

# **A Branch-and-Price Algorithm for Crowdsourcing Vehicle Routing Problem with Cross-Docking in Platform-Driven Crowdsourced Manufacturing Environment**

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## **Abstract**

Crowdsourced manufacturing through a platform-driven manner has been observed as an emerging trend towards Industry 4.0 by paving the way of delivering Manufacturing-as-a-Service (MaaS). It utilizes a cyber platform and crowdsourcing to reach external partner's manufacturing knowledge and resources while allowing companies to focus on their core competencies. It addresses an underlying logic that maximizing the reuse of resources by searching similarities among prolific product, process, and manufacturing resources varieties. It also challenges traditional logistic service for manufacturing industries by expanding a simple material flow to a complex networked and fluctuate one. Cross-docking has been widely recognized as a logistic solution to complex material flow by splitting service routes to pickups and deliveries for maximizing vehicle reuse. It adopts a platform-driven strategy by exchanging loads at the cross-docking. This study formulates the logistic service problem in platform-driven crowdsourced manufacturing as an Open Vehicle Routing Problem with Cross-Docking (O-VRPCD), which integrates logistic solution provider crowds into the manufacturing service process. This study considers the logistic provider as a capacitated homogeneous vehicle starting at various pickup points and times in a logistic service. The vehicles are scheduled in a route to visit service requesters synchronously and arriving cross-dock center simultaneously for load exchanging. Thus, this study formulates a mixed-integer programming (MIP) model for OVRPCD to minimize a total cost of crowdsourcing fleet, which considers logistic solution provider hiring costs and vehicle operation costs. A branch-and-price (B&P) algorithm is proposed to solve this problem using Pulse Algorithm-based column generation.

## **Keywords**

Crowdsourced manufacturing, Open vehicle routing problem, Cross-docking, Branch-and-price

## **1. Introduction**

Manufacturing industries are challenged by absorbing disruptive changes impacted by sustainability issues, volatile customer preferences, macro-environmental fluctuations, and effective delivery of manufacturing services in Manufacturing-as-a-Service (MaaS) paradigm (Kusiak, 2019). Therefore, platform-driven crowdsourced manufacturing has been proposed to deliver service-oriented manufacturing through crowdsourcing and integrating resources into a manufacturing value chain on a cyber platform (Gong et al., 2021). It provides systematic solutions for manufacturers to peel their peripheral manufacturing activities and thus achieving economies of scale by offering substitutive services. To manage material flow across a manufacturer network, providing optimal decision support on logistic services is critical, which enables the flow to serve materials and work-in-progress (WIP) transportation synchronously.

Since a cyber platform in crowdsourced manufacturing operates as a two-side peer-to-peer marketplace to match the open innovators and manufacturers, multiple value chains will be initiated and executed simultaneously by a logistic service system. Considering the fulfillment of each value chain requires collaboration among a group of manufacturers, several outstanding manufacturers may be awarded by multiple value chains. Thus, these manufacturers can be viewed as common vertexes in the networked material flow, and the logistic service system should have a service-oriented scheduling mechanism to fulfill two essential functions. Firstly, it should deliver the corresponding materials and WIP just-in-time for synchronization of manufacturing activities. Secondly, because the highly innovative products can be characterized as large variety yet small volume, the system is also required to handle the product and production variety.

From the material flow management perspective, logistic services in platform-driven crowdsourced manufacturing aim to send and pick up the required materials, WIP, subassemblies, or final products on time. Due to the large variety of value chains and the corresponding process variety, one manufacturer can play the role of downstream partners that work with a set of upstream partners. Thus, the material and WIP delivery services for this manufacturer has multiple destinations. Similarly, after the accomplishment of the manufacturing tasks, the pick-up services will send the material and WIP to multiple downstream manufacturers. Also, the process variety will propagate from process domain to the logistics one, requiring the companies to keep a reasonable cost and align customers, products, processes, and logistics domain for delivering an increasing product variety (Jiao et al., 2007). From a platform-based perspective, the cyber platform will collect the information from the manufacturing crowds, formulate the origins and destinations of the service demands, find the common routes in the corresponding transportation service tasks, and synchronize the manufacturing activities to achieve just-in-time (Qu et al., 2016).

Participating in platform-driven crowdsourced manufacturing implies that manufacturers are open to external partners and allow the integration with partner crowds. Moreover, because recent advancement of information service system enables digitization of manufacturing activities and streaming of process data, the logistic service system can retrieve real-time data on the shop floor and make optimal decisions. The new synergy of the Internet of Things (IoT) and cloud computing architecture enables the visualization of the logistic on the shop floor and big data analysis of the material flow inner the manufacturers, thus paves the way towards a holistic optimal logistic plan that balances the inner- and inter-manufacturers material flows (Zhong et al., 2015), synchronizing transportation tasks and manufacturing activities. Therefore, the time gaps between the manufacturing task accomplishment and picking up as well as the materials or WIP deliveries and the start of order execution can be minimized. The smaller the time gaps are, the better inventory level management on the shop floor can achieve.

