Production Optimisation Using Linear and Integer Programming in an Aluminium Company

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Abstract

The product mix problem is one of the salient issues faced by SMMEs. The study aims to establish a product mix optimisation model to address the problem. To achieve this goal, QM for Windows software is deployed to ascertain an optimal production level to maximise profit for NAYO Aluminum Company fabricators of aluminium products in Enugu (Eastern Nigeria). This software uses raw material and production requirements data to perform optimisation calculations. The analysis includes (1) the number of windows and doors to be produced using the available resources to maximise total income, (2) total costs of production, and (3) gross income. The findings indicate unused resource levels, which helps to manage input resource procurement and minimise waste. The analysis result using QM for Windows, after linear and integer programming, shows an optimal production: 28 aluminium doors and 2 casements ($P_1 = 0; P_2 = 2; P_3 = 28, P_4 = 0$) to make the highest income of 119720.00 Naira ($Max\ Z = 1720P_1 + 4000P_2 + 3990P_3 + 2970P_4$). This application can improve productivity, inventory management and company profitability.

Keywords
Optimisation, linear programming model, QM POM, integer programming.

1. Introduction
Business process optimisation is an economic, sustainability and continuous improvement imperative as organisations strive to remain competitive in a constantly dynamic operating environment. Predominantly, management decisions are focused on how best to achieve the firm's objectives, subject to the resource constraints and other restrictions prevailing in their domain of operation. Hence, making the most effective use of an organisation's finite resources, such as labour, machinery, materials, capital, time, and many others, is a vital requirement of all management decisions (Render et al. 2020). Knowing that what is committed in one operation cannot be reused in another, linear programming (LP) aims to help managers make the best resource combination choices and optimise corporate outcomes.

In a country ravaged by high youth unemployment, inflation, and economic and political uncertainty leading to increased poverty levels (Olorunfemi et al. 2021), SMMEs seemingly are a primary source of employment in Nigeria as the state-owned enterprises and corporations offer no significant work potential to the unemployed (Ilori et al. 2019). NAYO is a small-scale aluminium company located at Nnamdi Azikiwe Avenue in the cosmopolitan city of Enugu- Eastern Nigeria. NAYO and other SMMEs in the industry are struggling to break even, given the increased cost of production. Given the uncertainty of demand, they have a lot of finished product inventory as they experience varying sales (Company 2022 Annual Report). Further, the researcher observed they are not using any quantitative planning tool to manage their production. Figure 1 shows the aluminium products.

 Sliding Aluminum Window	 Casement Aluminum Window	 Aluminum Projected Window	 Aluminum Door

Figure 1. Products of the case company: NAYO Aluminium.

1.1 Objectives

Although linear programming (LP) has been deployed to attain the best decision in organisations, saving the day for most ailing companies, Akpan (2016) reported that many organisations, especially Nigerian production concerns, are yet to unleash the full potential of linear programming due to a lack of awareness of its numerous applications. Therefore, to optimise resource utilisation and minimise waste associated with keeping inventory and overproduction, the research objective is to use an LP model to identify the quantity of each product to produce to make an optimal return for NAYO Aluminum Company.

2. Literature

Depending on the situation, the LP objective usually involves minimising factors of negative implication like cost or maximising positive factors like audience exposure and profit. Essentially, the LP technique ensures businesses allocate financial and material resources effectively (Taylor 2018) and finish the manufacturing process within the allotted time.

Invented during World War II to optimise military logistics, the LP technique illustrates how a given optimality criterion allocates limited resources to a set of competing products or activities for a specific organisation (Dantzig 2002). Derived from a real-life problem, an LP mathematical model consists of a linear objective function, subject to inequality and equality constraints, including the specific quantity of resources available to help managers improve productivity. The LP model can be solved using the graphical method (if there are only two decision variables), simplex algorithm or other computer software to determine the optimal feasible solution for the formulated model (Render et al. 2020).

Linear programming lends itself to a wide range of application contexts because of the ease at which variables and constraints depicting the company's operating conditions can be incorporated into the linear programming model. LP is applied in various sectors: healthcare, agriculture, manufacturing, maritime and military operations, and social and behavioural science in organisations of different sizes. Planning production, investment portfolios, logistics and transportation routes for shipment of products, scheduling of workforce, and establishing a dietary mix to meet nutritional requirements are examples of complex areas of operations where the management and microeconomic applications of linear programming are prevailing (Render et al. 2020; Taylor 2018).

