were studied by Nandakumar and Morton (1993), Liu and Lian (1999), and others. By studying the myopic bounds of the order quantity for a perishable inventory system, Nandakumar and Morton (1993) evaluated their heuristics performance. Through Markov renewal processes, Liu and Lian (1999) analyzed an (s, S) continuous perishable inventory system with instantaneous replenishment. The expiry date or lifetime is provided on the package. After its expiry date or end of lifetime, the product should be disposed (Kouki et al. 2013). As indicated by Chaudhary et al. (2018), around one third of publications on perishable inventory problems considered perishable products with a fixed shelf life or lifetime, which are from the food processing industry. For example, in the yoghurt industry, perishability appears in every stage of the supply chain, from the raw milk that enters the dairy factory and has to be processed within strict time limits, to the intermediate products that are highly perishable, and finally the different final products which are all stamped with a best-before-date, indicating its shelf life.

A variable shelf life or lifetime product is the one whose exact shelf life or lifetime cannot be predetermined. For example, fresh fruits and vegetables whose decay rates are uncertain. Their shelf lives or lifetimes are assumed to be random variables. Analysis of random shelf life or lifetime products is much more difficult, compared to fixed ones. Most inventory models with continuous review assume instantaneous supply of products. Kalpakam and Arivarignan (1988), Liu (1990), and Pal (1990) used Markovian systems to study the perishable inventory problems. By developing a stochastic dynamic programming model, Jain and Silver (1994) obtained the optimal ordering policy for perishable products with random lifetime. The product lifetime follows an arbitrary probability distribution. Assuming the customer demand follows a Poisson distribution, Krishnamoorthy and Varghese (1995) studied a continuous review inventory problem. They derived steady-state probabilities of the perishable products with no shortage and zero lead time. Kalpakam and Sapna (1996) analyzed positive supply lead time of an (s, S) perishable inventory system with Poisson demand. There are many perishable products whose useful shelf life or lifetime cannot be predicted in advance. Fresh agri-food products, meat, fowl and fish are in this category. Perishable inventory models of perishable products with uncertain shelf life or lifetime can be used to formulate inventory problems of perishable products that are subject to obsolescence. There has been increasing research interest in the area of inventory management and optimization with random shelf life and random lifetime.

The impact of the supply lead time on the inventory cost was analyzed in an inventory system with single product in continuous time by Song (1994). The variables of interest are the inventory level and the behavior of average long-term cost. Isotupa and Samanta (2013) analyzed a lost sales (s, q) inventory policy with two types of customers and random supply lead times. The two types of customers arrive according to two independent Poisson processes and the supply lead times are modeled by using an Erlang distribution and constant supply lead times, respectively. Their numerical results on the total cost illustrate that inventory rationing policies are better than taking both types of customers the same.

For the inventory problems with random supply lead times, the single period case was considered in the most of the existing literature. Due to the uncertainty of supply lead times, it is hard to capture the crossover effect in the planning horizon (He et al. 1998). Because the placed orders may cross over the periods, it is very difficult to formulate the recursive relations of inventory states over the periods. With the special assumption that the placed orders are not allowed to cross over the periods, the supply lead times for the orders placed in the successive periods become dependent random variables (Lee and Hahmias 1993). Ould and Dolgui (2009) considered another special case where the demand is constant, the order is placed in each period, and the order size is the same for every period. For this special case, there is no need to know which order is actually delivered as all of those placed orders are the same. Gurnani et al. (1996) considered another multiple period inventory problem for assembly systems with two types of components, where the supply lead times are either zero or one with certain probabilities.

The uncertainties from both customer demands and supply lead times lead to managing inventory more difficult and complex. Gurnani et al. (1996) considered the special case with uncertainties from both customer demands and supply lead times, where the supply lead times only have two values, 0 or 1. As far as we know, Song et al. (2000) is the first to investigate the general inventory replenishment model for assembly systems with uncertainties from both random customer demands and supply lead times. The work was motivated by an industry problem from the computer and electronic industries.

This paper considers an inventory optimization problem of perishable raw materials which are supplied by multiple suppliers with different random supply lead times. Each raw material has an uncertain shelf life upon it is received. A stochastic programming is formulated to model the inventory optimization problem. The numerical experiments are conducted by using the dataset from the eye care device manufacturer to demonstrate the proposed inventory model can help the company to reduce the inventory cost significantly.

