Fair Assignment of Matchdays in the Turkish Super League under a Rest Mismatch Minimization Objective

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Abstract

Turkish Super League matchday assignments have been criticized heavily over the years by the stakeholders with arguments regarding economic losses and performance issues. With the objective to improve upon the current scheduling effort by the federation, we develop a multi-objective approach to achieve dynamically fair matchday assignments throughout season that also accounts for the rest time between consecutive matches. Given the seasonal fixture, we develop a nonlinear binary integer program that is solved optimally for each round feeding information on previous rounds. Our results offer promising improvements over the existing schedule generating better results when a greater degree of weight is given to seasonal matchday distribution fairness rather than obtaining ideal rest time.

Keywords
OR in sports, Sports scheduling, Matchday scheduling, Integer programming

1. Introduction

Applications of operations research on sports scheduling have gradually become more and more common over the last two decades. Researchers have demonstrated the utility of state-of-the-art linear and non-linear programming methodologies in arranging tournament fixtures of this huge enterprise. Arguably, the best measure of the popularity of these tournaments is the multi-billion-dollar worth of the top sports clubs and enormous payments made for broadcasting rights. It is largely due to this popularity that game schedules are critical elements of the entire organization. Unfortunately, determination of these schedules is no easy undertaking because it should strike a reasonable balance between the often-conflicting demands of the stakeholders in the presence of a wide variety of constraints. This is indeed where operations research has made some impact.

Broadly speaking, the problem we discuss in this article is the improvement of matchday schedules in soccer. This is a problem with two phases: the first phase is the determination of the seasonal fixture where we identify games to be played in each round of the tournament whereas the second phase is the assignment of each game to specific time slots. There is no easy solution because we need to satisfy a multitude of criteria in each phase of this problem. Among the criteria that have been commonly considered in previous work are carryover effect which is related to a potential performance reduction caused by a successive play with relatively strong opponents, breaks that occur when two home or away matches are scheduled consecutively, and the rest mismatches that are observed when the rest periods of participating teams vary. The main concerns that lead to the formulation of these criteria are somewhat related and cannot be accounted for independently from each other. However, fulfillment of the first two criteria is more often a subject of study in seasonal fixture determination whereas the last criterion is included in research on matchday assignment.

In this paper, we discuss the design of the top soccer tournament in Turkey with a fair matchday assignment. The governing body of Turkish soccer is the Turkish Football Federation (TFF). Turkish Super League (TSL) is the most prestigious tournament that regularly includes 18 teams. Although TFF cited the COVID-19 pandemic for the redesign of the league at the beginning of 2020-21 season to increase the participating teams to 21, the number of teams will gradually reduce to 18 again by the start of 2023-24 season. The procedure for arranging the seasonal schedule in TSL is as follows: TFF officials first prepare the seasonal fixture, therefore determine the matches in each round of the season. At this stage, there is also a tentative and rough time schedule for each round lacking the fine details on the matchdays. The reason for this is the need to coordinate the final arrangement for TSL with other tournaments organized by UEFA and FIFA (organizations that administrate soccer in Europe and the world, respectively) as well as the domestic Turkish Cup. The progress of teams in these tournaments imposes uncertainty to TSL scheduling in that the future results in a single-elimination format cannot be known a priori. Hence, TFF organizers must take the most recent information on teams in those tournaments into account and finalize the TSL matchday schedule dynamically. Each season in the 18-team format continues for 34 rounds. Games in each round are scheduled from Friday to Monday with very few exceptions in recent years of TSL scheduling with 18 teams.

To recap, the problem is to determine the day of each game in the entire TSL season. The objective is to minimize deviation from a fair distribution of gamedays among all the participating clubs. Since rest durations of the teams depend on the TSL schedule as well as several domestic and international tournaments, the solution procedure we propose should be dynamic in nature accounting for the latest information on the knockout stages of the other single elimination tournaments. This dependence introduces nonlinear constraints on the potential matchdays and consequently our formulation becomes a nonlinear binary integer programming problem.

