The Effects of Ambiguity on Evaluations of Multiple Prospects

Dolchai La-ornual
Mahidol University International College
Mahidol University, Thailand
dolchai.lar@mahidol.ac.th

Abstract

This paper examines whether individuals perceive benefits from diversification when prospects are ambiguous and how those benefits compare to analogous situations under risk. Based on Gilboa and Schmeidler's (1989) maxmin expected utility model, the hypothesis is that only risk-seeking individuals would receive non-zero diversification benefits. In particular, these decision-makers would receive negative benefits from diversification that would increase in the gains domain and further decrease in the losses domain as the degree of ambiguity increases. Results from a series of experiments suggest that people may perceive less benefit from diversification under ambiguity than under risk. However, individuals tend to diversify over a greater number of prospects when the prospects are ambiguous than when they are risky. This may be due to people's attempt to compensate for the lower marginal benefit of diversifying under ambiguity than under risk. It also implies a decrease in tolerance toward aleatory uncertainty in presence of epistemic uncertainty.

Keywords
Judgment and decision making, Ambiguity, Uncertainty, Diversification and Behavioral finance.

1. Introduction
We often have to make decisions in life that involve taking on several uncertain prospects simultaneously. For example, people often apply to many job openings, they may also buy bundles of different products, or most straightforwardly, they generally invest in several financial assets. In these situations, we normally do not know the exact probability distribution for all possible outcomes. Thus, we are essentially making decisions that involve multiple 'ambiguous' prospects.

Since the seminal paper by Ellsberg (1961), researchers have become well-aware that people can have distastes for unknown probability distributions. In particular, individuals will generally choose prospects with known probability distributions of outcomes over comparable ones with imprecisely known distributions. This behavioral phenomenon has become known as aversion to ambiguity. The distinction between known risk and unknown uncertainty or ambiguity was postulated by Knight (1921) and recently elaborated on with organizational perspectives by Kleindorfer (2010). There has been substantial work, both theoretical and empirical, on this topic of decision-making under ambiguity (see Camerer and Weber 1992, Nau 2007, and Epstein and Schneider 2010 for related reviews).

In regard to decisions concerning multiple prospects, there are two streams of relevant literature. First, there have been some studies in the marketing field that have studied choice and valuations of product bundles (Gaeth et al. 1991, Yadav 1994, Leszczyc et al. 2008). However, uncertainties in these investigations are with respect to values or outcomes rather than probabilities. The field of research that has examined decisions involving multiple ambiguous prospects most extensively is finance specifically, regarding portfolio choice and asset pricing. However, most of the experimental papers in this area have focused on comparative demands between risky and ambiguous assets (Ahn et al. 2009, Bossaerts et al. 2010).

In this paper, weintended to consider the scenario in which two or more prospects are equally ambiguous. The goal is to investigate the effect of ambiguity on individuals' valuations and selections of multiple prospects in a bundle. More specifically, weare interested in examining how the increasing degree of ignorance regarding the probability distributions of prospects can affect people's preferences for these bundles.

The paper is organized as follows. The next section discusses some of the decision models that have been proposed to accommodate ambiguity aversion and their implications on portfolio choice. Wedevelop some specific hypotheses to be tested experimentally using Gilboa and Schmeidler's maxmin expected utility or the multi-priors model in Section 3. Sections 4, 5, and 6 contain three experimental studies that examine how people price and choose single versus multiple ambiguous prospects. Wediscuss the results and conclude in Section 7.

2. Models of Decisions under Ambiguity and their Implications on Portfolio Choice

There have been many decision models that have tried to accommodate the behavioral phenomenon of ambiguity aversion. (See Camerer and Weber 1992, Nau 2007, and Epstein and Schneider 2010 for reviews.) Some of the most well-known models include Schmeidler's (1989) Choquet expected utility, Gilboa and Schmeidler's (1989) maxmin expected utility or the multiple-priors model, Ghirardato et al.'s (2004) α-maxmin expected utility, Klibanoff et al.'s (2005) smooth model, and Chateauneuf et al.'s (2007) neo-additive capacity model. Choquet expected utility relies on the concept of capacities to evaluate expected utility that is non-linearly weighted in probabilities in a rank-dependent manner. Maxmin expected utility calculates expected utility of prospects based on the least favorable subjective probability distributions. This is generalized by the α-maxmin model, which computes the value a prospect based on a linear combination of the highest and the lowest expected utilities that the decision-maker deem possible. The smooth model is based on second-order subjective probabilities and employs two transformations to capture the decision-maker's risk attitude and the ambiguity attitude independently. The neo-additive capacities model is a special case of the Choquet integral, which allows for both ambiguity-averse and ambiguity-seeking attitudes.

Applying Choquet expected utility, Dow and Werlang (1992) showed that ambiguity aversion can lead to portfolio inertia or an interval of prices (rather than a singleton) in which people would not be willing to either buy or sell an asset. Bossaerts et al. (2010) verified the α-maxmin expected utility experimentally that sufficiently ambiguity-averse individuals will refuse to hold an ambiguous portfolio or one in which their wealth is not distributed evenly over the states with unknown probabilities. In an example, Klibanoff et al. (2005) illustrated using their smooth model that the interaction between risk attitude and ambiguity attitude may cause people to under-diversify, i.e., in the case of home bias where investors prefer to hold stocks that they are more familiar with. Gollier (2009) examined this smooth model and found that higher degree of ambiguity aversion will not always result in a lower demand for an ambiguous asset compared to another risky one. Chateauneuf et al. (2007) proved using their neo-additive capacity model that the equity premium puzzle, where the returns for stocks are excessively higher than those for bonds, can arise when investors are ambiguity-averse in addition to being risk-averse.

