Managing Projects Under Different Payment Schemes

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Abstract

Project management in practice has grown very fast in recent years, and the risks managers face become much more complexed. As project complexity increases, the randomness of project completion time makes it more difficult for manufacturers to incentivize suppliers. In this paper, we consider a situation when a manufacturer outsources two parallel tasks to two different suppliers, because of the randomness of the project completion time, one supplier will often complete the project earlier (faster) and one will complete the project later (slower). Based on this, we propose two different dimensions of the incentive mechanism: one concerns the faster supplier and the other concerns the slower supplier. We obtain nine kinds of contracts that a manufacturer can offer to suppliers by combining these two dimensions, and different payment schemes manufacturer can offer to the suppliers are included in our analysis. Suppliers' optimal work rates are determined and compared among the nine contract types.

Keywords
Project Management, Incentive Contracts, Parallel Tasks, Delayed Payment Scheme, Stochastic Completion Time

1. Introduction

With the accelerating pace of market change, enterprises must constantly change the operation mode to adapt to the new competitive environment. Rapid launch of new products or customized products has become an important means of operation for many enterprises to cater for the rapid change of consumer demand. Research data of Pricewaterhouse Coopers and MIT show that from 2010 to 2012, the frequency of changes in enterprise operation network structure increased by 94%, and the speed of launching new products increased by 87%. Under new environment, the traditional supply chain of mass production is gradually evolving into a new supply chain structure composed of individual projects, which is called project-driven supply chain (hereinafter referred to as project supply chain) in academia. At the same time, many enterprises begin to turn from internal independent innovation to external collaborative innovation, due to the constraints of their own resources and capabilities, and in order to respond to market changes and reduce risks more quickly. Seeking cooperation of supply chain members and joint innovation with supply chain partners has become the main choice of most enterprises.

Project supply chain usually requires knowledge and technology complementation among supply chain members, division of labor and collaboration, and joint completion of specific tasks. The unique characteristics of project supply chain bring great difficulties to the management of project supply chain, and among all, the most challenging problem is time coordination among all parties with random project duration time. In particular, Boeing has entrusted over 65%

of the R&D and production tasks of 787 Dreamliner to more than 100 suppliers from 12 countries. The 787 Dreamliner utilizes a different supply chain and has been outsourced to multiple suppliers; however, Boeing faces a series of delays in its schedule for maiden flight and plane delivery to customers. Consequently, Boeing imposes a delayed payment regime (also known as the "risk-sharing" contract) on its strategic suppliers in which all suppliers are required to receive payments from Boeing to recoup their development costs until the first 787 plane is delivered to the customer (see Nolan and Kotha 2005). Boeing believe that the delayed payment regime provides a proper incentive for the suppliers to coordinate their tasks better respect to the trade-off between time and cost metrics, which can be beneficial to his own benefit (Tang et al. 2009).

However, the fast delivery of some parts by the supplier who finishes earlier have no additional benefits other than holding costs for the manufacturer. Manufacturer's schedule depends on the slowest delivery in settings where the manufacturer can't start working until all parts are delivered. Thus, time synergy is the main consideration for the manufacturer. Intuitively, we are wondering that why the supplier who finishes earlier is only punished. We can incorporate another punishment scheme concerning the supplier who finishes later as an incentive mechanism for the manufacturer to increase suppliers' work rates, other than raising prices paid to them to shorten the total project duration time. In this occasion, how does the interaction among those different factors influence the manufacturer's profit is unclear. Therefore, it is important for the manufacturer to explore these questions when designing incentive contracts.

In this paper, we consider a setting where a manufacturer outsources two parallel subtasks to two different suppliers with stochastic project completion time. Owing to suppliers' technical problems, differences in abilities and some other random factors, the two suppliers may end tasks with different durations. Thus, the result ex post could be that one supplier finishes earlier (hereinafter referred as faster supplier) and one later (hereinafter referred as slower supplier), and the manufacturer's profit may be harmed by this time asynchronization. Furthermore, because of the stochasticity of project duration times, both suppliers don't know exactly who will be the one who finishes earlier and who will be the one who finishes later ex ante. We mainly consider the adjustable work rate assumption in our context, and we assume that the manufacturer can't start to work until all parts are delivered. The manufacturer (she) acts as the Stackelberg leader to make her decision on the optimal price she offers to the supplier, and each supplier (he) has to decide on the optimal work rate to best his expected discounted profit.

Besides the no-delayed and delayed payment schemes considered by Boeing, we make a more general extension here: partial delayed payment scheme is introduced in our work. Under each payment scheme offered to the faster supplier by the manufacturer, three payment schemes offered to the slower supplier as a punishment to him are considered: full (no punishment), two-part partial (mild punishment), and partial (total punishment) payment schemes. The remainder of this paper is organized as followed, in Section 2, we provide a related literature review on project management. Section 3 gives the research methodology and preliminaries. Section 4 gives the specific models to compare different kinds of payment schemes. In Section 5, we summarize our work and give a conclusion of this paper.

2. Literature Review

Regarding single-agent project management, optimization is the main way to solve these problems and we refer the readers to Klastorin (2004) and Nahmias and Cheng (2009) for a further understanding. Hall (2016) provides a comprehensive review and gives a discussion of research and teaching opportunities within this discipline. In this section, we mainly focus on multi-agent project management, and we review some important contributions researchers have made.

Project bidding.

