Modeling FDM Gear Printing: An Experimental Design-Based Mathematical Approach

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Abstract

Gears, critical technological components found in most machines, require effective optimization during their manufacturing. This study examines the production of gears through the additive manufacturing process, specifically the Fused Deposition Modeling (FDM) technique. The two responses under scrutiny are manufacturing time and the amount of material consumed. The primary objective is to develop a mathematical model that correlates these two responses with four specific parameters (layer thickness, number of shells, and infill density). To achieve this objective, an experimental design was devised, followed by a statistical study aimed at formulating an appropriate mathematical model. Additionally, the mathematical model has been validated through residual verification, enhancing its reliability and suitability to the experimental data.

Keywords
Gears, Additive Manufacturing; Fused Deposition Modeling; Parameters Optimization; Experimental plan design and Mathematical model.

1. Introduction
Polymer gears are gaining popularity in various industrial sectors, including the food and textile industries, due to a better recognition of their economic and technical advantages. Among these benefits are the ability to operate without the need for grease or oil lubrication, affordable production costs, low density, high resilience, and internal damping capability (Mao et al.2009). The recently adopted method for manufacturing polymer gears is additive manufacturing, a process that involves building components layer by layer until the entire design is realized (Gebhardt 2007).
The technique utilized in our study is Fused Deposition Modeling (FDM), chosen due to its widespread popularity, emphasizing the importance of optimizing this method. As depicted in Figure 1, this manufacturing process entails building a component by layering successive sheets with a thickness ranging from 0.08 to 3 mm. The material is heated to a temperature of 200°C and then extruded through a nozzle with a diameter ranging from 0.4 to 1.2 mm. Commonly used materials include PLA or ABS, with a diameter ranging from 1.75 to 3 mm. After completing each layer, the platform or extruder moves upward along the z-axis to initiate the printing of the next layer (Isksioui et al. 2020).

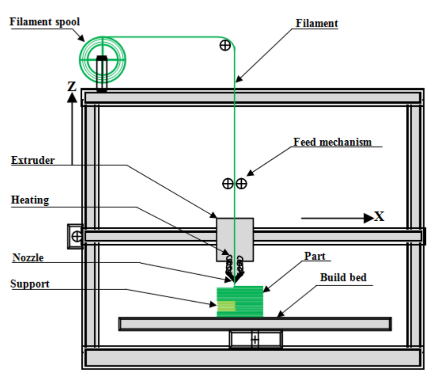

Figure 1. FDM manufacturing process.

1.1 Objectives
This study aims to model the FDM process for printing a gear, with the primary goal of determining a mathematical model that links specific printing parameters to two key responses: manufacturing time and material consumption. By developing this model, we seek to enhance our understanding of the intricate interactions between process parameters and the ultimate performance of the model, contributing to advancements in 3D printing knowledge.

2. Literature Review
The optimization of additive manufacturing parameters, particularly in the context of Fused Deposition Modeling (FDM), is a critical challenge to positively influence the characteristics of produced gears. Among the research studies addressing this issue, the work conducted by Muminovic et al. (2022) laid the groundwork, while Isksioui et al. (2020) focused on the specific optimization of FDM printing parameters for standardized samples, aiming to minimize both manufacturing time and material consumption.
Kumar and Regalla (2012) conducted a factorial experimental study to assess the impact of various parameters, including layer thickness, orientation, and frame angle, on manufacturing time and support material consumption. The study's findings suggest that both layer thickness and the specific orientation of the part during machining contribute to a reduction in manufacturing time.
Nancharaiah (2011) used the Taguchi design matrix (the orthogonal matrix L9) and ANOVA (analysis of variance) technique to examine the relationship between manufacturing time and FDM printing parameters. The study demonstrated that the parameters with the most influence on manufacturing time are the layer thickness and the nozzle-to-platform gap.

3. Methodology and gears specifications
The gear wheel's technical illustration utilized in our investigation was created using the Catia V5 design software. The detailed specifications of the gear wheel are itemized in Table 1. Ultimaker Cura software is employed to transform the drawing file of the designed component into STL format. An instance of a gear wheel produced through the FDM process is showcased in Figure 2.