There is a stream of operation solutions for a logistic network with a high material variety and tight time constraints. The service-oriented logistic system installs an agile control mechanism, which entails a user-friendly, flexible, scalable, and widely connected engineering system architecture to link internal and external transportation (Evers et al., 2000). Traditional logistic solutions like direct shipping and milk-run shipment from an origin to one or multiple destinations in a tour have been observed with a limited capability of serving small shipment size and geographically dispersed customers (Buijs et al., 2014).

To overcome these shortcomings, warehousing and cross-docking are developed by using a centralized depot. According to our knowledge, an analytical solution for crowdsourcing vehicle routing problem with cross-docking (CVRPCD) is not existed. This study aims to formulate the CVRPCD with a mathematical model and proposes a branch-and-price-based approach as an analytical solution.

In this regard, the rest of this study proceeds as follows. Section 2 reviews related work regarding CVRPCD and branch-and-price. Section 3 presents problem context and mathematical formulation of CVRPCD. Section 4 proposes a branch-and-price algorithm for CVRPCD. The computational results are presented in Section 5. Conclusions are made in Section 6.

## **2. Related Work**

### **2.1 Crowdsourcing Vehicle Routing Problem with Cross Docking**

Cross-docking allows the inbound trucks to unload the freight and transport it directly to the outbound trucks with no or simple storage infrastructure (Wen et al., 2009). Compared to the warehousing that holds an inventory of products to act as a shortage buffer, cross-docking groups similar shipping requirements that are fulfilled by immediate recombination to a delivery tour in a centralized freight terminal, which is known as cross-dock (Bozer and Carlo, 2008). The cross-docking addresses a platform-driven approach by operating in similarity exploration and consolidating freight with the same downstream manufacturers utilizing less handling efforts to serve product varieties (Ladier and Alpan, 2016). It has been widely accepted as a solution to serve the complex logistic network with a short delivery lead time and to reduce the storage space (Van Belle et al., 2012). Cross-docking requires synchronization of pickup and delivery routes to achieve a just-in-time paradigm by having no or less storage buffer (Vogt, 2010). Therefore, a successful cross-docking operation meets the demands of a holistic approach for modeling, quick response to uncertainty, and precise decision-making for resource planning.

Vehicle routing planning with time window (VRPTW) for pickup and delivery truck fleet management builds up the foundation of cross-docking operations (Shakeri et al., 2012). A large variety of algorithms have been developed to solve the cross-docking planning problem based on VRPTW formulation with time constraints and other management concerns (Buijs et al., 2014). The heuristic algorithm solution for vehicle routing problem with cross-docking (VRPCD) includes tabu search (Lee et al., 2006), multi-objective population-based heuristics (Arabani et al., 2011), large neighborhood metaheuristics (Grangier et al., 2017), to name but a few. Analytical solutions for VRPCD can be derived from mixed integer programming (MIP) formulation by Lee et al. (2006), and it can be accelerated by adopting branch-and-price approach, which utilizes column generation techniques to divide the VRPTW problem into a pair of master problem and subproblem and update the route pool iteratively (Santos et al., 2011).

Other fleet management issues rising along with the instantiation of cross-docking have also been studied, including arrival uncertainty (Konur and Golias, 2013), pickup and deliveries with cross-docking (Santos et al., 2013), split

deliveries (Moghadam et al., 2014), resource constraints (Grangier et al., 2021), and queue model-based multi-door facilities (Goodarzi et al., 2021). Platform-driven crowdsourced manufacturing searches a large amount of logistic service providers for material and WIP deliveries, which brings a challenge for opening conventional vehicle routes and allowing the participation of logistic service provider crowds. Open vehicle routing problem has been proposed to accommodate third party logistic provider (Schopka and Kopfer, 2016). Vincent et al. (2016) formulates open VRPCD and solve it by simulated annealing.

## 2.2 Branch-and-Price for Vehicle Routing Problem

B&P is an approach to solve large-scale mixed integer programming problems that applies column generation techniques throughout branch-and-bound (B&B) with linear programming relaxations (Barnhart et al., 1998). Compared to the network flow-based formulation with linear programming-based B&B method, B&P-based formulation provides stronger bounds with a smaller gap between upper bound and lower bound (Santos et al., 2011).

Two main topics studied for B&P-based vehicle routing are column generation techniques and branching strategies. Column generation is used to solve the subproblem, which can be formulated as the elementary shortest path problem with resource constraints (ESPPRC). Since it is a NP-hard problem, several exact algorithms have been studied to accelerate the search. In 2004, a label correcting algorithm is proposed that finds the optimal path by extending the labels from the start node to the ending node (Feillet et al., 2004). Later, a bidirectional labeling algorithm that relies on state-space relaxation is proposed to reduce computational time (Righini and Salani, 2008). In 2011, this algorithm is further improved by including bounding functions on state-space relaxation (Baldacci et al., 2011). In 2016, Leonardo et al. proposed a new exact algorithm faster than labeling algorithms based on the pulse propagation concept with a novel bounding scheme. Different from labeling algorithms with dynamic programming, it firstly obtains a lower bound matrix with limited time resources, which is used to prune subspace in the depth-first search.