Auctions usually occur before projects begin and involve multiple bidders. Some researchers have focused their attention on the bidding strategies that emerge in the construction industry or problems arising from new product development activities in a firm. Gupta et al. (2015) focus on the A+B transportation procurement mechanism, which is used by American transportation agencies to provide incentives for faster completion. This mechanism contains two parts: the proposed cost for component A, and the proposed completion time for component B to score the contractor's bid. Managers use time-based incentive contracts to encourage contractors to exert greater effort toward faster completion. Tang et al. (2015) extend the analysis in Gupta et al. (2015) and compare two different time-related project management contracts (C1 and C2) when the manager conducts a reverse auction. The contractor with the lowest quoted price will win under the C1 contract, and with the lowest composite score under C2 contracts. They reveal that the more sophisticated C2 contract does not add additional value to the manager, and the simpler C1 contract would suffice.

Moral hazard and teamwork.

Information asymmetry is a very common phenomenon in project management, and an agency-theoretic perspective is often applied in joint development, services, and production activities that involve two or more agents to participate. Bayiz and Corbett (2005) establish a moral hazard model to study the impact of contract design on subcontractors' efforts under asymmetric information. They tried to solve the problem that the project manager faced when two sub-projects are outsourced to different subcontractors with an unobservable subcontractor's effort level by the manager, and they derived the optimal incentive contracts the project manager offers to the subcontractors when the two sub-projects are conducted in parallel or in series, respectively. Zhang (2016) examines the value of deadlines from the agency-theoretic perspective and considers a firm that pays an agent to lead product development activities. The chance of success depends on the viability of the project and the agent's effort. This study offers managerial recommendations on when to impose hard deadlines. They find that hard deadlines are more suitable if the agent's cost of effort is higher, the firm's yield from success is lower, the agent is more patient, the firm is less patient, or the prior belief of the project is more pessimistic. Fu et al. (2016) develop a model in which a manager controls a team of workers who execute a project. The project's total payoff equals the output generated by the workers' effort, which is unobservable by the manager, plus random, normally distributed noise, whose variance represents the project's risk. They use the model to show how agency conflicts, free-rider effects, and monitoring costs interact to affect the optimal size and workers' incentive contracts.

Project supply chain.

In contrast to project bidding or teamwork cooperation, when multiple parties enter into a stable partnership with each other, the problems the project owner faces will be different. In settings where a manufacturer outsources subtasks to different subcontractors, incentive mechanism design is the main focus. In this sub-section, we pay attention to incentive issues and gaming behaviors between a project owner and its subcontractors, and our research is one that is included in this scope. We list several papers in which incentive issues play a prominent role and project contract design is a crucial mechanism when managing projects. Kwon et al. (2010a) is the first study in this area to consider the problem of project contract coordination. In their work, they analyze the contract-design problem faced by a principal outsourcing a single task, and project price (e.g., fixed price, time-based, cost-based, rate-based) is the main contractual term in project contracts, and derive the optimal coordination contract under this scenario when the project completion time is exponentially distributed. Kwon et al. (2011) consider a situation in which a project manager has to decide which tasks to keep in-house and which to outsource when conducting a project with two parallel or two serial tasks. They focus on certain simple contracts and examine the impact of different sourcing decisions of project tasks with exponential completion times on operating profits. Chen and Lee (2017) consider a coordination problem of a project supply chain with a stochastic task completion time assumption, where the manufacturer carries out a series of tasks and each task needs to procure a certain material from a supplier. They propose time-based incentive contracts to mitigate incentive misalignment between firms in a project supply chain and finally solve the coordination problem. Dawande et al. (2019) study the contract design problem faced by a firm in conducting a project consisting of multiple tasks. They study strategic interactions among multiple agents in a more complicated project structure. This study derives the optimal coordination contracts for both parallel and sequential tasks under the exponential completion time assumption.

We elaborate on two papers that are mostly related to our research. One is an article that considers a strategy to reduce the risks manufacturers face when they are engaged in the outsourcing activities of some parallel subtasks, and it takes an incentive mechanism about the supplier who completes faster into account. Kwon et al. (2010b) study incentive contracts for projects with parallel tasks. They consider a situation in which the project manager can offer two different payment schemes to the suppliers when multiple tasks are outsourced to different suppliers, that is, no delayed payment and delayed payment regimes. Under the delayed payment regime, each supplier will be paid after all tasks are completed. The other article takes into account the strategy of a penalty mechanism for the suppliers. Chen et al. (2015) propose an incentive payment contract for serial stochastic projects and compare the contract to several variations, including a fixed-price contract, a hybrid price contract, a dynamic price contract, and two payment timing options.

3. Research Methodology

We suppose that the manufacturer outsources two parallel sub-tasks to two different suppliers in a project management. The manufacturer will receive her revenue of $2q$ when the all the tasks are completed and this value is given exogenously. We assume that the completion time of each task X_i is exponential distributed with parameter λ_i, and the work rate λ_i is selected at the beginning of each task by the supplier. Let the continuous time discount rate be $\alpha > 0$ due to the effective decreasing reward in the project completion time, because each supplier would like to receive his payment earlier. We assume that the suppliers have equal capacity to finish the task, so that we assume the payment p manufacturer offered to the suppliers is the same and $q \geq 0, p \geq 0$. We assume that two suppliers have the capacity to adjust their work rates over time.

While working at work rate λ, each supplier will occur an operating cost of $K(\lambda)$ per unit time, we assume that the operating cost $K(\lambda)$ is a convex-increasing function with $K(0) = 0$ and for simplicity, we let $K(\lambda) = k\lambda^2$ with

$k > 0$. Next, we present two formulas which will be used in the following part of the paper. Because the p.d.f of random variable X_i $(i = 1,2)$ is $\lambda_i e^{-\lambda_i t}$ and the c.d.f of the random variable X_i is $F_i(t) = 1 - e^{-\lambda_i t}$, then we can formulate that the discount factor $\beta_i = E[e^{-\alpha X_i}] = \int_0^\infty \lambda_i e^{-\alpha t} e^{-\lambda_i t}\, dt$ equals to $\lambda_i/\alpha + \lambda_i$. The supplier i's expected discounted total operating cost is $E[\int_0^{X_i} K(\lambda_i) e^{-\alpha t} dt] = K(\lambda_i)\,(1 - E[e^{-\alpha X_i}])/\alpha$, which is equal to $k\lambda_i^2/\lambda_i + \alpha$.