Recent studies indicate robust LP applications as researchers integrate the LP model with other quantitative statistical techniques. An et al. (2021) used LP to solve a booking limit problem to improve revenue management. The authors composed an LP-solvable model for an established maximum regret problem in an airline company. They developed a genetic algorithmic heuristic solution for calculating the airline booking limits, thus minimising loss due to a forgone alternative. Aimed at cost minimisation amidst constraining future production demand, fixed raw materials capacity, and limited labour hours amid recruitment barriers, Adriantantri and Indriani (2021) developed a model for optimising aggregate production. The study demonstrated how production concerns can integrate linear programming with forecasting and aggregate planning to maximise output.

While other softwares are used, most authors adopt the Simplex algorithm. Syifa et al. (2023) conducted an LP optimisation study in an Indonesian ceramic company where production is based on intuition, resulting in excess inventory, some of which is non-recyclable. Using the Simplex technique, the researchers found that producing less of one of the products - glass type 1 compared to others will maximise the company's profit. Hence, the new product mix offered an optimised solution for the company. Maurya et al. (2015) demonstrated how linear programming can help management improve production and profit margins in an Ethiopian government chemical company manufacturing sulphuric acid and aluminium phosphate. The study recommends that companies forecast production capacity and revenue using quantitative tools like linear programming, trend and regression models rather than trial and error to ensure objectivity and accuracy.

3. Methods

The aluminium company's production data inform the LP Model formulation. The basic assumptions and requirements of linear programming (certainty, proportionality, additivity, divisibility) were observed to establish the LP application for NAYO Aluminum factory business optimisation. The non-negativity constraint dictates that the decision variables cannot take on negative values besides meeting the linear requirement, as having negative resources is infeasible. The QM for Windows software deployed used input raw material and production data to perform optimisation calculations. The results are analysed, and the findings are discussed.

4. Data Collection

A questionnaire designed for data collection was sent to the company executives (operations manager and CEO). They were interviewed to ascertain product names, the finite quantity of resources available, and other production information. Also, the production cost, selling price and profit contribution per product unit during April, May and June 2023 were obtained. The data is organised to identify the decision variables and formulate a Linear Programming model for the problem. Table 1 summarises the company products and the accompanying inputs required to produce the products, while Table 4 enumerates the income from each of the four products.

Table 1. Production Cost (Naira)

Product / Inputs	Sliding window	Casement	Door	Projected
Aluminum sheet	21 430	32 030	21 630	13 760
Processing	1000	1000	1000	1000
Accessories	3550	4350	4100	3850
Water	300	300	300	300
Total	26 280	37 680	27 030	18910

Source: Production Cost Report: May 2023 for NAYO Aluminum

Table 2 . Daily Income

Product	Production Cost (Naira)	Selling Price (Naira)	Income (Naira)
Sliding window	26 280	28 000	1 720
Casement	37 680	41 680	4 000
Projected	18 910	22 900	3990
Door	27 030	30 000	2 970

Source: NAYO Aluminum Production Cost Report: May 2023

4.1 Objective function

The objective function is tailored to maximise the income based on the number of units of products produced, while $P_1, P_2, P_3,$ and P_4 are decision variables required to maximise revenue.
Maximise income: Max $Z = 1720P_1 + 4000P_2 + 3990P_3 + 2\,970P_4$
Where: Z = Maximum income
P_1= number of sliding windows
P_2 = number of casement windows
P_3 = number of projected windows
P_4 = number of aluminium doors

4.2 Constraint Function

The data required for this study was collected from NAYO Aluminium Company, Nigeria. The resource data consist of the total quantity of aluminium (primary raw material consumed) and machine time (cutting, drilling and milling machine) available for daily production of three window types (P_1 -Sliding window, P_2 -Casement, and P_2 - Projected) and one door (P_2). The first constraint, the consumption of aluminium material per window and door unit, is shown below.

Aluminium:
The total amount of aluminium available is 30 square meters.
Each unit of sliding window requires 1.8 square meters of aluminium.
Each unit of casement window requires 0.9 square meters of aluminium.
Each unit of projected window requires 1.0 square meters of aluminium.
Each door requires 2.0m^2 of aluminium.
The other constraint, Machine Time for cutting, drilling and milling, is depicted in Table 3.