The branching strategy of VRPCD is different from normal VRPTW. Conventional branching in VRPTW conducts branching on vehicles, arcs, edges, and generalized upper bounds (Røpke, 2012). For VRPCD, since upstream and downstream vehicles are to arrive at the cross-dock and freight exchange brings costs, a new variable that indicates if freight is exchanged between two vehicles can be used for branching (Santos et al., 2011).

## 3. Problem Definition and Mathematical Model

### 3.1 Problem Context for CVRPCD

This study focuses on the logistic service system for platform-driven crowdsourced manufacturing, which is driven by the advantages of cross-docking following a platform-driven strategy and a crowdsourcing fulfillment strategy to utilize external logistic providers. The challenges of optimal scheduling in a crowdsourcing environment, which instantiates platform-driven strategies through cross-docking is solved by the model of CVRPCD. Figure 1 conceptually illustrates CVRPCD by incorporating three logistic providers to fulfill logistic service for crowdsourced manufacturing. A crowdsourcing logistic service process starts from a combination of different pickup routes, which has various locations and service time window. Logistic providers hired from a logistic crowd are viewed as homogeneous and are indexed as  $\gamma$ ,  $\gamma \in [1, \Gamma]$ ,  $\Gamma \in \mathbb{Z}^+$ . The vehicles are scheduled to visit every manufacturer in a crowdsourcing network synchronously to exchange the WIP and materials they picked and then load for delivery routes. The WIP and materials for manufacturers in pickup and delivery routes can be different to fulfill various innovative product projects. From a perspective of service quality control, all manufacturers should be visited exactly once per pickup and delivery routes.

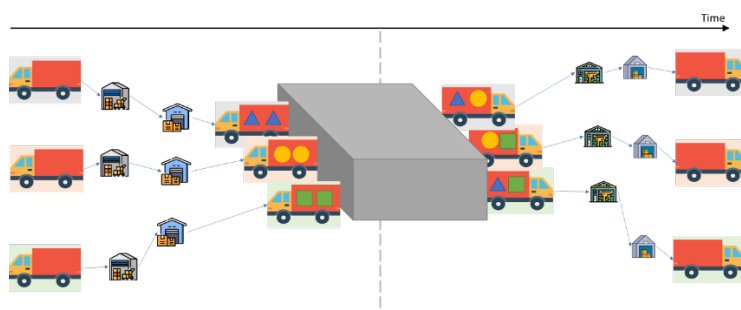


Figure 1. Crowdsourcing Vehicle Routing Problem with Cross-Docking

The overall operational objective for this problem is to seek a minimized transportation cost following scheduled routes and an optimal number of hired logistic providers. It is a variation of the VRPCD which requires all vehicle starts from the cross-docking depot. The CVRPCD integrates logistic provider crowd via crowdsourcing, which implies that services of vehicles are started and ended by various time and locations. From a management perspective, this optimal decision-making process can be further decomposed as a series of VRP problems and an optimal combinatorial problem of combining possible routes to a fleet plan. An architecture of a master problem and subproblems entails a negotiation process of a pricing and bidding process to plan logistic providers. As an analytical method of solving large-scale MIP, B&P updates potential solution space through a column generation technique (Barnhart et al., 1998). B&P can solve VRPCD by modeling the route selection problem as a master problem and individual routing problems as subproblems (Santos et al., 2011).

### 3.2 Mathematical Formulation of CVRPCD

CVRPCD assumes logistic providers are heterogenous vehicles which park in a dummy cross-dock, which can access to all possible manufacturers with no distance, collect all vehicles without the limit on the number of vehicles, and be viewed as a logistic crowd. All hired vehicles arrive at the cross-dock simultaneously to enable a division of routes to pickup and delivery process. The nomenclature and MIP model for CVRPCD is formulated as follows.