We assume that the work rate can be adjustable and the suppliers have ability and capacity to satisfy this assumption. So the completion time of the supplier who finishes first is $S = \min(X_1, X_2)$ with its distribution of $F^S(t) = 1 - [1 - F_1(t)][1 - F_2(t)] = 1 - e^{-(\lambda_1+\lambda_2)t}$, and the corresponding expected discounted factor associated with completion time S is $\beta_s = E[e^{-\alpha S}] = \int_0^\infty e^{-\alpha t}\, d\big(S(t)\big) = \frac{\lambda_1+\lambda_2}{\alpha+\lambda_1+\lambda_2}$. Table 1 shows the specific elements used in our analysis. Payment timing and amounts under different contracts are two key concerns in our model.

Table 1. The classification of nine different cases

Faster supplier's payment \ Contract type \ Slower supplier's payment	**Full** $(0, p)$	**Two-part Partial** $(\beta\omega p, (1-\beta)\omega p)$	**Partial** $(0, \omega p)$
No-delayed $(p, 0)$	I NDF	II NDTP	III NDP
Delayed $(0, p)$	IV DF	V DTP	VI DP
Partial-delayed $(\gamma p, (1-\gamma)p)$	VII PDF	VIII PDTP	IX PDP

4. Results and Discussion

4.1. No-delayed Payment to the Faster Supplier

We first analyze the occasion when the manufacturer offers a no-delayed payment scheme to the supplier who finishes earlier, that is—the faster supplier will receive a payment of p at the instant when he finishes his task. In case I, the manufacturer does not offer any incentive to the supplier who finishes later, and she offers a full payment scheme to him; the slower supplier receives a payment p at the instant when he completes his own task. In case II, the manufacturer imposes a degree of moderate punishment on the supplier who finishes later, and she offers a two-part partial payment scheme to the slower supplier; he receives a partition of $\beta\omega p$ at the instant when the faster supplier has finished the subtask, and obtains the remaining part of $(1-\beta)\omega p$ when he completes his own subtask. In case III, the manufacturer imposes a complete punishment on the supplier who finishes later and offers a partial payment scheme to him; the slower supplier will be punished and only gets a payment of ωp after both tasks are completed.

Case I: Full Payment (NDF) to the Slower Supplier

In this setting, there is no incentive contract yet, and we define this as our benchmark case. Contract NDF depicts a scenario that both suppliers start to work at time $t = 0$, and they both make the work rate decision λ_i $(i = 1,2)$. After each subtask is completed, he receives a payment p at the instant when he completes his own task. We can express each supplier's expected discounted profit as

$$\Pi_i^1(p; \lambda_i) = p\mathrm{E}[\mathrm{e}^{-\alpha X_i}] - \frac{c\lambda_i^2}{\lambda_i+\alpha} = p\beta_i - \frac{c\lambda_i^2}{\lambda_i+\alpha}, i = 1,2. \quad (1)$$

Proposition 1. Under contract NDF, both suppliers' optimal work rates are the same, and are given by

$$\lambda^{\mathrm{I}*}(p) = \sqrt{\alpha^2 + \frac{p\alpha}{c}} - \alpha. \quad (2)$$

which is strictly positive, strictly increasing, and strictly concave in p. With some simple calculations, we can infer that supplier i's expected discounted profit is given by

$$\Pi_i^1(\lambda^{\mathrm{I}*}) = \Pi^1(\lambda^{\mathrm{I}*}) = \frac{c[\lambda^{\mathrm{I}*}]^2}{\alpha}, i = 1,2. \quad (3)$$

Furthermore, for a given p, the manufacturer's expected discounted profit satisfies the following:

$$\max_p \Pi_m^1(q; p) = (2q - p)\frac{\mu_1^*}{\alpha+\mu_1^*}\frac{2\lambda^{\mathrm{I}*}}{\alpha+2\lambda^{\mathrm{I}*}} - p\frac{2\lambda^{\mathrm{I}*}}{\alpha+2\lambda^{\mathrm{I}*}} \quad (4)$$
$$\text{s.t.}\, \Pi_i^1(p; \lambda_i) \geq 0 \text{ for } i = 1,2.$$

Case II: Two-part Partial Payment (NDTP) to the Slower Supplier

Under this scenario, the manufacturer will pay a partition of the total payment after punishment to the slower supplier at time $t = S = \min(X_1, X_2)$. We define L as the additional completion time for the continuing supplier to complete his task since the time when one of the two suppliers has completed the subtask. As the exponential distribution has the memoryless property, none of the suppliers would change their work rate until one of them finishes his subtask first. At this moment, the remaining supplier will have an incentive to change his work rate. Hence, we consider this problem to be a two-stage non-cooperative game between the two suppliers.