3. Experimental Predictions based on the Multiple-priors Model

To develop some predictions for the experiments in the following sections, wewill rely on Gilboa and Schmeidler's (1989) maxmin expected utility (MEU) or the multiple-priors model. This model is among the most popular for decisions under ambiguity and provides relative clear predictions for the scenario of interest in this paper. Intuitively, MEU captures ambiguity aversion by assuming that people are extremely pessimistic and presume the least favorable probability distribution out of all subjective possibilities, resulting in the lowest expected utility (EU) for any prospect.

Consider the following situation. A decision-maker faces a choice between prospect *A* that will give payoff x with some probability $p \in [p_L, p_H]$ and a bundle of two prospects *B* and *C* each of which will give payoff $x/2$ also with some probability $p \in [p_L, p_H]$. (Although the results do not differ for this analysis, wewill assume for simplicity that the underlying probabilities in prospects *B* and *C* are identical.) The alternative outcomes for all these prospects are gaining or losing nothing. The choice between these two options can be represented by preferences over gambles $f = (p, x; 1\text{-}p, 0)$ and $g = (p^*, x; 2(p\text{-}p^*), x/2; 1\text{-}2p\text{+}p^*, 0)$ where $0 \leq p^* \leq p$. By letting p^* be a variable and preserving only the marginal probabilities of incurring the outcome $x/2$ in each of the prospects *B* and *C*, wehave not restricted what the decision-maker may assume regarding the dependence between the two prospects in the bundle.

For $x > 0$, we can assume without loss of generality, that $u(x) = 1$, $u(x/2) = u$, and $u(0) = 0$. Thus, MEU(f)is simply equal to p_L, which decreases as the probability interval $[p_L, p_H]$ expands around a certain point. In other words, we expect that the decision-maker's valuation and preference for a single prospect *A* will decrease with increasing level of ambiguity.

For the bundle of prospects *B* and *C* represented by gamble g, its value according to MEU depends on the value of u, or the decision-maker's risk attitude. For risk-averse and risk-neutral individuals or when $u \geq ½$, EU(g) = $p^*(1\text{-}2u)$ +

$2pu$ is minimized when $p^* = p_L$ and $p = p_L$ and thus, MEU(g) = p_L. For risk-seeking individuals or when $u < ½$, MEU(g) is equal to $2p_Lu$ as $p^* = 0$ and $p = p_L$ in the EU(g) expression. The insight is that according to MEU, risk-averse averse agents would suppose that the two prospects B and C are positively correlated, while risk-seeking agents would believe that they are negatively correlated. However, regardless of their risk attitudes, these results suggest that the value of the bundle will decrease as the degree of ambiguity increases.

What we are most interested in, however, are the benefits from diversification, which can be determined by the differences between valuations of the bundle and those of the single prospect. It is easy to see that there are essentially no benefits from diversification according to MEU to either risk-averse or risk-neutral individuals. In particular, MEU(f) = MEU(g) = p_L for these agents. However, for risk-seeking individuals, the (dis)benefit can be expressed as MEU(g) – MEU(f) = $p_L(2u\text{-}1) < 0$. Thus, with increasing degree of ambiguity, we should expect an increase in benefits from diversification for these people.

Similar analyses can be performed for prospects in the losses domain where $x < 0$. Letting $u(x) = -1$, $u(x/2) = -u$, and $u(0) = 0$, the expected value of the single prospect A would then be equal to $-p_H$ according to MEU. This will also decrease with increasing degree of ambiguity as in the gains domain. The value of the bundle of prospects B and C for risk-averse and risk-neutral individuals would be equal to $-p_H$, while that for risk-seeking ones would be $-2p_Hu$. We can easily see that these values would decrease with the widening of the probability interval or increase in the ambiguity level, which is identical to the situation in the gains domain. In addition, risk-averse and risk-neutral agents will also not derive any benefits from diversification MEU(f) = MEU(g) = $-p_H$ for these individuals. In contrast, risk-seeking individuals will actually receive decreasing benefits, represented MEU(g) – MEU(f) = $p_H(1\text{-}2u) < 0$, by as the degree of ambiguity increases.

To summarize, according to MEU, diversification is not expected to create any benefits or disbenefits for risk-averse and risk-neutral agents. However, we anticipate that risk-seeking agents will be adversely affected by diversification. Furthermore, we also hypothesize that with increasing degree of ambiguity, these disbenefits would be ameliorated in the gains domain, but would be further exacerbated in the losses domain.

4. Experiment 1: Pricing of One versus Two Uncertain Prospects

4.1 Design and Procedure

The subjects in this experiment were 141 undergraduate engineering students in international programs at Chulalongkorn University in Thailand. Through a paper-and-pencil questionnaire in a classroom test setting, each subject was asked to state his or her certainty equivalent (CE) for 8 sets of gambles, presented in three different orders. Each participant received course credit and two subjects were randomly selected to each play out one set of gamble(s) for real money using a Becker-DeGroot-Marschak (1964) incentive compatible mechanism.

The design of the experiment was based on 3 factors: 2 domains (gains, losses) × 2 set types (single, double) × 3 probability precisions (risky, ambiguous, unknown). The domain factor was between-subject, while the set type and probability precision were within. In addition, each subject also evaluated two additional ‘mixed’ gamble sets where each set consisted of two gambles with distinct probability precisions (risky and ambiguous, risky and unknown).