This model uses a graph-based presentation of potential routes. Consider a directed graph  $G = (V, A)$ , where  $V$  collects all possible locations as vertices and  $A$  collects all possible trips as arcs. The vertices set  $V$  can be further decomposed as  $V = \{0\} \cup \{0\} \cup M^U \cup M^D$ , where  $\{0\}$  represents cross-dock depot,  $\{0\}$  represents dummy cross-dock,  $M^U$  collects all upstream manufacturers who provide WIP and material, and  $M^D$  collects all downstream manufacturers who receive WIP and material. The total number of upstream manufacturers  $M^U$  is  $n^U$  and total number of downstream manufacturers  $M^D$  is  $n^D$ . The arcs can be further decomposed into  $A = A^U \cup A^D$ , and  $A^U \cap A^D = \emptyset$ .  $A^U = \{(i^U, j^U): i^U, j^U \in \{0, 1, \dots, n^U\}\}$  denotes all possible arcs connecting upstream manufacturers, the cross-dock depot, and dummy cross-docks. Similarly,  $A^D = \{(i^D, j^D): i^D, j^D \in \{0, 1, \dots, n^D\}\}$  denotes all possible arcs connecting downstream manufacturers, the cross-dock depot, and dummy cross-docks. Transportation cost  $c_{ij}$  are attached to these arcs,  $\{c_{ij} \geq 0: (i, j) \in A\}$ . Logistic service requests are modeled as triples  $S = \{s_{i^U} := (i^U, i^D, q_{i^U}): i^U \in \{1, \dots, n^U\}\}$ , where each triple regulating the start and end positions and the undividable load  $q_{i^U} \geq 0$ . These loads are shipped from requester upstream manufacturer  $i^U$  to receiver downstream  $i^D$ . A fleet size of  $K$  homogeneous vehicles are responsible for fulfillment all service request. In our model, every vehicle is required to stop at cross-dock depot  $\{0\}$  before delivering to downstream manufacturer  $M^D$  for possible loads exchange. If the loads  $q_{i^U}$  from pickup routes  $R^P$  and delivery routes  $R^D$  use different vehicle  $k$ , an exchanging cost  $c_{i^U}^k$  is generated.

Several decision variables and binary parameters are used for problem modeling. Decision variable  $\beta_r^k$  assumes 1 to indicate a pickup tour  $r$  utilizes vehicle  $k$ , 0 for otherwise, and a transportation cost  $c_r$  are attached to it. Correspondingly, a decision variable  $\gamma_{r'}^k$  assumes 1 for a delivery route  $r'$  uses vehicle  $k$ , 0 otherwise, and cost is  $c_{r'}$ . Exchanging decision variable  $\tau_{i^U}^k$  is introduced to indicate the load  $i^U$  in vehicle  $k$  is exchanged in the cross-dock depot or not, and a cost  $c_{i^U}^k$  is attached. Two binary parameters  $a_r^{i^U}$  and  $b_{r'}^{i^D}$  describes the pickup route  $r$  and delivery route  $r'$  in the form of whether it visits upstream manufacturer  $i^U$  and downstream manufacturer  $i^D$  or not, respectively.

The MIP mathematical formulation of CVRPCD is Equation (1) – (8).

$$\text{Min} \sum_{r \in R^P} c_r \sum_{k \in K} \beta_r^k + \sum_{r' \in R^D} c_{r'} \sum_{k \in K} \gamma_{r'}^k + \sum_{k \in K} \sum_{i^U \in S} c_{i^U}^k \tau_{i^U}^k \quad (1)$$

s. t.

$$\sum_{r \in R^P} \beta_r^k \leq 1 \quad \forall k \in K \quad (2)$$

$$\sum_{r' \in R^D} \gamma_{r'}^k \leq 1 \quad \forall k \in K \quad (3)$$

$$\sum_{r \in R^P} a_r^{i^U} \sum_{k \in K} \beta_r^k = 1 \quad \forall i^U \in M^U \quad (4)$$

$$\sum_{r' \in R^D} b_{r'}^{i^D} \sum_{k \in K} \gamma_{r'}^k = 1 \quad \forall i^D \in M^D \quad (5)$$

$$\sum_{r \in R^P} \beta_r^k a_r^{i^U} - \sum_{r' \in R^D} \gamma_{r'}^k b_{r'}^{i^D} + \tau_{i^U}^k \geq 0, \forall i^U \in S, \forall k \in K \quad (6)$$

$$- \sum_{r \in R^P} \beta_r^k a_r^{i^U} + \sum_{r' \in R^D} \gamma_{r'}^k b_{r'}^{i^D} + \tau_{i^U}^k \geq 0, \forall i^U \in S, \forall k \in K \quad (7)$$

$$\beta_r^k, \gamma_{r'}^k, \tau_{i^U}^k, a_r^{i^U}, b_{r'}^{i^D} \in \{0,1\} \quad (8)$$

Equation (1) formulates the objective function of C-VRPCD, which minimizes total cost incurred in pickup, delivery, and exchanging operations through crowdsourcing logistic service process. Equation (2) and (3) are convexity constraints to enforce all vehicle are used at most once in pickup and delivery routes. Equation (4) and (5) ensure that every request is covered without overlap by pickup and delivery routes. Equation (6) and (7) guarantee  $\tau_{i^U}^k$  equals to 1 if a service load  $i^U$  uses different vehicle. Equation (8) requires all decision variables and binary parameters are either 0 or 1.