In this study, we propose a rate-based incentive contract for the manufacturer to incentivize the supplier to expend more effort to complete the project earlier. We define our rate-based penalty function $\omega(\mu)$ as follows: $\omega(\mu) = 1 - \delta/\mu, \delta > 0$, where μ is the slower supplier's second-stage work rate and δ represents the incentive factor. We first analyze the second-stage problem: at the time of $S = \min(X_1, X_2)$, one supplier has completed his task and we assume that the continuing supplier has to decide his second-stage work rate μ. We have that the continuing supplier's expected discounted profit back to time $t = S$ can be expressed as

$$R(\mu) = \frac{(1-\beta)\omega(\mu)p\mu}{\alpha+\mu} - \frac{c\mu^2}{\alpha+\mu}. \quad (5)$$

Proposition 2. The optimal work rate μ_2^* under contract NDTP satisfies:

$$\mu_2^*(p, \beta, \delta) = \sqrt{\alpha^2 + \frac{(1-\beta)(p\alpha+\delta p)}{c}} - \alpha. \quad (6)$$

We have that $\mu_2^*(p, \beta, \delta)$ strictly increases in δ and decreases in β. We can obtain $R(\mu_2^*) = \frac{c[\mu_2^*]^2-(1-\beta)\delta p}{\alpha}$, which strictly decreases in δ. We have that $\omega_2^* = 1 - \frac{\delta}{\mu_2^*}$.

We then pay attention to the first stage of the game: at time $t = 0$, suppliers 1 and 2 are engaged in a Nash game, and they have to decide their stage 1 work rate. We suppose that supplier i $(i = 1,2)$ selects his own work rate at λ_i, while the other supplier selects the work rate at λ_{-i}, and the duration of stage 1 satisfies $S = \min(X_1, X_2)$. With a simple calculation, we can infer that the probability of supplier i $(i = 1,2)$ finishing first is $\text{Prob}(X_i < X_{-i}) = \frac{\lambda_i}{\lambda_i + \lambda_{-i}}$. It is easy to express supplier i's expected discounted profit (back to time $t = 0$) as follows:

$$\Pi_i^2(p; \lambda_i, \lambda_{-i}) = -\frac{c\lambda_i^2}{\alpha+\lambda_i+\lambda_{-i}} + \frac{\lambda_i+\lambda_{-i}}{\alpha+\lambda_i+\lambda_{-i}} \cdot \left[\frac{\lambda_i}{\lambda_i+\lambda_{-i}} \cdot p + \frac{\lambda_{-i}}{\lambda_i+\lambda_{-i}}\left(\beta\omega_2^* p + R(\mu_2^*)\right)\right], i = 1, 2.$$

Proposition 3. In equilibrium, each supplier has a symmetric solution at the beginning of the project, and the optimal work rate at this stage is given by:

$$\lambda^{\text{II}*}(p, \beta, \delta) = \frac{1}{6c}\{-[2\alpha c + R(\mu_2^*) + \beta\omega_2^* p - p] + \sqrt{[2\alpha c + R(\mu_2^*) + \beta\omega_2^* p - p]^2 + 12\alpha c p}\} \quad (7)$$

We can easily get that $\lambda^{\text{II}*}(p, \beta, \delta)$ strictly increases in δ and $\lambda^{\text{II}*}(p, \beta, \delta) > \mu_2^*(p, \beta, \delta)$.
The manufacturer's problem can be expressed as:

$$\max_p \Pi_m^2(q; p) = [2q - (1-\beta)\omega_2^* p] \frac{\mu_2^*}{\alpha+\mu_2^*} \frac{2\lambda^{\text{II}*}}{\alpha+2\lambda^{\text{II}*}} - (\beta\omega_2^* p + p) \frac{2\lambda^{\text{II}*}}{\alpha+2\lambda^{\text{II}*}}$$
$$\text{s.t.}\ \Pi_i^2(p; \lambda_i, \lambda_{-i}) \geq 0 \text{ for } i = 1,2. \quad (8)$$

Case III: Partial Payment (NDP) to the Slower Supplier

We first analyze the second-stage problem, when one supplier has completed his task and another supplier continues to work on his own task at the rate of μ. We define the penalty function is $\omega(\mu) = 1 - \delta/\mu, \delta > 0$ as above. The continuing supplier's expected profit, discounted back to the beginning of stage 2, can be characterized as

$$R(\mu) = \frac{\omega(\mu)p\mu}{\alpha+\mu} - \frac{c\mu^2}{\alpha+\mu}. \quad (9)$$

Proposition 4. With simple differentiation, we obtain the optimal work rate during the second stage, which is characterized as

$$\mu_3^*(p, \delta) = \sqrt{\alpha^2 + \frac{p\alpha+\delta p}{c}} - \alpha. \quad (10)$$

We have $\mu_3^*(p, \delta) > \lambda^*(p)$ and $\mu_3^*(p, \delta) > \mu_2^*(p, \beta, \delta)$. Using simple algebra, we obtain $R(\mu_3^*) = \frac{c[\mu_3^*]^2 - \delta p}{\alpha}$. Furthermore, $\mu_3^*(p, \delta)$ strictly increases in δ and $R(\mu_3^*)$ strictly decreases in δ.

We can infer that supplier i's expected discounted profit (back to the beginning of stage 1; that is, time $t = 0$) can be expressed as follows:

$$\Pi_i^3(p; \lambda_i, \lambda_{-i}) = -\frac{c\lambda_i^2}{\alpha+\lambda_i+\lambda_{-i}} + \frac{\lambda_i+\lambda_{-i}}{\alpha+\lambda_i+\lambda_{-i}} \cdot \left[\frac{\lambda_i}{\lambda_i+\lambda_{-i}} p + \frac{\lambda_{-i}}{\lambda_i+\lambda_{-i}} R(\mu_3^*)\right], i = 1,2.$$

Proposition 5. The optimal work rate under contract NDP for both suppliers at the beginning of the project satisfies the following condition:

$$\lambda^{\mathrm{III}*}(p,\delta)=\frac{1}{6c}\left\{-[2\alpha c+R(\mu_3^*)-p]+\sqrt{[2\alpha c+R(\mu_3^*)-p]^2+12\alpha cp}\right\}. \quad (11)$$