2. Stochastic Programming Model

3.1 Model Description

Every material has a fixed shelf life which is provided by its supplier. The shelf life starts to count at the moment when the supplier issues the material to the company. Since the supply lead-time is random, the actual available shelf life to the company is also random, which is equal its original shelf life minus the supply lead-time. We assume there are *M* materials with different shelf lives. There are *S* suppliers which supply a certain range of *M* materials. The same material may come from the different suppliers with the same fixed shelf life but different supply lead-time. To be more specific, we denote material *m* has a fixed shelf life $l_m, m = 1, 2, \dots, M$. The lead-time that Supplier *s* supplies the materials to the company is $L_s, s = 1, 2, \dots, S$, where L_s is a random variable with cumulative distribution function $F_s(x), x \geq 0, 1, 2, \dots, S$.

The materials are purchased for the production of finished goods. The company produces various finished goods with different BOM (Bill of Materials). That is, the different finished goods may need the different amounts of the materials. The customer demand for each finished goods is random. The random customer demands for the finished goods can be converted to the random production demands for the materials through the BOM structures of the finished goods. In the context of our interest, we assume the production demand for each material is $D_m, m = 1, 2, \dots, M,$ where D_m is a random variable with cumulative distribution function $G_m(x), x \geq 0, 1, 2, \dots, M.$ For the production demands, there is no differentiation on the different shelf lives of the same material. For example, for production demand D_m, there is no specific requirement about the shelf life of material *m*. However, with economical consideration, the shorter the remaining shelf life of material *m*, the earlier to be used to fulfil the production demand.

In the warehouse of materials, there are *M* different materials with different shelf lives. For material *m*, its shelf life in the warehouse is τ_m, $0 \leq \tau_m \leq l_m$, $m = 1, 2, \dots, M$. It is readily known that when $\tau_m = l_m$, the supply is instantaneous and the supply lead-time is zero. When $\tau_m = 0$, the corresponding material *m* has to be discarded.

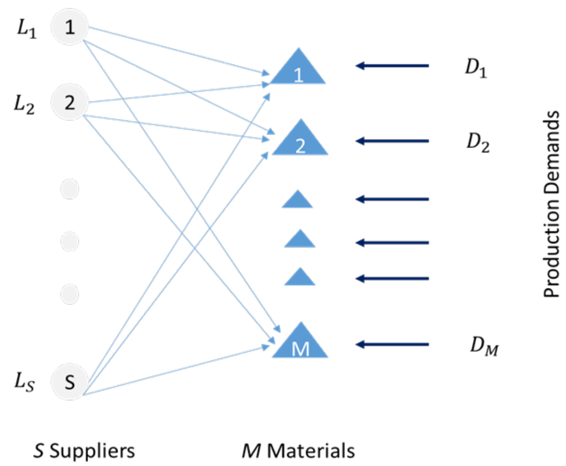

Figure 1: Inventory System of *M* Perishable Materials with *S* Suppliers

3.2 Model Parameters

We consider an inventory planning problem with a finite planning horizon *T* which is composed of *T* planning periods, starting with period $t = 1$ and ending with period $t = T$. We define the system parameters as follows.

$D_{t,m}$: Production demand for material *m* in period *t*;

$c_{t,m,s}$: Unit purchasing price for material *m* from supplier *s* in period *t*;

$K_{t,m,s}$: Ordering cost for material *m* from supplier s at the beginning of period t;

$h_{t,m}$: Unit holding cost for material *m* in period t;

$p_{t,m}$: Unit penalty/shortage cost for material *m* in period t;

$w_{t,m}$: Unit disposal cost for material *m* at the beginning of period t;

x_{t,m,τ_m}: On-hand inventory level of material *m* with shelf life τ_m in the warhouse at the beginning of period *t*;

z_{t,m,τ_m}: In-transit quantity of material *m* at the beginning of period *t* with shelf life τ_m when received.

3.3 Decision Variables

We have the following decision variables:

$y_{t,m,\tau_m,s}$: Ordering quantity of material *m* from supplier *s* at the beginning of period *t* with shelf life τ_m when received;

$\delta_{t,m,s}$: Binary variable equal to 1 if and only if there is an order placed for material *m* from supplier *s* in period *t*; Otherwise, 0;

For material m, we assume production demand $D_{t,m}$ is time homogeneous, and i.i.d. with $D_m, m = 1, 2, \dots, M$. For period *t*, the inventory related costs can be expressed as follow:

The holding cost: $\sum_{m=1}^{M} h_{t,m} E\left(\sum_{\tau_m=1}^{l_m} x_{t,m,\tau_m} + \sum_{s=1}^{S} y_{t,m,l_m,s}\, \delta_{t,m,s} - D_m\right)^{+}$;

The penalty cost: $\sum_{m=1}^{M} p_{t,m} E\left(\sum_{\tau_m=1}^{l_m} x_{t,m,\tau_m} + \sum_{s=1}^{S} y_{t,m,l_m,s}\, \delta_{t,m,s} - D_m\right)^{-}$;

The purchase cost: $\sum_{m=1}^{M} \sum_{s=1}^{S} c_{t,m,s}\, y_{t,m,\tau_m,s}\, \delta_{t,m,s} E\left(I_{\{\tau_m = l_m - L_s \geq 1\}}\right)$, where $I_{\{\}}$ Is an indicator function;

The disposal cost: $\sum_{m=1}^{M} w_{t,m} x_{t,m,0}$;

The ordering cost: $\sum_{m=1}^{M} \sum_{s=1}^{S} K_{t,m,s}\, \delta_{t,m,s}$.