Following a branch-and-price modeling approach, the master problem modeled in Equation (1) – (8) are changed to Restricted Master Problem (RMP) by replacing pickup  $R^P$  and delivery routes  $R^D$  with a restricted route pool. B&P utilizes linear programming relaxation of RMP. A series of dual variables  $\{v^k: k \in K\}$ ,  $\{\varphi^k: k \in K\}$ ,  $\{v_{i^U}: i^U \in M^U\}$ ,  $\{\mu_{i^D}: i^D \in M^D\}$ ,  $\{\pi_{i^U}^k: i^U \in M^U\}$ ,  $\{\chi_{i^U}^k: i^U \in M^U\}$  are assigned to constraints (2) – (7). The routes for RMP are updated by seeking routes with negative reduced costs in two pricing problems modeled in (9) and (10).

$$\arg \min_{r \in R^P} c_r - \sum_{i^U \in M^U} a_r^{i^U} v_{i^U} - \sum_{i^U \in M^U} a_r^{i^U} \pi_{i^U}^k + \sum_{i^U \in M^U} a_r^{i^U} \chi_{i^U}^k - v^k, \forall k \in K \quad (9)$$

$$\arg \min_{r' \in R^D} c_{r'} - \sum_{i^D \in M^D} b_{r'}^{i^D} \mu_{i^D} + \sum_{i^U \in M^U} b_{r'}^{i^D} \pi_{i^U}^k - \sum_{i^U \in M^U} b_{r'}^{i^D} \chi_{i^U}^k - \varphi^k, \forall k \in K \quad (10)$$

#### 4. Branch-and-Price algorithm for C-VRPCD

B&P utilizes column generation techniques in an iterative way and combines with B&B to solve the linear relaxation of large-scale optimization models that involve massive volume of variables and associated columns (Choi and Tcha, 2007). B&P utilize the observation that the optimal solution of a combinatorial problem only includes a small subset of columns. Thus, an iteratively updating scheme can significantly limit the scale of master problem, and a pricing problem can be solved to seek fast cost reduction.

##### 4.1 Algorithm Architecture of B&P for C-VRPCD

The flowchart of B&P for C-VRPCD is shown in Figure 2. It starts with a feasible subset of the pickup route set  $R^P$  and the delivery route set  $R^D$ . A full B&P algorithm has two cycle, column generation and branching. In each iteration of column generation, it solves linear programming relaxation of RMP considering a subset of the columns and yields dual variables, which include  $v^k$ ,  $\varphi^k$ ,  $v_{i^U}$ ,  $\mu_{i^D}$ ,  $\pi_{i^U}^k$ ,  $\chi_{i^U}^k$ . These dual variables are used to find routes with negative reduced costs by solving pricing problems modeled in Equation (9) - (10). If either pricing problems explored negative reduced cost columns, they can be added to the RMP. A new set of dual variable values can be explored by re-solve the RMP with linear programming relaxation. Otherwise, the current RMP solution founds its optimal and terminates the column generation cycle.

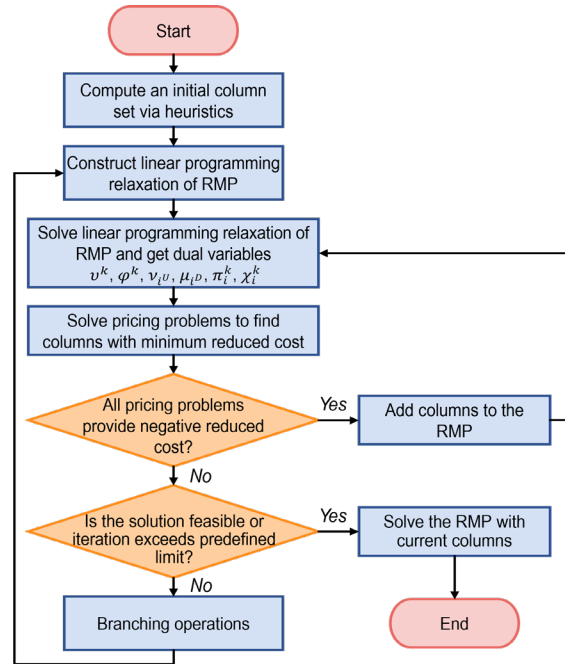


Figure 2. Flowchart of branch-and-price algorithm for C-VRPCD

The second cycle is the branching operation to add bounding constraints following the branching-based tree search. It branches variables from the master problem and finds a feasible integer optimal solution of the current RMP. After an integer solution is found, this solution from RMP with current columns can be viewed as the optimal solution of the master problem. A predefined limit can be added to restrain the iteration of branching operations.

#### 4.2 Pulse Algorithm for Pricing Problem

The objective of the pricing problem is to generate high-quality columns attached to variables that have the potential to improve the solution performance based on the current variables until no such columns can be found. This is done by exploring the variables with minimum negative reduced costs using the dual solution of the current linear programming relaxation of restricted master problem. Pricing problems are modelled in Equation (9) and (10), which can be formulated as an elementary shortest path problem with resource constraints (ESPPRC). It checks the feasibilities of visiting nodes in a certain precedence and calculates objective values based on resource extension functions along a route.