We can easily verify that $\lambda^{\mathrm{I}*}(p)<\lambda^{\mathrm{III}*}(p,\delta)<u_3^*(p,\delta)$, and $\lambda^{\mathrm{III}*}(p,\delta)$ strictly increases in δ.
The manufacturer's expected discounted profit can be expressed as follows:

$$\max_p \Pi_m^3(q;p)=(2q-\omega_3^*p)\frac{\mu_3^*}{\alpha+\mu_3^*}\frac{2\lambda^{\mathrm{III}*}}{\alpha+2\lambda^{\mathrm{III}*}}-p\frac{2\lambda^{\mathrm{III}*}}{\alpha+2\lambda^{\mathrm{III}*}}$$
$$\text{s.t. } \Pi_i^3(p;\lambda_i,\lambda_{-i})\geq 0 \text{ for } i=1,2. \quad (12)$$

4.2. Delayed Payment to the Faster supplier

This sub-section shows our consideration of incentive mechanism offered to the supplier who finishes earlier; the faster supplier's payment is totally delayed, which is known as the delayed payment scheme. The supplier who completes earlier will not get paid at the instant when he finishes his own task and he can receive a payment of p until the slower supplier has finished the subtask. Under case IV, the manufacturer offers a full payment scheme to the supplier who finishes later, and he can receive payment p when both tasks are completed; under case V, the manufacturer offers the slower supplier a two-part partial payment scheme: he receives a partition of $\beta\omega p$ at the instant when the faster supplier has finished the subtask, and obtains the remaining part of $(1-\beta)\omega p$ when he completes his own subtask; under case VI, the slower supplier is offered a partial payment scheme, and he will face a complete punishment by the manufacturer and will get his payment of ωp after both tasks are completed.

Case IV: Full Payment (DF) to the Slower Supplier

Using the same approach discussed above, we first solve the slower supplier's second-stage problem, and his expected discounted (back to time $t=S$) profit can be written as follows: $R(\mu)=\frac{p\mu}{\alpha+\mu}-\frac{c\mu^2}{\alpha+\mu}$. With simple differentiation, we obtain the optimal work rate during the second stage, which is characterized as $\mu_4^*(p)=\sqrt{\alpha^2+\frac{p\alpha}{c}}-\alpha$. Then, we can calculate that $R(\mu_4^*)=\frac{c[\mu_4^*]^2}{\alpha}$. We then pay our attention to the first stage of the game. We can express supplier i's expected discounted profit (back to the beginning of stage one, that is, time $t=0$) as follows:

$$\Pi_i^4(p;\lambda_i,\lambda_{-i})=-\frac{c\lambda_i^2}{\alpha+\lambda_i+\lambda_{-i}}+\frac{\lambda_i+\lambda_{-i}}{\alpha+\lambda_i+\lambda_{-i}}\left[\frac{\lambda_i}{\lambda_i+\lambda_{-i}}\cdot\frac{\mu_4^*}{\alpha+\mu_4^*}p+\frac{\lambda_{-i}}{\lambda_i+\lambda_{-i}}R(\mu_4^*)\right], i=1,2.$$

Proposition 6. Under contract DF, there exists one symmetric equilibrium, and the optimal work rate for both suppliers at the beginning of the project satisfies:

$$\lambda^{\mathrm{IV}*}(p)=\frac{1}{6c}\left\{-\left[2\alpha c+R(\mu_4^*)-\frac{\mu_4^*}{\alpha+\mu_4^*}p\right]+\sqrt{\left[2\alpha c+R(\mu_4^*)-\frac{\mu_4^*}{\alpha+\mu_4^*}p\right]^2+12\alpha cp\frac{\mu_4^*}{\alpha+\mu_4^*}}\right\}. \quad (13)$$

We have $\lambda^{\mathrm{IV}*}(p)<\mu_4^*(p)=\lambda^{\mathrm{I}*}(p)$.
The manufacturer's expected discounted profit can be expressed as:

$$\max_p \Pi_m^4(q;p)=(2q-2p)\frac{\mu_4^*}{\alpha+\mu_4^*}\frac{2\lambda^{\mathrm{IV}*}}{\alpha+2\lambda^{\mathrm{IV}*}} \quad (14)$$

$$\text{s.t.}\, \Pi_i^4(p;\lambda_i,\lambda_{-i}) \geq 0 \text{ for } i = 1,2.$$

Case V: Two-part Partial Payment (DTP) to the Slower Supplier

Using the same approach discussed above, we can express the continuing supplier's expected discounted profit (back to time $t = S$) as: $R(\mu) = \frac{(1-\beta)\omega(\mu)p\mu}{\alpha+\mu} - \frac{c\mu^2}{\alpha+\mu}$. We can obtain the slower supplier's optimal work rate in the second stage: $\mu_5^*(p,\beta,\delta) = \sqrt{\alpha^2 + \frac{(1-\beta)(p\alpha+\delta p)}{c}} - \alpha$. We obtain $R(\mu_5^*) = \frac{c[\mu_5^*]^2-(1-\beta)\delta p}{\alpha}$. We then express supplier i's expected discounted (to time $t = 0$) profit as:

$$\Pi_i^5(p;\lambda_i,\lambda_{-i}) = -\frac{c\lambda_i^2}{\alpha+\lambda_i+\lambda_{-i}} + \frac{\lambda_i+\lambda_{-i}}{\alpha+\lambda_i+\lambda_{-i}} \cdot \left[\frac{\lambda_i}{\lambda_i+\lambda_{-i}} \cdot \frac{\mu_5^*}{\alpha+\mu_5^*} p + \frac{\lambda_{-i}}{\lambda_i+\lambda_{-i}}\left(\beta\omega_5^* p + R(\mu_5^*)\right)\right], i = 1,2.$$