3.4 Constraints

The following constraints need be satisfied:

$\sum_{s=1}^{S} \delta_{t,m,s} \le 1$, represents that in period *t*, if we need to order material *m*, the order will be placed to the most suitable supplier among the *S* suppliers, for all $m = 1, 2, \cdots, M; t = 1, 2, \cdots, T$;

$z_{t,m,\tau_m} = \sum_{s=1}^{S} y_{t,m,\tau_m,s}$, for all $\tau_m = 1,\ 2, \cdots, l_m; m = 1, 2, \cdots, M; t = 1, 2, \cdots, T$;

$x_{t,m,\tau_m} = \left(\sum_{l=1}^{\tau_m+1} x_{t-1,m,l} - D_m\right)^{+} + \sum_{s=1}^{S} z_{t-L_s,m,\tau_m} \delta_{t-L_s,m,s}$, for all $\tau_m = 1,\ 2, \cdots, l_m; m = 1, 2, \cdots, M; t = 1, 2, \cdots, T$;

$\sum_{l=1}^{\tau_m^*+1} (x_{t,m,l} + \sum_{s=1}^{S} z_{t-L_s,m,\tau_m} \delta_{t-L_s,m,s}) \ge D_m$, and $x_{t+1,m,l} = 0, 1 \le l \le \tau_m^*$ if there exists $\tau_m^* = \max\{\tau_m |\, D_m > \sum_{l=1}^{\tau_m} (x_{t,m,l} + \sum_{s=1}^{S} z_{t-L_s,m,\tau_m} \delta_{t-L_s,m,s})\}$, for all $m = 1, 2, \cdots, M; t = 1, 2, \cdots, T$. These constraints ensure that the shorter the shelf life, the earlier to be used to fulfil the production demand.

3.4 Stochastic programming inventory model

In summary, the stochastic programming inventory model formulation can be presented as follows.

$$\text{Minimize} \sum_{t=1}^{T} \Big\{ \sum_{m=1}^{M} \sum_{s=1}^{S} K_{t,m,s}\, \delta_{t,m,s} + \sum_{m=1}^{M} \sum_{s=1}^{S} c_{t,m,s}\, y_{t,m,\tau_m,s}\, \delta_{t,m,s} E\left(I_{\{\tau_m = l_m - L_s \ge 1\}}\right) + \sum_{m=1}^{M} h_{t,m} E\left(\sum_{\tau_m=1}^{l_m} x_{t,m,\tau_m} + \sum_{s=1}^{S} y_{t,m,l_m,s}\, \delta_{t,m,s} - D_m\right)^{+} + \sum_{m=1}^{M} p_{t,m} E\left(\sum_{\tau_m=1}^{l_m} x_{t,m,\tau_m} + \sum_{s=1}^{S} y_{t,,m,l_m,s}\, \delta_{t,m,s} - D_m\right)^{-} + \sum_{m=1}^{M} w_{t,m} x_{t,m,0} \Big\}$$

Subject to

(1) $z_{t,m,\tau_m} = \sum_{s=1}^{S} y_{t,m,\tau_m,s}\,; \tau_m = 1,\ 2, \cdots, l_m; m = 1, 2, \cdots, M; t = 1, 2, \cdots, T;$

(2) $x_{t,m,\tau_m} = \left(\sum_{l=1}^{\tau_m+1} x_{t-1,m,l} - D_m\right)^{+} + \sum_{s=1}^{S} z_{t-L_s,m,\tau_m} \delta_{t-L_s,m,s}; \tau_m = 1,\ 2, \cdots, l_m; m = 1, 2, \cdots, M; t = 1, 2, \cdots, T;$

(3) $\sum_{l=1}^{\tau_m^*+1} (x_{t,m,l} + \sum_{s=1}^{S} z_{t-L_s,m,\tau_m} \delta_{t-L_s,m,s}) \ge D_m$, and $x_{t+1,m,l} = 0, 1 \le l \le \tau_m^*$ if there exists $\tau_m^* = \max\{\tau_m |\, D_m > \sum_{l=1}^{\tau_m} (x_{t,m,l} + \sum_{s=1}^{S} z_{t-L_s,m,\tau_m} \delta_{t-L_s,m,s})\}, m = 1, 2, \cdots, M; t = 1, 2, \cdots, T;$

(4) $\sum_{s=1}^{S} \delta_{t,m,s} \le 1;\ m = 1, 2, \cdots, M; t = 1, 2, \cdots, T.$

The stochastic programming inventory model captures the dynamics of the perishable material system over the planning horizon *T*. This stochastic programming formulation optimizes the perishable material inventory system of the eye care manufacturer. In the next section, we shall implement the stochastic programming by using Python and find the solution to the stochastic programming.