Among the exact solutions for ESPPRC, the labeling algorithm is the most widely used solution for pricing problems in B&P, which iteratively calculates the label (a tuple to represent a route) following the dynamic programming approach and utilizes dominance rule to reduce searching space (Costa et al., 2019). Recently, the Pulse Algorithm has been proposed for VRPTW as a faster exact solution to the pricing problem (Lozano et al., 2016). It firstly finds lower bounds on the cost given an amount of resource consumed, and recursively explores paths connecting vertices based on inexplicit enumeration in a graph through the pulse propagation, which addresses a depth-first search of a directed graph. Pulse algorithm incorporates three pruning strategies: 1) feasibility pruning, which prunes infeasible paths by using structural constraints; 2) bound pruning, which utilize primal and lower bounds to discard suboptimal partial paths; 3) rollback pruning, which compares pulses with and without the latest vertex visited to discard suboptimal partial paths.

A general pulse algorithm is shown in Table 1. Lines 1-4 in Table 1 initialize value for the partial path  $\mathcal{P}$ , the cumulative reduced cost  $r(\mathcal{P})$ , cumulative path load  $q(\mathcal{P})$ , cumulative path time consumption  $t(\mathcal{P})$ . Line 5 calls Bound function to find lower bound for every node of the question, which is shown in Table 2. Line 6 triggers recursive pulse which propagates from the start nodes  $v_s$  (dummy cross-dock  $\{0\}$  in pickup routes and cross-dock depot  $\{0\}$  in delivery routes). This function will explore all of the information of feasible path from  $v_s$  to end node  $v_e$  (cross-dock depot  $\{0\}$  in pickup routes and dummy cross-dock  $\{0\}$  in delivery routes).

Pulse Algorithm starts with Bound function to obtain a lower bound matrix for bound pruning. Then the pulse search begins. Three pruning functions, namely Feasible, Bounds, RollBack, which are called in line 7, 8, and 9, respectively when the path is extended to a new node. These pruning functions ensure that Pulse Algorithm can find an optimal elementary path in a limited space efficiently. If the path is not pruned, the current partial path adds the current node  $v_i$  in line 10. Line 11 updates the vehicle loads. Line 12 to line 16 forms a for-loop to propagate the pulse by invoking the pulse procedure to every possible node  $v_j \in \mathcal{A}_i^+$ , where  $\mathcal{A}_i^+$  is the set of accessible nodes set of current one  $v_i$ .

Table 1. Pseudocode of general pulse algorithm for ESPPRC

<b>Algorithm 1:</b> general pulse algorithm for ESPPRC
<b>Input:</b> $\mathcal{G}$ directed graph; $v_s$ start node; $v_e$ end node; $\delta$ bound step size; $[\underline{t}, \bar{t}]$ time bound for planning; $r(\mathcal{P})$ path reduced cost; $q(\mathcal{P})$ path load; $t(\mathcal{P})$ path time; $v_i$ current node; $\mathcal{A}_i^+$ set of accessible nodes set of current one $v_i$ .
<b>Output:</b> $\mathcal{P}^*$ optimal path
<pre> 1: <math>\mathcal{P} \leftarrow \{0\}</math>; 2: <math>r(\mathcal{P}) \leftarrow 0</math>; 3: <math>q(\mathcal{P}) \leftarrow 0</math>; 4: <math>t(\mathcal{P}) \leftarrow 0</math>; 5: bound(<math>\mathcal{G}, \delta, [\underline{t}, \bar{t}]</math>); 6: pulse(<math>v_s, r(\mathcal{P}), q(\mathcal{P}), t(\mathcal{P}), \mathcal{P}</math>); 7:   if Feasible(<math>v_i, q(\mathcal{P}), t(\mathcal{P})</math>) = true 8:     if Bounds(<math>v_i, t(\mathcal{P}), r(\mathcal{P})</math>) = false 9:       if RollBack(<math>v_i, t(\mathcal{P}), r(\mathcal{P}), \mathcal{P}</math>) = false 10:        <math>\mathcal{P} \leftarrow \mathcal{P} \cup \{v_i\}</math>; 11:        <math>q(\mathcal{P}) \leftarrow q(\mathcal{P}) + q_i</math>; 12:        for <math>v_j \in \mathcal{A}_i^+</math> do 13:          <math>r(\mathcal{P}') \leftarrow r(\mathcal{P}) + r_{ij}</math>; 14:          <math>t(\mathcal{P}') \leftarrow \max\{a_j, t(\mathcal{P}) + t_{ij}\}</math>; 15:          pulse(<math>v_s, r(\mathcal{P}), q(\mathcal{P}), t(\mathcal{P}), \mathcal{P}</math>); 16:        end for 17:      end if 18:    end if 19:  end if 20: end pulse 21: return <math>\mathcal{P}^*</math> </pre>

Once the pulse algorithm reached the end node  $v_e$ , the best-performed path  $\mathcal{P}^*$  will be updated. The algorithm will be terminated till the current node reaches the end node  $v_e$  or the current path is pruned.