Proposition 7. In equilibrium, both suppliers have the same optimal first-stage work rate, and can be characterized as

$$\lambda^{\mathrm{V}*}(p,\beta,\delta) = \frac{1}{6c}\{-\left[2\alpha c + R(\mu_5^*) + \beta\omega_5^* p - p\frac{\mu_5^*}{\alpha+\mu_5^*}\right] + \sqrt{\left[2\alpha c + R(\mu_5^*) + \beta\omega_5^* p - p\frac{\mu_5^*}{\alpha+\mu_5^*}\right]^2 + 12\alpha c p\frac{\mu_5^*}{\alpha+\mu_5^*}}\}. \tag{15}$$

We have that the manufacturer's expected discounted profit can be expressed as:

$$\max_p \Pi_m^5(q;p) = [2q - (1-\beta)\omega_5^* p - p]\frac{\mu_5^*}{\alpha+\mu_5^*}\frac{2\lambda^{\mathrm{V}*}}{\alpha+2\lambda^{\mathrm{V}*}} - \beta\omega_5^* p\frac{2\lambda^{\mathrm{V}*}}{\alpha+2\lambda^{\mathrm{V}*}}$$
$$\text{s.t.}\, \Pi_i^5(p;\lambda_i,\lambda_{-i}) \geq 0 \text{ for } i = 1,2. \tag{16}$$

Case VI: Partial Payment (DP) to the Slower Supplier

The continuing supplier's expected profit discounted back to the beginning of stage 2 can be characterized as: $R(\mu) = \frac{\omega(\mu)p\mu}{\alpha+\mu} - \frac{c\mu^2}{\alpha+\mu}$. We simply calculate the slower supplier's second-stage optimal work rate, which has the following expression: $\mu_6^*(p,\delta) = \sqrt{\alpha^2 + \frac{p\alpha+\delta p}{c}} - \alpha$. We can obtain $R(\mu_6^*) = \frac{c[\mu_6^*]^2-\delta p}{\alpha}$. We express supplier i's expected discounted (to time $t = 0$) profit as:

$$\Pi_i^6(p;\lambda_i,\lambda_{-i}) = -\frac{c\lambda_i^2}{\alpha+\lambda_i+\lambda_{-i}} + \frac{\lambda_i+\lambda_{-i}}{\alpha+\lambda_i+\lambda_{-i}} \cdot \left[\frac{\lambda_i}{\lambda_i+\lambda_{-i}} \cdot \frac{\mu_6^*}{\alpha+\mu_6^*} p + \frac{\lambda_{-i}}{\lambda_i+\lambda_{-i}} R(\mu_6^*)\right], i = 1,2.$$

Proposition 8. Both suppliers' optimal work rates in the first stage of the game can be characterized as

$$\lambda^{\mathrm{VI}*}(p,\delta) = \frac{1}{6c}\left\{-\left[2c\alpha + R(\mu_6^*) - p\frac{\mu_6^*}{\alpha+\mu_6^*}\right] + \sqrt{[2c\alpha + R(\mu_6^*) - p\frac{\mu_6^*}{\alpha+\mu_6^*}]^2 + 12c\alpha p\frac{\mu_6^*}{\alpha+\mu_6^*}}\right\}. \tag{17}$$

We have $\lambda^{\mathrm{VI}*}(p,\delta)$ strictly increases in δ and $\lambda^{\mathrm{VI}*}(p,\delta) < \mu_6^*(p,\delta)$.
The manufacturer's expected discounted profit can be expressed as:

$$\max_p \Pi_m^6(q;p) = (2q - \omega_6^* p - p)\frac{\mu_6^*}{\alpha+\mu_6^*}\frac{2\lambda^{\mathrm{VI}*}}{\alpha+2\lambda^{\mathrm{VI}*}}$$
$$\text{s.t.}\, \Pi_i^6(p;\lambda_i,\lambda_{-i}) \geq 0 \text{ for } i = 1,2. \tag{18}$$

4.3. Partial-delayed Payment to the Faster Supplier

In this sub-section, the manufacturer will offer a partial-delayed payment scheme to the supplier who finishes earlier: the faster supplier can only obtain a partition of the total payment at the instant when he completes his own subtask and let this partition be $\gamma p \ (0 < \gamma < 1)$; thus, he will receive the remaining part $(1-\gamma)p$ when both tasks are completed. Under case VII, the manufacturer offers a full payment scheme to the supplier who finishes later; under case VIII, the manufacturer offers the slower supplier a two-part partial payment scheme; under case IX, the slower supplier is offered a partial payment scheme.

Case VII: Full Payment (PDF) to the Slower Supplier

We first solve the second-stage problem, and the slower supplier's optimal work rate at the second stage satisfies: $\mu_7^*(p) = \sqrt{\alpha^2 + \frac{p\alpha}{c}} - \alpha$. Furthermore, we can see that $R(\mu_7^*) = \frac{c[\mu_7^*]^2}{\alpha}$. We can express supplier i's expected discounted profit (back to time $t = 0$) as

$$\Pi_i^7(p; \lambda_i, \lambda_{-i}) = \frac{\lambda_i + \lambda_{-i}}{\alpha + \lambda_i + \lambda_{-i}} \left\{ \frac{\lambda_i}{\lambda_i + \lambda_{-i}} \left[\gamma p + \frac{\mu_7^*(1-\gamma)}{\alpha + \mu_7^*} p \right] + \frac{\lambda_{-i}}{\lambda_i + \lambda_{-i}} R(\mu_7^*) \right\} - \frac{c\lambda_i^2}{\alpha + \lambda_i + \lambda_{-i}}, i = 1,2.$$