With respect to the set type, there was only one gamble in the single set and two unrelated ones in the double and mixed sets. The potential gain or loss of the gamble in the single set was 5,000 Thai baths, which was equivalent to about €100. In the double and mixed sets, the potential gain or loss for each of the two gambles was 2,500 Thai baths or approximately €50. The probability precisions of receiving these gains or paying these losses for the risky, ambiguous, and unknown levels were 50%, 40-60%, and 0-100% respectively. The alternative outcome in all cases was gaining or losing nothing.

4.2 Results and Analyses

In Table 1 below, we categorized subjects according to their risk and ambiguity attitudes inferred from their valuations for certain gamble sets. In particular, if a subject’s CE for the single risky gamble is less than, more than, or equal to its expected value, then that subject is considered to be risk-averse, risk-seeking, or risk-neutral respectively. Also, if a subject’s CE for the single unknown gamble is less than, more than, or equal to that of the single risky one, then that subject is determined to be ambiguity-averse, ambiguity-seeking, or ambiguity-neutral respectively. In this regard, the majority of the subjects were risk-averse in the gains domain (63%) and risk-seeking in the losses domain (63%). With respect to their ambiguity attitudes, the highest percentages of subjects were ambiguity-averse in both the gains (51%) and the losses (42%) domains. These characteristics are consistent with prior findings in literature in general.

Table 1. Classification of Subjects according to their Risk-Ambiguity Attitudes in Experiment 1

GAINS / LOSSES	**Ambiguity-averse**	**Ambiguity-neutral**	**Ambiguity-seeking**	Total
Risk-averse	**22 (32%)** 5 (7%)	3 (4%) 1 (1%)	18 (26%) 5 (7%)	**43 (63%)** 11 (15%)
Risk-neutral	8 (12%) 6 (8%)	4 (6%) 7 (10%)	5 (7%) 3 (4%)	17 (25%) 16 (22%)
Risk-seeking	5 (7%) **20 (27%)**	- (0%) 8 (11%)	3 (4%) 18 (25%)	8 (12%) **46 (63%)**
Total	**35 (51%)** **31 (42%)**	7 (10%) 16 (22%)	26 (38%) 26 (36%)	68 (100%) 73 (100%)

Below in Table 2 and Table 3 are the mean CE's for each of 8 gamble sets in the gains domain and the losses domain respectively.

Table 2. Mean Certainty Equivalents (S.D.) in Thai Baht for Gambles Sets in Gains ($N = 68$)

Set Type	**Probability Precision**		
	Risk	*Ambiguity*	*Unknown*
SINGLE	2334 (349)	2434 (511)	2261 (985)
DOUBLE	2491 (439)	2494 (524)	2258 (957)
MIXED	-	2503 (415)	2411 (717)

Table 3. Mean Certainty Equivalents (S.D.) in Thai Baht for Gambles Sets in Losses ($N = 73$)

Set Type	**Probability Precision**		
	Risk	*Ambiguity*	*Unknown*
SINGLE	-2412 (400)	-2491 (464)	-2439 (790)
DOUBLE	-2367 (393)	-2432 (468)	-2421 (765)
MIXED	-	-2391 (456)	-2431 (649)

We analyzed the experimental results for each domain, gains and losses, separately. First, we conducted a two-way ANOVA with repeated measures on both variables, set type, and probability precision. In the gains domain, the main effect of set type was marginally significant ($F_{1, 67} = 3.741$, $p = 0.057$). This implied that the double gamble sets seemed to be valued higher than the single ones. More specifically, subjects perceived benefits in diversifying over risk, ambiguity, and unknown probability precisions collectively. The other main effect of probability precision was not significant ($F_{1.321, 88.490} = 2.653$, $p = 0.097$). (Because this factor failed the Mauchly sphericity test, implying a violation of the homogeneity of covariance assumption, we have reported the more conservative Greenhouse-Geisser modified F-test.) In addition, there was also no significant interaction between probability precision and set type ($F_{2, 134} = 1.922$, $p = 0.150$). These results imply that different probability precisions did not affect subjects' valuation of the gamble sets overall. Nevertheless, closer inspection of planned contrasts on this factor revealed that although valuations of the gamble sets did not significantly differ when comparing risk to ambiguity levels ($M = 2418$ vs. 2464, $F_{1, 67} = 0.832$, $p = 0.365$), they did decrease significantly from ambiguity to unknown levels ($M = 2464$ vs. 2260, $F_{1, 67} = 4.049$, $p < 0.05$).

Second, because we are interested in measuring each subject's perceived benefits from diversification, we computed the differences in valuations of the double gamble set over their single counterpart for each probability precision. For example, the CE for the double risky set is approximately 147 Baht higher than that of the single risky one as illustrated in Figure 1. These differences decline as the probability precision changes from risk to ambiguity to unknown. To analyze these transformed data, weperformed a one-way ANOVA on the probability precision factor. Although the main effect of this factor was not significant ($F_{2, 134} = 1.922$, $p = 0.150$), more detailed planned contrasts revealed that the difference is significant between risk and unknown ($M = 147$ vs. -3,$F_{1, 67} = 3.988$, $p = 0.05$) even if it is not between risk and ambiguity ($M = 60$ vs. -3, $F_{1, 67} = 1.455$, $p = 0.232$). From these results, we can deduce that subjects perceive less benefit from diversification when the probability of gaining is unknown than when it is risky.