3. Numerical experiments

This section uses the stochastic programming inventory model to conduct the simulation experiments by Python, and compares the experiment results with the current practice of the eye care manufacturer. As an illustration, we consider the special case with one type of material and three suppliers.

The company data are presented in Table 1. The unit for all the costs is dollar ($). The supply lead times of the three suppliers are random variables with statistics provided in the table. The shelf life of the material is 5 months.

Table 1. System parameters

Parameter	Supplier #23	Supplier #25	Supplier #29
Unit Purchasing Cost	0.1	0.15	0.2
Unit Holding Cost	0.03	0.03	0.03
Unit Shortage Cost	0.06	0.06	0.06
Unit Wastage Cost	0.2	0.2	0.2
Supply Lead Time	1 month: 25%	1 month: 20%	1 month: 10%
	1.5 months: 50%	1.5 months: 60%	1.5 months: 80%
	2 months: 25%	2 months: 20%	2 months: 10%

With the forecasted demand over the future 12 months, the order quantities for the material are obtained by the stochastic programming model simulation, as shown in Figure 1.

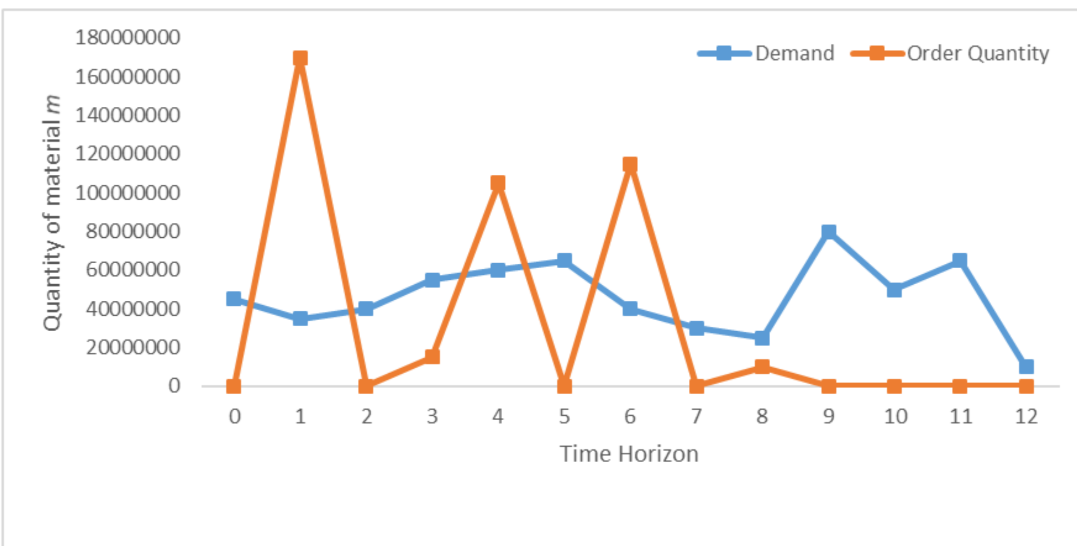

Figure 1. Forecasted demand and order quantity obtained for 12 months planning horizon

The total cost from the stochastic programming model is 58.98 million dollars. Compared with the total cost 81.15 million dollars by the current planning approach, the cost saving is about 27.32%. With numerical experiments for more scenarios, more cost savings can be expected from the stochastic programming model.

3. Concluding Remarks

This paper studied the perishable material inventory problem from a global leading eye care manufacturer. With consideration of uncertain demand and multiple suppliers, the perishable material inventory problem was formulated as the stochastic programming inventory model. Based on the stochastic programming inventory model, the numerical experiments were conducted to obtain the order quantities for all the periods over the planning horizon. The special case of the company problem with one type of material demonstrated that the stochastic programming inventory model can save the total cost by 27.32%.

With the stochastic programming inventory model, we shall continue to conduct the numerical experiments with more scenarios. In another note, to develop algorithms to solve the stochastic programming inventory model will be our future research along this direction.

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