Proposition 9. Under contract PDF, both suppliers have a symmetric equilibrium at the beginning of the project, and the optimal work rate at this stage is given by

$$\lambda^{\mathrm{VII}*}(p,\gamma) = \frac{1}{6c} \Big\{ -\left[2c\alpha + R(\mu_7^*) - p\left(\gamma + \frac{\mu_7^*(1-\gamma)}{\alpha + \mu_7^*}\right) \right] + \sqrt{[2c\alpha + R(\mu_7^*) - p\left(\gamma + \frac{\mu_7^*(1-\gamma)}{\alpha + \mu_7^*}\right)]^2 + 12c\alpha p\left(\gamma + \frac{\mu_7^*(1-\gamma)}{\alpha + \mu_7^*}\right)} \Big\}. \quad (19)$$

We can infer that $\lambda^{\mathrm{VII}*}(p,\gamma)$ strictly increases with γ, and $\lambda^{\mathrm{VII}*}(p,\gamma) < \mu_7^*(p) = \lambda^{\mathrm{I}*}(p)$.
The manufacturer's expected discounted profit can be expressed as:

$$\max_p \Pi_m^7(q;p) = (2q - 2p + \gamma p) \frac{2\lambda^{\mathrm{VII}*}}{\alpha + 2\lambda^{\mathrm{VII}*}} \frac{\mu_7^*}{\alpha + \mu_7^*} - \gamma p \frac{2\lambda^{\mathrm{VII}*}}{\alpha + 2\lambda^{\mathrm{VII}*}}$$
$$\text{s.t.}\ \Pi_i^7(p; \lambda_i, \lambda_{-i}) \geq 0 \text{ for } i = 1,2. \quad (20)$$

Case VIII: Two-part Partial Payment (PDTP) to the Slower Supplier

First, we solve the slower supplier's second-stage decision variable μ, and we can easily get that $\mu_8^*(p,\beta,\delta) = \sqrt{\alpha^2 + \frac{(1-\beta)(p\alpha + \delta p)}{c}} - \alpha$. Furthermore, we have $R(\mu_8^*) = \frac{c[\mu_8^*]^2 - (1-\beta)\delta p}{\alpha}$. We then solve the first-stage problem. Supplier i's expected discounted profit (back to time $t = 0$) can be expressed as follows:

$$\Pi_i^8(p; \lambda_i, \lambda_{-i}) = -\frac{c\lambda_i^2}{\alpha + \lambda_i + \lambda_{-i}} + \frac{\lambda_i + \lambda_{-i}}{\alpha + \lambda_i + \lambda_{-i}} \cdot \left[\frac{\lambda_i}{\lambda_i + \lambda_{-i}} \cdot \left(\gamma p + \frac{\mu_8^*(1-\gamma)}{\alpha + \mu_8^*} p \right) + \frac{\lambda_{-i}}{\lambda_i + \lambda_{-i}} \left(\beta \omega_8^* p + R(\mu_8^*) \right) \right], i = 1, 2.$$

Proposition 10. The optimal work rate in the first stage is the same for both suppliers, and it satisfies

(21)

$$\lambda^{\mathrm{VIII}*}(p,\beta,\gamma,\delta)=\frac{1}{6c}\Bigg\{-\left[2\alpha c+R(\mu_8^*)+\beta\omega_8^*p-p\left(\gamma+\frac{\mu_8^*(1-\gamma)}{\alpha+\mu_8^*}\right)\right]+\sqrt{\left[2\alpha c+R(\mu_8^*)+\beta\omega_8^*p-p\left(\gamma+\frac{\mu_8^*(1-\gamma)}{\alpha+\mu_8^*}\right)\right]^2+12\alpha cp\left(\gamma+\frac{\mu_8^*(1-\gamma)}{\alpha+\mu_8^*}\right)}\Bigg\}.$$

The manufacturer's expected discounted profit can be expressed as:

$$\max_p \Pi_m^8(q;p)=[2q-(1-\beta)\omega_8^*p-(1-\gamma)p]\frac{\mu_8^*}{\alpha+\mu_8^*}\frac{2\lambda^{\mathrm{VIII}*}}{\alpha+2\lambda^{\mathrm{VIII}*}}-(\beta\omega_8^*p+\gamma p)\frac{2\lambda^{\mathrm{VIII}*}}{\alpha+2\lambda^{\mathrm{VIII}*}}.$$
$$\text{s.t.}\,\Pi_i^8(p;\lambda_i,\lambda_{-i})\geq 0 \text{ for } i=1,2. \tag{22}$$

Case IX: Partial Payment (PDP) to the Slower Supplier

Using the same approach above, we first analyze the second-stage problem, in which one supplier has completed his task and the other supplier continues to work on his own task at the rate of μ. we have that the optimal work rate during the second stage is characterized as $\mu_9^*(p,\delta)=\sqrt{\alpha^2+\frac{p\alpha+\delta p}{c}}-\alpha$. We obtain $R(\mu_9^*)=\frac{c[\mu_9^*]^2-\delta p}{\alpha}$. Regarding the first-stage problem, supplier i's expected discounted profit (back to time $t=0$) can be expressed as:

$$\Pi_i^9(p;\lambda_i,\lambda_{-i})=\frac{\lambda_i+\lambda_{-i}}{\alpha+\lambda_i+\lambda_{-i}}\left\{\frac{\lambda_i}{\lambda_i+\lambda_{-i}}\left[\gamma p+\frac{\mu_9^*(1-\gamma)}{\alpha+\mu_9^*}p\right]+\frac{\lambda_{-i}}{\lambda_i+\lambda_{-i}}R(\mu_9^*)\right\}-\frac{c\lambda_i^2}{\alpha+\lambda_i+\lambda_{-i}},i=1,2.$$