Table 3. Machine time available

	Cutting	Drilling	Milling
Number of machines	4	1	3
Maximum operating hours	7	\`7	7
Total machine time available	28	7	21

Table 4. Total requirements

Requirements / Product	Aluminum (M^2)	Cutting Machine Time (Hours)	Drilling Machine Time (Hours)	Milling Machine Time (Hours)
Sliding window	1.8	0.5	0.7	0.2
Casement	0.9	0.8	0.7	0.5
Projected	1.0	0.3	0.2	0.2
Door	2.0	0.6	0.5	0.3
Resource Capacity	30	28	7	21

The cutting time needed to process the four products is captured in hours. We assume 1 day as 7 hours of machine time. The maximum time available is 28 hours of cutting machine time, 7 hours for drilling, and 21 hours of milling per day, as depicted in Table 4.

NAYO Linear Programming Model

The linear programming model is as follows:
Maximise income: Max Z = $1720P_1 + 4000P_2 + 3990P_3 + 2\,970P_4$
Subject to the following resource constraints
$1.8P_1 + 0.9\,P_2 + 1P_3 + 2P_4 \leq 30\text{m}^2$:Total aluminium available
$0.5P_1 + 0.8P_2 + 0.3P_3 + 0.6P_4 \leq 28$ hours: Total Cutting machine time
$0.7P_1 + 0.7P_2 + 0.2P_3 + 0.5P_4 \leq 7$ hours: Total drilling machine time
$0.2P_1 + 0.5P_2 + 0.2P_3 + 0.3P_4 \leq 21$ hours: Total milling machine time
Non-negativity constraint: $P_1, P_2, P_3, P_4 \geq 0$

5. Results and Discussion

The analysis includes (1) the number of windows and doors to produce using the available resources to maximise total income, (2) total costs of production, and (3) gross income. The analysis results from QM for Windows, after linear and integer programming, show an optimal production: 28 aluminium doors and 2 casements ($P_1 = 0; P_2 = 2; P_3 = 28, P_4 = 0$) to make the highest income of 119720.00 Naira.

Table 5. Illustration of the problem in QM for Windows.

Cj	Basic Variables	Quantity	1720P1	4000P2	3990P3	2970P4	0slack 1	0slack 2	0slack 3	0slack 4
Iteration 1										
0	slack 1	30	1.8	0.9	1	2	1	0	0	0
0	slack 2	28	0.5	0.8	0.3	0.6	0	1	0	0
0	slack 3	7	0.7	0.7	0.2	0.5	0	0	1	0
0	slack 4	21	0.2	0.5	0.2	0.3	0	0	0	1
	zj	0	0	0	0	0	0	0	0	0
	cj-zj		1,720	4,000	3,990	2,970	0	0	0	0
Iteration 2										
0	slack 1	21.0	0.9	0	0.7429	1.3571	1	0	-1.2857	0
0	slack 2	20.0	-0.3	0	0.0714	0.0286	0	1	-1.1429	0

4000	P2	10.0	1	1	0.2857	0.7143	0	0	1.4286	0
0	slack 4	16.0	-0.3	0	0.0571	-0.0571	0	0	-0.7143	1
	zj	40,000	4000	4000	1142.86	2857.14	0	0	5714.29	0
	cj-zj		-2,280	0	2,847.…	112.8571	0	0	-5,714	0
Iteration 3										
3990	P3	28.2692	1.2115	0	1	1.8269	1.3462	0	-1.7308	0
0	slack 2	17.9808	-0.3865	0	0	-0.1019	-0.0962	1	-1.0192	0
4000	P2	1.9231	0.6538	1	0	0.1923	-0.3846	0	1.9231	0
0	slack 4	14.3846	-0.3692	0	0	-0.1615	-0.0769	0	- 0.6154	1
	zj	120,48	7449.42	4000	3990	8058.65	3832.69	0	786.54	0
	cj-zj		-5,729	0	0	-5,088.…	-3,832	0	-786.5387	0

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Table 6. Linear Programing Results

Linear Programming Results

	X1	X2	X3	X4		RHS	Dual
Maximize	1720	4000	3990	2970			
Constraint 1	1.8	.9	1	2	<=	30	3832.69
Constraint 2	.5	.8	.3	.6	<=	28	0
Constraint 3	.7	.7	.2	.5	<=	7	786.54
Constraint 4	.2	.5	.2	.3	<=	21	0
Solution	0	1.92	28.27	0		120486.5	