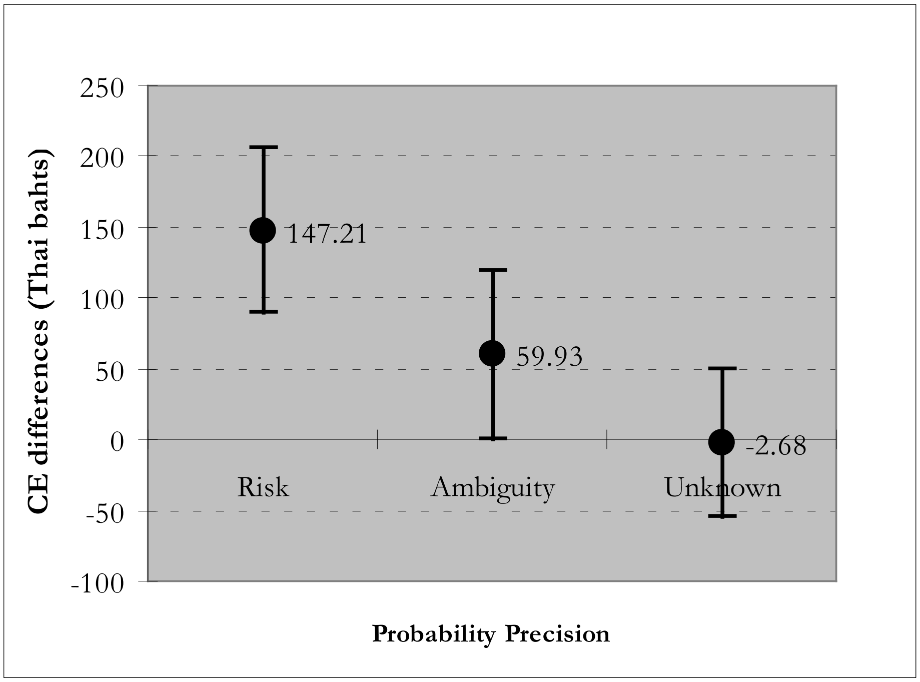

Figure 1. Differences in CE's for Double over Single Gamble Sets in Gains ($N = 68$)

Identical analyses were performed on data in the losses domain. A two-way ANOVA with repeated measures found that both of the two main effects were not significant: set type ($F_{1, 72} = 0.709$, $p = 0.403$) and probability precision ($F_{1.290, 92.853} = 0.669$, $p = 0.452$). Moreover, neither was the interaction between the two factors ($F_{1.668, 120.128} = 0.213$, $p = 0.768$). (Because both probability precision and its interaction with set type failed the Mauchly sphericity test, wehave again reported the more conservative Greenhouse-Geisser modified *F*-test for these two cases.) However, planned contrasts on the probability precision factor revealed a significant decrease in valuation from risk to ambiguity ($M = -2389$ vs. -2461, $F_{1, 72} = 4.787$, $p < 0.05$), but not from ambiguity to unknown ($M = -2461$ vs. -2429, $F_{1, 72} = 0.211$, $p = 0.684$). Collectively, these results suggest that subjects did not value gamble sets involving losses differently whether they were of single or double type. In other words, participants did not perceive any benefit from diversifying in losses under risk, ambiguity, or uncertainty.

Below in Figure 2 is the illustration of differences between CE for double versus single gamble sets in the losses domain. Although, all the mean differences were higher than zero, a one-way repeated measures ANOVA did not find the main effect of probability precision to be significant (Greenhouse-Geisser $F_{1.668, 120.128} = 0.213$, $p = 0.768$).

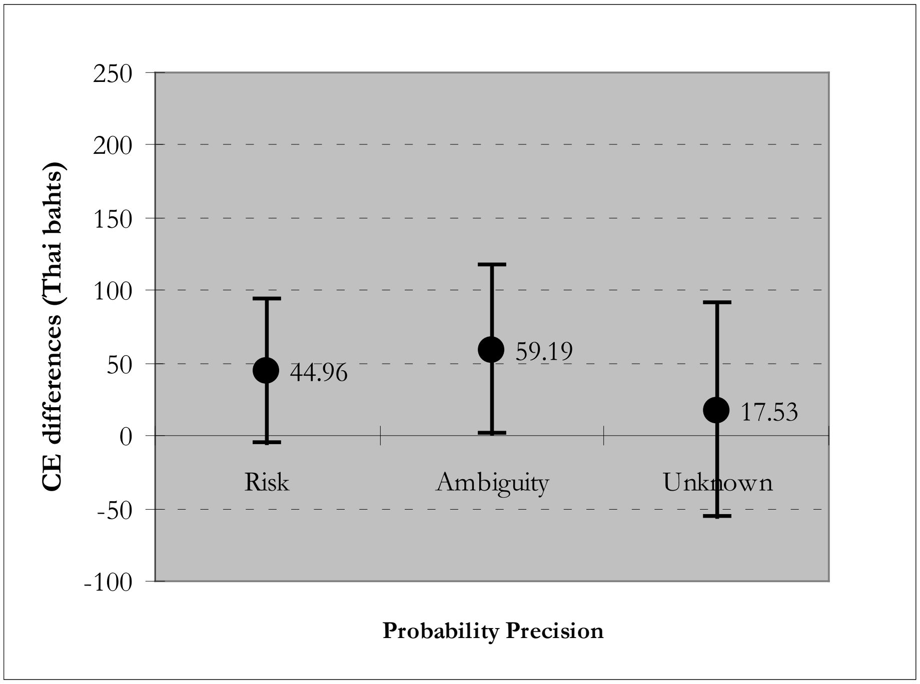

Figure 2. Differences in CE's for Double over Single Gamble Sets in Losses ($N = 73$)

5. Experiment 2: Choice between One versus Two Uncertain Prospects

5.1 Design and Procedure

In this study, 268 MBA participants at INSEAD in France were asked to answer two questions regarding their preferences between two (sets of) gambles. The experiment was conducted using a paper-and-pencil questionnaire in a classroom setting.