Proposition 11. Under the contract PDP, both suppliers have a symmetric equilibrium at the beginning of the project, and the optimal work rate in the first stage is given by

$$\lambda^{\mathrm{IX}*}(p,\gamma,\delta)=\frac{1}{6c}\Bigg\{-\left[2c\alpha+R(\mu_9^*)-p\left(\gamma+\frac{\mu_9^*(1-\gamma)}{\alpha+\mu_9^*}\right)\right]+\sqrt{[2c\alpha+R(\mu_9^*)-p\left(\gamma+\frac{\mu_9^*(1-\gamma)}{\alpha+\mu_9^*}\right)]^2+12c\alpha p\left(\gamma+\frac{\mu_9^*(1-\gamma)}{\alpha+\mu_9^*}\right)}\Bigg\}. \tag{23}$$

We can easily confirm that $\lambda^{\mathrm{IX}*}(p,\gamma,\delta)<\mu_9^*(p,\delta)$, and $\lambda^{\mathrm{IX}*}(p,\gamma,\delta)$ strictly increases in both δ and γ.
The manufacturer's expected discounted profit can be expressed as:

$$\max_p \Pi_m^9(q;p)=[2q-\omega_9^*p-(1-\gamma)p]\frac{\mu_9^*}{\alpha+\mu_9^*}\frac{2\lambda^{\mathrm{IX}*}}{\alpha+2\lambda^{\mathrm{IX}*}}-\gamma p\frac{2\lambda^{\mathrm{IX}*}}{\alpha+2\lambda^{\mathrm{IX}*}}$$
$$\text{s.t.}\,\Pi_i^9(p;\lambda_i,\lambda_{-i})\geq 0 \text{ for } i=1,2. \tag{24}$$

In our study, we focus on the situation in which the manufacturer outsources two parallel subtasks to two different suppliers. However, scholars can also consider serial subtasks when attempting to solve project-related problems. In addition to pure parallel or serial tasks, we can analyze a project that involves a network of both parallel and serial tasks with exponential completion times. Our model can be used as a building block for further study, to examine the influence of incentive mechanisms on the operating performance of different parties.

5. Conclusions

The goal of this study is to compare the effect of incentive mechanisms from two different dimensions: one is an incentive mechanism concerning the supplier who finishes earlier, and the other concerns the supplier who finishes later. The manufacturer can outsource two parallel tasks to two different suppliers considered in our analysis, and we assume that the two subtasks have the same workload and difficulty. With a combination of different payment timings and payment amounts, we obtain nine types of contracts that the manufacturer offers to the suppliers.

There are a number of extensions we can further continue to study when we pay attention to the field of project management. It needs us scholars to pay attention and time to do research and show some managerial implications to lead guidance to the practical world. We believe that in the near future, great advances will be achieved in academic research by scholars working on this topic and this will further give guidance to the real-world business applications of project management.

References

Bayiz, M., and Corbett, C.J., Coordination and incentive contracts in project management under asymmetric information, *Working paper, UCLA Anderson School,* 2005.

Chen, S., and Lee, H., Incentive alignment and coordination of project supply chains, *Management Science*,vol. 63, no.4, pp. 1011–1025, 2017.

Chen, T., Klastorin, T., and Wagner, M.R., Incentive contracts in serial stochastic projects, *Manufacturiing & Service Operations Management*, vol. 17, no.3, pp. 290–301, 2015.

Dawande, M., Janakiraman, G., Qi, A., and Wu, Q., Optimal incentive contracts in project management, *Production & Operations Management*, vol. 28, no.6, pp. 1431-1445, 2019.

Fu, R., Subramanian, A., and Venkateswaran, A., Project characteristics, incentives, and team production, *Management Science,, vol.* **62, no.3,** pp. 785-801, 2016.

Gupta, D., Snir, E.M., and Chen, Y., Contractors and agency decisions and policy implications in A+B bidding, *Production & Operations Management*, vol. 24, no.1, pp. 159–177, 2015.

Hall, N.G., Research and Teaching Opportunities in Project Management, *Optimization Challenges in Complex, Networked and Risky Systems,* 2016.

Klastorin, T. (2004). *Project Management: Tools and Trade-offs*, John Wiley and Sons, Inc., Somerset, New Jersey.

Kwon, H.D., Lippman, S.A., McCardle, K., and Tang, C.S., Managing project contracts with delayed payments, *Manufacturing & Service Operations Management*, vol. 12, no.4, pp. 692–707, 2010.

Kwon, H.D., Lippman, S.A., and Tang, C.S., Optimal time-based and cost-based coordinated project contracts with unobservable work rates, *International Journal of Production Economics*, vol. 126, no.2, pp. 247–254, 2010.

Kwon, H.D., Lippman, S.A., and Tang, C.S., Sourcing decisions of project tasks with exponential completion times: Impact on operating profits, *International Journal of Production Economics*, vol. 134, no.1, pp. 138–150, 2011.

Nahmias, S., and Cheng, Y., *Production and Operations Analysis*, vol. 6 (McGraw-Hill, New York), 2009.

Nolan, R.L., and Kotha, S., Boeing 787: The Dreamliner, *HBS Case 9-305-101, Harvard Business School*, Boston, 2005.

Tang, C.S., Zhang, K., and Zhou, S.X., Incentive contracts for managing a project with uncertain completion time, *Production & Operations Management*, vol. 24, no.12, pp. 1945-1954, 2015.

Tang, C.S., Zimmerman, J.D., and Nelson, J.I., Managing new product development and supply chain risks: The Boeing 787 case, *Supply Chain Forum*, vol. 10, no.2, pp. 74–86, 2009.

Zhang, Juanjuan., Deadlines in product development, *Management Science*, vol. 62, no.11, pp. 3310-3326, 2016.

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