Ranging

Variable	Value	Reduced Cost	Original Val	Lower Bound	Upper Bound
X1	0	5729.42	1720	-Infinity	7449.42
X2	1.92	0	4000	3591	13965
X3	28.27	0	3990	1204.63	4444.44
X4	0	5088.65	2970	-Infinity	8058.65
	Dual Value	Slack/Surplus	Original Val	Lower Bound	Upper Bound
Constraint 1	3832.69	0	30	9	35
Constraint 2	0	17.98	28	10.02	Infinity
Constraint 3	786.54	0	7	6	23.33
Constraint 4	0	14.38	21	6.62	Infinity

The findings in Table 6 indicate unused resource levels, which helps to manage input resource procurement and minimise waste. By reducing the excess resources (17.98 and 14.38 unused cutting and milling machine hours, respectively), NAYO can sustain the same production volume while reducing costs and increasing gross income. However, aluminium and drilling machine hours were exhausted; hence, acquiring additional units for a price less than the marginal value, within a feasible limit ($9 \leq q_1 \leq 35$ for aluminium; $6 \leq q_3 \leq 23.3$ for drilling machine hours) will increase their return.
the marginal value of one additional m^2 of aluminium Y_1 = #3832.69
the marginal value of one additional cutting machine hour $Y_2 = 0$
the marginal value of one additional drilling machine hour Y_3 = #786.54
the marginal value of one additional milling machine hour $Y_4 = 0$

5.2 NAYO Integer Programming (IP.)

Notably, products are sold as finished windows and doors; hence, fractional values are unacceptable as they will have no real contribution to profit. Therefore, the need to deploy integer programming to convert this relaxed optimal solution obtained from linear programming to an optimal all-integer solution will bring income.

Integer Programming (IP)

Integer programming (Schrijver 1986; Render et al. 2020) is of three major types:
1) Total Integer Model where all decision variables must have integer values.
2) 0-1 Integer Model where all decision variables must have zero or one integer value.
3) Mixed Integer Model: some but not all decision variables must be integer(s).
The integer model objective function and constraints can be illustrated as follows:

$$Z = \sum_{j=1}^{n} c_j p_j$$

Subject to constraints:

$$Z = \sum_{j=1}^{n} a_{ij} p_j = b_i \quad (i = 1,2, \dots, m)$$

$p_j \geq 0 \quad (j = 1,2, \dots, n)$ for a mixed integer

However, for the scenario under study, which requires an all-integer solution:

$p_j \geq 0 \; and \; integer \quad (j = 1,2, \dots, n)$

Module/sub model: Integer & Mixed Integer Programming Problem title: Production Optimisation @ NAYO Aluminum Company

Table 7. Integer Programming Iteration Results from QM Windows

Iteration	Level	Added constraint	Solution type	Solution Value	P1	P2	P3	P4
			Optimal	119720	0	2	28	0
1	0		Non-Integer	120486.5	0	1.92	28.27	0
2	1	P2<= 1	Non-Integer	120109	0	1	29.1	0
3	2	P3<= 29	Non-Integer	119858.5	0	1	29	0.05
4	3	P4<= 0	Non-Integer	119805.6	0.06	1	29	0
5	4	P1<= 0	Integer	119710	0	1	29	0
6	4	P1>= 1	Suboptimal	114647	1	1	27.3	0
7	3	P4>= 1	Suboptimal	115099	0	1	27.1	1
8	2	P3>= 30	Suboptimal	119700	0	0	30	0
9	1	P2>= 2	Integer	119720	0	2	28	0

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The analysis result using QM for Windows, after linear and integer programming, shows an optimal production: 28 aluminium doors and 2 casements ($P_1 = 0; P_2 = 2; P_3 = 28, P_4 = 0$) to make the highest income of 119720.00 Naira ($Max\ Z = 1720P_1 + 4000P_2 + 3990P_3 + 2970P_4$).

Limitation

This study operates on the current constraints and profit per product. It does not indicate how changes in the variables, for example, the product's selling price, can affect the LP algorithm. Further, it assumes that income is maximised based on already identified company constraints, costs and revenue estimates from recent months. However, the output from the algorithm shows the binding constraints (aluminium and drilling machine hours) that should be increased to boost the return and the excess resources (cutting and milling machine hours), which can be subsequently reduced to minimise cost.

6. Conclusion

QM for Windows was used to solve an LP problem modelled from the operations of an aluminium product fabrication company to determine how many units of each product must be produced given the current constraints to maximise income. The findings indicate unused resource levels, which helps to manage input resource procurement and minimise waste. Using LP and integer techniques, other production variables and constraints can be included in the model to generate an optimal solution in any given situation. This application can improve productivity, inventory management and company profitability.

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Biography

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