The feasibility pruning is proceeded through Feasible function. The paths that violate structural constraints can be identified and discarded. The constraints covered by this study includes time window, vehicle load capacity, and cycle constraints.

The bound pruning limits the search space by providing the lower bounds  $\underline{r}(v_i, t(\mathcal{P}))$  for every node, which is shown in Table 2. The bound contains minimum reduced cost of a path  $\mathcal{P}$  that reaches  $v_i$ . The time bound of planning horizon  $[\underline{t}, \bar{t}]$ , bound step size  $\delta$ , and directed graph are essential input for this function. The time windows for paths are gradually reduced by the give step size  $\delta$ . The output of this function is denoted as lower bound matrix  $\mathbf{B} = [\underline{r}(v_i, \tau)]$ ,



which stores all lower bounds for every node and time step. Lines 4 to 9 solves ESPPRC for every node in that consumption using pulse procedure. Lines 10 – 14 stores the optimal reduced cost value found as the lower bound for every node at given time  $\underline{r}(\nu_i, \tau)$ . If the optimal path is an empty set, the lower bound is set to infinity.

Table 2. Pseudocode of bounds function for pulse algorithm for ESPPRC

<b>Algorithm 2:</b> Bounds function for pulse algorithm for ESPPRC
<b>Input:</b> $\mathcal{G}$ directed graph; $\delta$ bound step size; $[\underline{t}, \bar{t}]$ time bound for planning.
<b>Output:</b> $\mathbf{B} = [\underline{r}(\nu_i, \tau)]$ , lower bound matrix.
<pre> 1: <math>\tau \leftarrow \bar{t}</math>; 2: <b>while</b> <math>\tau &gt; \underline{t}</math> <b>do</b>; 3:   <math>\tau \leftarrow \tau - \delta</math>; 4:   <b>for</b> <math>\nu_i \in \mathcal{A}</math> <b>do</b> 5:     <math>\mathcal{P} \leftarrow \{\}</math>; 6:     <math>r(\mathcal{P}) \leftarrow 0</math>; 7:     <math>q(\mathcal{P}) \leftarrow 0</math>; 8:     <math>t(\mathcal{P}) \leftarrow \tau</math>; 9:     pulse(<math>\nu_s, r(\mathcal{P}), q(\mathcal{P}), t(\mathcal{P}), \mathcal{P}</math>); 10:    <b>if</b> <math>\mathcal{P}^* = \{\}</math> <b>then</b>; 11:      <math>\underline{r}(\nu_i, \tau) \leftarrow \infty</math>; 12:    <b>else</b> 13:      <math>\underline{r}(\nu_i, \tau) \leftarrow r(\mathcal{P}^*)</math>; 14:    <b>end if</b> 15:  <b>end for</b> 16: <b>return</b> <math>\mathbf{B}</math> </pre>

Rollback pruning aims to avoid exploration of unpromising regions by making better decisions in the early searching stage through re-evaluation of the last node visited. Once a path  $\mathcal{P}_{s_j}$  reaches node  $\nu_j$  through node  $\nu_i$ , re-evaluation of a potential bypath  $\mathcal{P}'_{s_j}$  that skips node  $\nu_i$  is conducted. The reduced cost of the bypath will be calculated. If the bypath has smaller reduced cost and consumed time, it will dominate the original path (Feillet et al., 2004). Compared to conventional labelling algorithms, Pulse Algorithm's rollback pruning excels in using no storage for saving labels.

### 4.3 Branching Heuristics

B&P synergizes branch-and-bound and column generation by iteratively searching of the branching tree, as shown in Figure 2. The variable chosen to be branched in this study is  $\tau_{i^U}^k$ , which links the manufacturer requests of pickup and delivery routes by Equation (6) and (7). The branching of  $\tau_{i^U}^k$  can formulate a pair of  $k$  and  $i^U$  for a father node of the branching tree. A branching uncertainty index  $\kappa_{i^U}^k$  is introduced to determine the branching priority, which is defined in Equation (11).

$$\kappa_{i^U}^k := \sum_{r \in R^P} \min\{a_r^{i^U} \beta_r^k, 1 - a_r^{i^U} \beta_r^k\} + \sum_{r' \in R^D} \min\{b_{r'}^{i^U} \gamma_{r'}^k, 1 - b_{r'}^{i^U} \gamma_{r'}^k\}, \forall k \in K \quad (11)$$

It measures the uncertainty that a vehicle  $k$  to serve a request  $i^U$  or not. If a vehicle  $k$  is assigned or unassigned to a request certainly,  $\kappa_{i^U}^k$  will approach to zero. Otherwise,  $\kappa_{i^U}^k$  will increase to show a high uncertainty of assigned or unassigned to a request. Thus, branching the maximum  $\kappa_{i^U}^k$  in all  $K$  vehicles is perceived to be efficient search to deviate uncertainty.