Table 1. Specifications of gears.

Parameter	Value / type
Type	Spur gear
Module	3 mm
Number of teeth	18
Pressure angle	20°
Tooth and gear width	10 mm
Root fillet	0.75 mm

Figure 2. Photo of a gear wheel produced using the FDM process.

The selection of the four printing parameters, considered as factors in the experimental design, is based on a previous study (Isksioui 2020). Table 2 provides detailed information on these factors and their respective levels.

Table 2: Parameters and levels of varying Processing Parameters.

Symbols	Factors	Units	Levels	
			Low	High
A	Layer thickness	mm	0.12	0.24
B	Infill density	%	25	100
C	Infill pattern	--	Tri-hexagon	Lines
D	Number of shells	–	1	4

To optimize the number of experiments, an experimental plan was devised. Initially, a full experimental plan with four factors at two levels was implemented, totaling 16 experiments. During this first phase, it was observed that Factor C (Infill pattern) had no significant influence on the two responses under study.

To further refine the analysis while reducing the number of experiments, a new approach was adopted. A new experimental plan was designed, considering only three factors (A,B and D) at two levels, thereby reducing the total number of experiments to 8.

4. Data Collection

Table 3 presents the adopted design matrix for this new experimental plan. This matrix provides a solid foundation for a precise estimation of response variation based on the studied parameters. These successive adjustments in the design of experimental plans aim to maximize analysis efficiency while minimizing the number of required trials.

Table 3: Design matrix and collected data

RUN	Factors			Responses	
	A	B	D	**T** (mn)	**M** (g)
1	0.12	25	1	132	11
2	0.24	25	1	84	13
3	0.12	100	1	366	25
4	0.24	100	1	189	26
5	0.12	25	4	212	15
6	0.24	25	4	121	17
7	0.12	100	4	397	25
8	0.24	100	4	205	26

The selected response parameters to assess the variation of factors in the matrix include manufacturing time (T) and material consumption (M). From the results of 8 experiments, we derived a linear mathematical model that takes into account the interactions between the factors. The model development process was carried out using MATLAB software.

The linear regression model adopted for this study is generally described by Equation 1:

$$Y = \beta_0 + \sum_{i=1}^{k} \beta_i X_i + \sum_{i<j}^{k} \beta_{ij} X_i X_j + \varepsilon \qquad (1)$$

Based on the statistical results and Equation 1, the formulation of the linear model for this study regarding the manufacturing time (T) is given by the following equation 2:

$$T = 94.792\text{-}245.83A + 4.42B + 14.083D\text{-}13.167AB \qquad (2)$$

The predictive model demonstrated strong performance with a Root Mean Squared Error of 18.7, an impressive R-squared value of 0.989, and a robust Adjusted R-Squared of 0.973. Additionally, the low p-value of 0.00303 indicates the statistical significance of the model's predictors.

Equation 3 represents the linear model pertaining to material consumption (M):

$$M = 2.2222 + 19.444A + 0.21778B + 1.7778D\text{-}0.017778AB + 6.6216 \cdot 10^{-15} AD\text{-}0.017778BD \qquad (3)$$

The model exhibits excellent predictive accuracy, as evidenced by a Root Mean Squared Error of 1.78e-06. Furthermore, with a high R-squared value of 0.999 and an Adjusted R-Squared of 0.998, the model demonstrates a strong fit to the data. The p-value, standing at 1.98e-07, signifies the statistical significance of the model's coefficients.

5. Results and Discussion

In order to validate the previously described mathematical model, normal probability curves were plotted. The curves in Figure 3 (a and b) demonstrate that the residuals are aligned and follow a normal distribution. This affirms that the mathematical models represented by equations 2 and 3 fit well with the experimental results.