Each student was randomly assigned either to the gains or the losses domain. First, each subject was then asked if he or she preferred either a lottery with 50% chance of gaining (or losing) €100 and nothing otherwise or its expected value of receiving (or paying) €50 for sure. (Participants could have stated that they were indifferent between the two options.) Then, subjects were asked if they preferred one single gamble or two unrelated ones with identical characteristics or were indifferent between the two options. In particular, the probability of receiving the gain or incurring the loss for all gambles was either 50% (risk), 40-60% (low-ambiguity), or 10-90% (high-ambiguity) in a between-subject design. The potential gaining (or losing) amount for the single gamble and was €100, while that for each gamble in the double set was €50. Thus, this second question was essentially based on a 2 domains (gains, losses) × 3 probability precisions (risk, low-ambiguity, high-ambiguity) between-subject design.

5.2 Results and Analyses

The responses from the first question are presented in Table 4 below. Subjects were classified as being risk-averse, risk-seeking, or risk-neutral if they preferred the lottery, the sure amount, or were indifferent between the two options respectively. As in Experiment 1, the majority of the subjects were risk-averse in the gains domain (78%) and risk-seeking in the losses domain (67%).

Table 4. Classification of Subjects according to their Risk Attitudes in Experiment 2

Domain	**Risk-averse**	**Risk-neutral**	**Risk-seeking**	Total
GAINS	**105 (78%)**	21 (16%)	9 (7%)	135 (100%)
LOSSES	14 (11%)	30 (23%)	**89 (67%)**	133 (100%)

Preferences between the single gamble and the double set of two identical but unrelated gambles for the gains and the losses domains can be found in Figure 3 and Figure 4 respectively. For the gains domain, the highest percentage of subjects strictly preferred the double set over a single gamble for all probability precisions: risk (49%), low-ambiguity (58%), and high-ambiguity (66%). Similarly, the majority of subjects in the losses domain also chose the double set of gambles over its single counterpart whether the probability precision was risk (73%), low-ambiguity (66%), and high-ambiguity (71%).

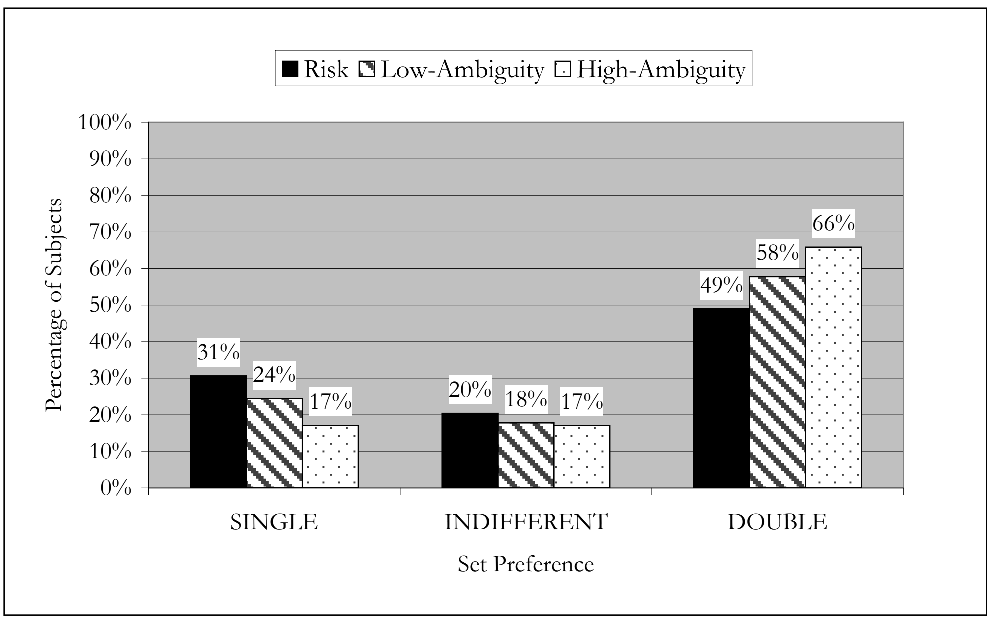

Figure 3. Preference between Single versus Double Gamble Sets in the Gains Domain ($N = 135$)