If the branch variable  $\tau_{i^U}^k$  are branched to zero, a service load  $i^U$  will not load to or unload from the vehicle  $k$  at cross-dock. Therefore,  $\tau_{i^U}^k = 0$  implies two scenarios, which are  $\sum_{r \in R^P} a_r^{i^U} \beta_r^k = 0$  and  $\sum_{r' \in R^D} b_{r'}^{i^U} \gamma_{r'}^k = 0$  that indicate the vehicle  $k$  never takes service load  $i^U$ , and  $\sum_{r \in R^P} a_r^{i^U} \beta_r^k = 1$  and  $\sum_{r' \in R^D} b_{r'}^{i^U} \gamma_{r'}^k = 1$  that indicate the vehicle  $k$  is to take service load  $i^U$ . On the other hand,  $\tau_{i^U}^k = 1$  implies that  $\sum_{r \in R^P} a_r^{i^U} \beta_r^k$  and  $\sum_{r' \in R^D} b_{r'}^{i^U} \gamma_{r'}^k$  have different values, meaning there will be load or unload at the vehicle  $k$  at the cross-dock. All scenarios are shown in Table 3.

Table 3. Four nodes of branching of a parent node

		Vehicle $k$ serves service load $i^U$ or not in pickup routes	
		$\sum_{r \in R^P} a_r^{i^U} \beta_r^k = 0$	$\sum_{r \in R^P} a_r^{i^U} \beta_r^k = 1$
Vehicle $k$ serves service load $i^U$ or not in delivery routes	$\sum_{r' \in R^D} b_{r'}^{i^U} \gamma_{r'}^k = 0$	$\tau_{i^U}^k = 0$	$\tau_{i^U}^k = 1$
	$\sum_{r' \in R^D} b_{r'}^{i^U} \gamma_{r'}^k = 1$	$\tau_{i^U}^k = 1$	$\tau_{i^U}^k = 0$

This branching rule makes C-VRPCD a quadtree, and a best-first strategy with maximum  $\kappa_{i^U}^k$  is used to search the branch tree in the depth-first manner.

## 5. Computational Results of C-VRPCD

To test the proposed B&P approach, two instances (R102, R101) from Solomon's VRPTW benchmark are used to build a C-VRPCD problem (Solomon, 1987). The first 25 customers from instances are used. Because this problem formulates an open network, the dock node is valid as the ending node for upstream vehicles and as the starting node for downstream vehicles. In this problem, the exchange cost is set to 50, and the travel cost is set as one fifth smaller than the original cost, so as the service time. The route cost is the sum of total travel distance. The maximum vehicle number is set to 5. The algorithms are implemented with C++ and the IBM ILOG CPLEX 20.10 optimization tool.

The calculated objective function value of the restricted master problem is 308.96. Figure 3 presents the optimization process of the objective function value from the linear relaxation problem.

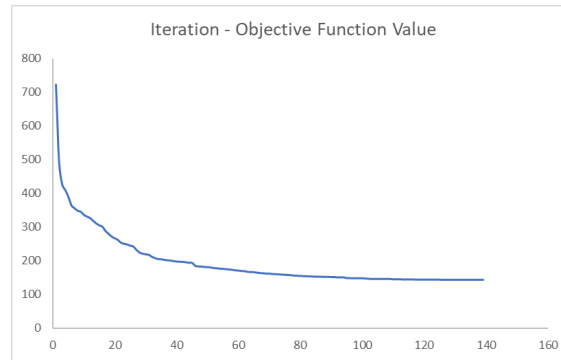


Figure 3. Objective Function Values of Linear Relaxation of the Restricted Master Problem

## 6. Conclusions

Logistic services for crowdsourced manufacturers entail networked material flow planning, which is to plan optimal vehicle service routes to link manufacturers as a material network. It enables manufacturers peeling off their logistic department and focusing on manufacturing activities.

There are two contributions of this study. The first contribution is that it proposes a cross-docking service method to use a platform-based strategy by splitting service routes into pickup and delivery ones and exploring maximum similarities among product variety. The vehicle can be used maximally by synchronizing pickup and delivery activities

to achieve no or few inventory in cross-dock depot. This service method shows the potential of handling a large number of manufacturers and volatile service requirements in platform-driven crowdsourced manufacturing.

The second contribution is that this study formulates the optimal decision-making model of logistics services for platform-driven crowdsourced manufacturing through cross-docking as C-VRPCD. The B&P algorithm is proposed, which utilizes a divide-and-conquer philosophy to decompose C-VRPCD into master problems and subproblems. The two problems are connected by dual values of the RMP, and subproblems are to update the column pool for size controlled RMP. Pulse Algorithm is applied as the fundamentals to solve subproblems. Branching rules on the freight exchange variables are used to search integral solutions in a quadtree. The proposed branch-and-price algorithm is tested through a problem built from two 25-customer instances in the Solomon's benchmark.

Future work of this study includes comparing the efficiency of different ESPPRC algorithms for solving the subproblems, applying cutting techniques in the branch-and-price framework to accelerate the branching process, and studying different branching strategies to reduce the number of branching nodes.

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