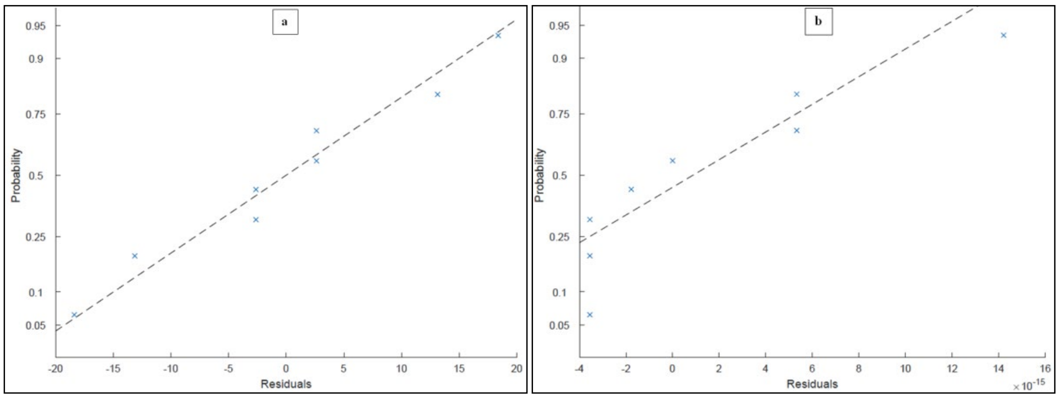

Figure 3. Normal probability curves for: a: T (manufacturing time); b: M (material consumption)

Figure 4 (a and b) represents another test for the chosen mathematical model. The comparison between experimental values and those predicted by the model demonstrates a good agreement. These results ensure the validation of the mathematical model to determine the relationship between the two responses (T and M) and the three studied parameters.

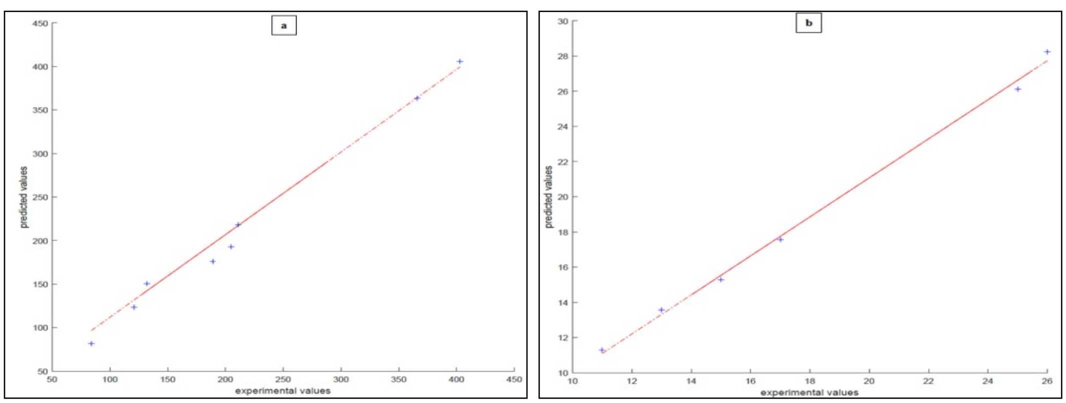

Figure 4: Model predictions and experimental results for: a: T (manufacturing time); b: M (material consumption)

6. Conclusion

The additive manufacturing of gears, particularly through the FDM process, requires careful optimization to compete with other manufacturing methods. The primary goal is to achieve a gear at minimal cost, involving the simultaneous reduction of both manufacturing time and material consumption, two criteria selected for this study. Crucial factors considered include process parameters such as layer thickness, infill density, and the number of contours. A mathematical model adapted to the experiment has been developed and validated to determine the relationship between these factors and the two response criteria. To further enhance this model, it is essential to broaden the range of parameters studied to fully capture the complex interactions among process variables.

Simultaneously, an in-depth optimization approach for printing the gear is undertaken. The objective is to determine optimal parameters that minimize both manufacturing time and material consumption. This phase involves systematic experiments to assess the impact of different parameter combinations on printing performance. Identifying ideal conditions, such as optimal layer thickness and the most efficient infill density, aims to maximize the efficiency of the FDM process for gear manufacturing by striking an optimal balance between quality, printing speed, and material savings.

Thorough analysis of the obtained results will validate and refine the existing mathematical model, while providing practical recommendations for optimal printing parameters in the production of gears through FDM.

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