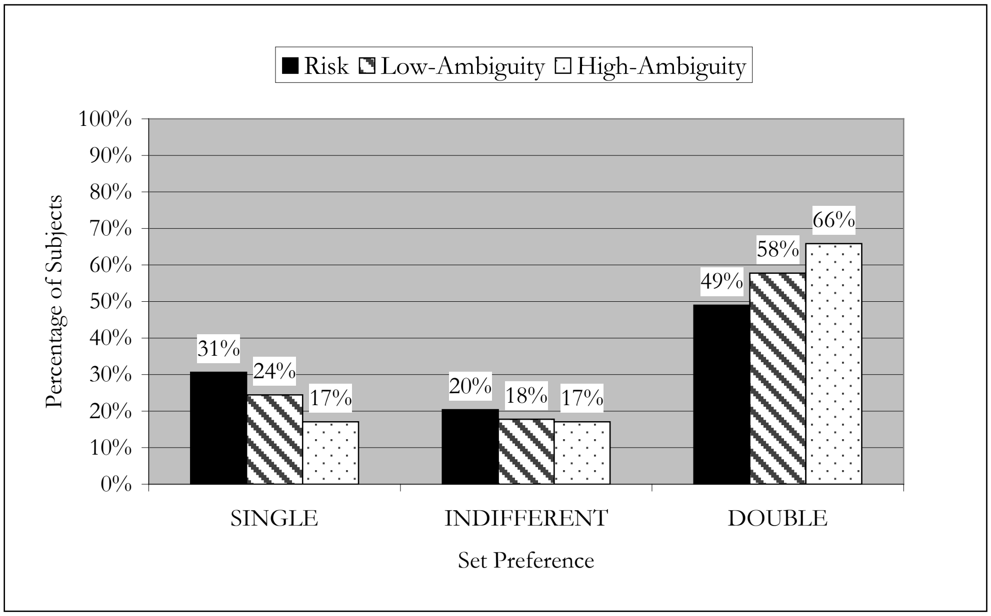

Figure 4. Preference between Single versus Double Gamble Sets in the Losses Domain (N = 133)

To perform more detailed analyses, we coded preferences for the single gamble, its double set counterpart, and indifference between the two as 1, 2, and 1.5 respectively. The mean results for both domains are presented in Table 5 and Table 6 below. For example, an average of 1.59 for the risky condition in gains denoted that subjects slightly preferred the double risky set over its single counterpart on average. However, a simple t-test revealed that this preference to diversify was only marginally significant ($t_{48} = 1.46$, $p = 0.076$). In contrast, the mean values for all other conditions were significantly below zero: low-ambiguity ($t_{44} = 2.62$, $p< 0.01$) and high-ambiguity ($t_{40} = 4.01$, $p< 0.001$) in gains; risk ($t_{40} = 5.56$, $p< 0.001$), low-ambiguity ($t_{46} = 7.38$, $p< 0.001$), and high-ambiguity ($t_{44} = 5.36$, $p< 0.001$) in losses. These results imply that people perceive benefits from diversification under risk and ambiguity in both the gains and losses domains.

Table 5. Preference between Single (1) versus Double (-1) Gamble Sets in Gains (N = 135)

	Probability Precision		
	Risk	*Ambiguity*	*Unknown*
Mean	1.59	1.67	1.74
S.D.	0.44	0.43	0.39
Difference	0.09	0.17	0.24
t-stat	1.46	2.62	4.01
DF	48	44	40
Sig.	7.57E-02	5.98E-03	1.28E-04

Table 6. Preference between Single (1) versus Double (-1) Gamble Sets in Losses (N = 133)

	Probability Precision		
	Risk	*Ambiguity*	*Unknown*
Mean	1.80	1.81	1.79
S.D.	0.35	0.29	0.36
Difference	0.30	0.31	0.29
t-stat	5.56	7.38	5.36
DF	40	46	44

Sig.	9.95E-07	1.22E-09	1.44E-06

Wealso performed a two-way independent ANOVA on the domain and probability precision factors. The main effect of domain was highly significant ($F_{1,\,262}$ = 4.741, p< 0.01). This meant that, in general, subjects preferred a higher number of prospects in the losses domain (M = 1.80) than in the gains domain (M = 1.66). However, neither the main effect of probability precision ($F_{2,\,262}$ = 0.708, p = 0.493) or its interaction with domain ($F_{2,\,262}$ = 1.076, p = 0.343) was significant. These results implied that the different probability precisions had no impact on people's diversification behavior regardless of the domain. Because of the slightly different number of subjects in each condition, wealso carried out the Gabriel post-hoc test on the probability precision. Nevertheless, the differences were not significant between any pair of probability precisions: risk and low-ambiguity (p = 0.753), risk and high-ambiguity (p = 0.431), and low-ambiguity and high-ambiguity (p = 0.945).

6. Experiment 3: Choice over Multiple Uncertain Prospects

6.1 Design and Procedure

Subjects in this experiment were 168 undergraduates at the University of Waterloo in Canada who participated in an online questionnaire for course credit.

There were two parts in this experiment, presented in random order. The first part contained four hypothetical questions where subjects were asked to choose between one of two prospects in each question. In particular, the four questions asked for preferences between i) gaining C$5,000 for sure or a lottery with 50% chance of gaining C$10,000 and nothing otherwise, ii) a lottery with 50% chance of gaining C$10,000 and nothing otherwise or a gamble with unknown probability of gaining C$10,000 and nothing otherwise, iii) paying C$5,000 for sure or a lottery with 50% chance of paying C$10,000 and nothing otherwise, iv) a lottery with 50% chance of losing C$10,000 and nothing otherwise or a gamble with unknown probability of losing C$10,000 and nothing otherwise. These questions were presented in random order and were used to determine each participant's risk and ambiguity attitudes in both the gains and the losses domain.

The second part was based on a 2 domains (gains, losses) × 3 probability precisions (risk, low-ambiguity, high-ambiguity) mixed design, where the domain factor was within-subject and the probability precision was between-subject. Each subject was asked two hypothetical questions, one concerning potential losses and the other containing potential gains. Specifically, they were asked to choose the most preferred option(s) out of five possible choices. The choices were either one single gamble with the possibility of gaining (or losing) C$10,000, two unrelated gambles with the possibility of gaining (or losing) C$5,000 in each, and so on up to five unrelated gambles with the possibility of gaining (or losing) C$2,000 in each. Subjects were allowed to specify more than one option as the most preferred if they wished. For each subject, the probability of gaining or losing in all the gambles was either 50% (risk), 40-60% (low-ambiguity), or 10-90% (high-ambiguity). Furthermore, wealso conducted a test of understanding for this part by asking each subject to identify all possible outcomes from a set of three unrelated gambles. Feedback was provided and subjects were asked to correct their mistakes if any.

6.2 Results and Analyses

Results from the first part that determined each subject's risk and ambiguity attitudes in both gains and losses domains are presented in Table 7 below. These results are similar to that from Experiments 1 and 2 where the majority of participants were risk-averse in gains and risk-seeking in losses. More specifically, 66% of the subjects preferred a sure gain of C$5,000 over a lottery with 50% chance of gaining C$10,000 and 52% of the people preferred a lottery with 50% chance of losing C$10,000 over the sure loss of C$5,000. In addition, the highest percentages of subjects were ambiguity-averse in gains, preferring the lottery with 50% chance of receiving C$10,000 over a gamble with unknown chance of receiving the same amount (47%). Conversely, most were ambiguity-seeking in losses, preferring a gamble with unknown chance of losing C$10,000 over the lottery with 50% chance of losing the same amount (48%).

Table 7. Classification of Subjects according to their Risk-Ambiguity Attitudes in Experiment 3

GAINS LOSSES	**Ambiguity-averse**	**Ambiguity-neutral**	**Ambiguity-seeking**	Total

Risk-averse	**60 (36%)** 12 (7%)	20 (12%) 9 (5%)	29 (17%) 18 (11%)	**109 (66%)** 39 (23%)
Risk-neutral	7 (4%) 10 (6%)	14 (8%) 19 (11%)	4 (2%) 12 (7%)	25 (15%) 41 (25%)
Risk-seeking	11 (7%) 23 (14%)	6 (4%) 14 (8%)	15 (9%) 49 (30%)	32 (19%) **86 (52%)**
Total	**78 (47%)** 45 (27%)	40 (24%) 42 (25%)	48 (29%) **79 (48%)**	166 (100%) 166 (100%)

The results from the second part are presented in Figure 5 and Figure 6 below. In both domains, the majority of the subjects either preferred the single gamble or a set of five unrelated gambles. Moreover, the percentage of people that preferred the set of five gambles the most increased with higher level of ambiguity, e.g., from 52% to 59% to 72% in the gains domain.

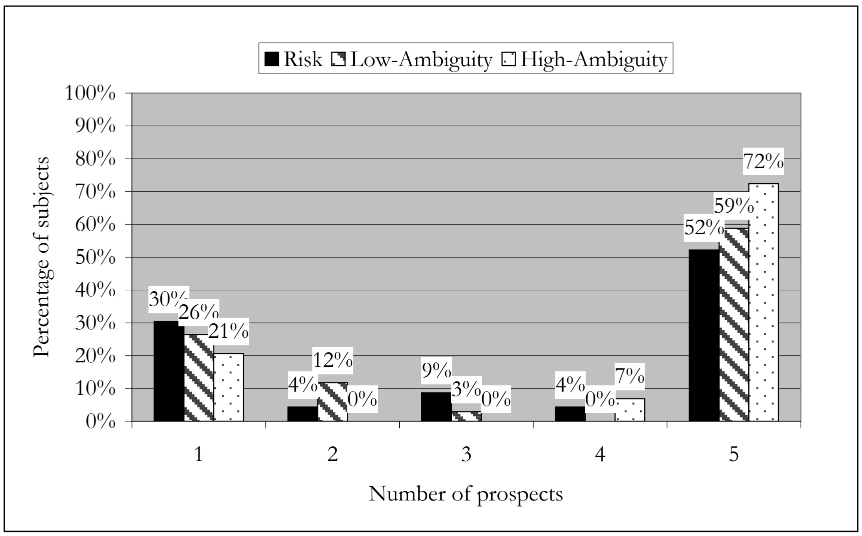

Figure 5. Number of Prospects Most Preferred in the Gains Domain (N = 166)

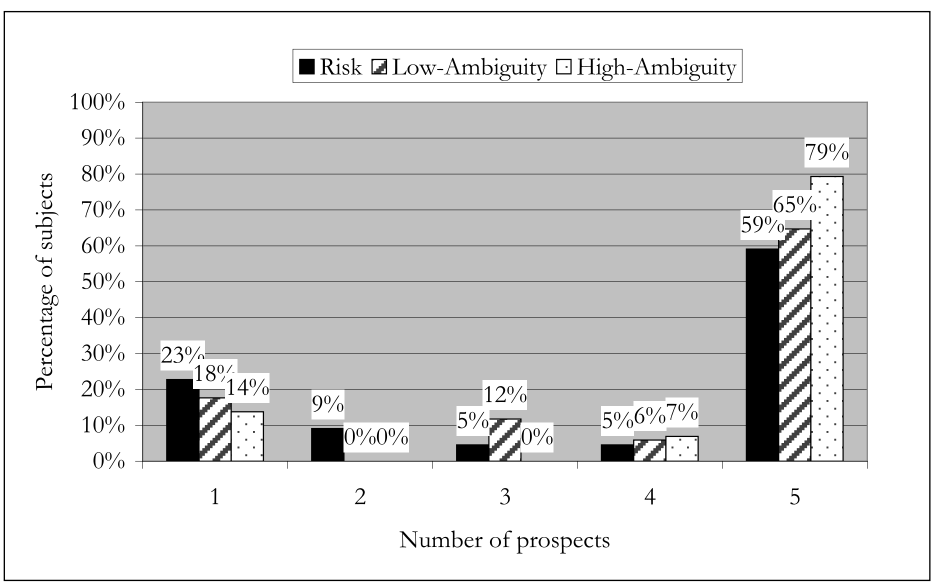

Figure 6. Number of Prospects Most Preferred in the Losses Domain (N = 166)

A two-way mixed ANOVA with domain as the within-subject factor and probability precision as the between-subject factor revealed a marginally significant effect for domain ($F_{1,\,164} = 3.686$, $p = 0.057$). This suggests that subjects may prefer a higher number of prospects in the losses domain ($M = 3.45$) than in the gains domain ($M = 3.80$) or that they diversify to a greater degree in losses than in gains. However, the interaction between domain and probability precision was not significant ($F_{2,\,164} = 1.014$, $p = 0.365$) and neither was the between-subject effect of probability precision ($F_{2,\,164} = 0.970$, $p = 0.381$). These results imply that the different probability precisions had no influence on people's diversification behavior regardless of the domain. Because of the slight differences in number of subjects in each condition, wealso ran the Gabriel post-hoc test on the probability precision. Nevertheless, the differences were not significant between any pair of probability precisions: risk and low-ambiguity ($p = 1.000$), risk and high-ambiguity ($p = 0.572$), and low-ambiguity and high-ambiguity ($p = 0.524$).

7. Discussion and Conclusion

Recall from Section 3, that we had expected that only people who are risk-seeking in both domains would be affected by diversification under ambiguity. For these individuals, we had predicted that the negative benefits from diversification in the gains domain would decrease, while the negative benefits from diversification in the losses domain would increase with increasing degree of ambiguity. We did not anticipate that risk-averse and risk-neutral individuals would receive any positive or negative benefits from diversification in any domains according to MEU.

Results from Experiment 1 seem to contradict our predictions with regards to pricing of prospects in both the gains and losses domain. As the predominant attitudes were risk-averse in gains and risk-seeking in losses, we should expect no effects in the former and decreasing diversification benefits in the latter. However, there seems to be a general benefit to diversification in the gains domain that decreases with increasing degree of ambiguity (Figure 1). In addition, there seems to be no significant disadvantages to diversification in the losses domain at any level of ambiguity (Figure 2). Results from Experiment 2 and Experiment 3 are particularly interesting in comparison to the findings from Experiment 1. In particular, even though individuals perceive less benefit from diversification under ambiguity than under risk in the gains domain from Experiment 1, the results in Experiments 2 and 3 imply that people may tend to diversify to an even greater extent under ambiguity than under risk. More specifically, the percentage of people preferring the double set of gambles in Experiment 2 (Figure 3) and the average number of prospects chosen in Experiment 3 were somewhat higher under ambiguity than under risk (Figure 5).

Although it is not uncommon to observe distinct ambiguity preferences under different tasks (see Du and Budescu 2005), the findings from the Experiments 1-3 may have plausible explanations. First, it may be more difficult for individuals to determine the value of diversification under ambiguity than under risk, especially when prospect concern potential losses. This may have lead to under-valuations of multiple ambiguous prospects in the gains domain and no effect of diversification in the losses domain as is found in Experiment 1. Second, if the marginal benefit from diversifying over an additional prospect is indeed higher under risk than under ambiguity, this may actually lead to individuals choosing a larger number of prospects to compensate in the latter condition. This implies that the results all the three experiments, at least in the gains domain, may in fact be consistent. More specifically, in presence of epistemic risk or the imprecisely known probability of outcomes, people may be less tolerant toward aleatory risk or the uncertainty of the resulting outcome.

One of the strongest and most interesting results from the experiments in this paper is the fact that people tend to diversify over potential losses and to an even greater extent than over potential gains. This is surprising as we had expected from the predictions that the predominantly risk-averse individuals in the gains domain would receive no benefits from diversification, while the predominantly risk-seeking individuals in the losses domain would actually receive disbenefits from the same process. In examining reasons subjects had given for their choices in Experiment 2, wehave found that most subjects who chose the double gamble set over their single counterpart gave some variations of diversification to reduce the uncertainty as the reason in both domains. However, it appeared that many participants were concerned about the maximum payoff in the losses domain. Thus, their preference to diversify may be in fact be driven by greater influence of loss aversion (Kahneman and Tversky 1979) in face of increasing ambiguity.

There are many aspects that warrant further investigation on the topic of decisions that involve multiple ambiguous prospects. For example, it is not clear how subjects perceive the relationship between two ambiguous prospects and how they may hedge against ambiguity or epistemic risk in comparison to aleatory risk or the uncertain outcomes. It may also be beneficial to investigate how people would allocate resources among various ambiguous prospects at different levels of ambiguity. Nevertheless, the findings in this paper proposes that rather than under-diversify, people may in fact diversify over a higher number of prospects in presence of ambiguity. This seems to be particularly true when individuals are faced with prospects that involve potential losses. Prescriptive managerial implications may include recommendations provided by financial advisors. In particular, when markets are relatively volatile, investors may actually be more attracted to allocate funds over many less-familiar financial products)

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Biographies

Dolchai La-ornual received his Ph.D. in Management (Decision Science) from INSEAD in France. He also holds master degrees in Engineering-Economic Systems and Operations Research from Stanford University and Industrial and Operations Engineeringfrom the University ofMichigan at Ann Arbor, U.S.A. He is currently an Assistant Professor of Business Administration at Mahidol University International College, Mahidol University, Thailand. His primarily research interest is in judgment and decision making, i.e., decision under ambiguity. He has published in journals such as Journal ofRisk and Uncertainty, Journal ofBehavioral Decision Making, and Tourism Management.