In the remaining sections of this paper, we first summarize the sports scheduling literature, largely on soccer. The presentation of the problem formulation for each round of the season is provided in the third section. A comparative analysis of the schedule generated by our formulation against the TFF schedule is performed in Section 4. We do this using data from the 2018-2019 season and offer an extended discussion of the results as well. In Section 5, we state our concluding remarks.

2. Literature

Roughly speaking, there is a vast body of literature that goes back to five decades for constructing sports schedules. Our review is not intended to be comprehensive. Some attempts have been made, though, to provide an extensive coverage of past work on the problem (see Kim 2019, Rasmussen and Trick 2008, Ribeiro 2012, and Kendall et al. 2010). Most of this research has been centered around organizing round robin tournaments in which teams face each other a fixed number of times every season. TSL season is broken into two halves, where each half is a mirrored replica of the other in that home and away teams change roles. Accordingly, it is a double round robin tournament. For the most part, we set this section to cover round robin scheduling.

There is no single study that solves the problem with an objective that collectively accounts for all aforementioned concerns. It is possible to categorize these studies based on the various objectives that have been employed. For example, Atan and Çavdaroğlu (2018) and Çavdaroğlu and Atan (2020) minimize rest mismatches, Rasmussen and Trick (2007), Durán, Guajardo and Sauré (2017), Recalde, Torres and Vaca (2013) and Bulck and Goossens (2020) minimize breaks, Kendall and Westphal (2013) minimizes travel distances, Guedes and Ribeiro (2011), Januario and Urrutia (2016), and Günneç and Demir (2019) minimize carry over effects. Some other studies weigh these performance related objectives against more economic ambitions such as gate revenue and attendance maximization subject to constraints that arise from league organization in different countries (see Durán, Guajardo and Wolf-Yadlin 2012, Rasmussen 2008, Goossens and Spieksma 2009, Ribeiro and Urrutia 2012, Durán et al. 2007, Bartsch, Drexl and Kröger 2006).

The structure of the sports scheduling problems lend themselves to the use of linear programming formulations including many binary or integer valued decision variables. Therefore, most of the related literature presents state-of-the-art applications of integer programming that seek to achieve a delicate balance between various objectives as well as to address multiple concerns at the same time. Some examples are in order. Briskorn and Drexl (2009) offers a solution to scheduling of round robin tournaments in the presence of constraints on breaks, carryover effects as well as constraints that reflect team and organization preferences. A non-soccer application in the context of Argentina's professional basketball leagues by Durán et al. (2019) minimizes the total traveling distance which is certainly an important performance metric for such a congested tournament. An even greater degree of congestion where teams regularly have to play multiple times in a week is common for baseball leagues around the world. Thus, Kim (2019) analyzes the Korean baseball league with a similar travel distance metric as well as baseball specific match and attendance fairness metrics. With a break minimization objective in mind, Recalde, Torres and Vaca (2013) proposes a large-scale integer programming formulation for scheduling the professional soccer league in Ecuador. Both Kim (2019) and Recalde, Torres and Vaca (2013) develop heuristics as their formulations do not offer practical solutions. Other soccer related integer programming applications include Durán, Guajardo and Sauré (2017) that offers an alternative to the usual double round robin FIFA World Cup South American qualifiers schedule, which was well received by the tournament organizers, Guedes and Ribeiro (2011) that minimizes the weighted sum of the carry over effects and Günneç and Demir (2019) that presents a scheduling application in the Turkish Super League which minimizes the carry over effect while satisfying a maximum break constraint.

As indicated in the title of this paper, one objective of this study is the minimization of the rest mismatches among the participating teams in one season of a soccer tournament. Accordingly, we should highlight in this review two recent publications that investigate the tournament scheduling problem under this objective. In Atan and Çavdaroğlu (2018), the authors introduce integer programming models and a heuristic that uses the "circle method" for generating an initial solution which in later iterations can be updated to obtain a zero-mismatch schedule under certain instances. In a follow-up study, authors modify the objective of their previous work to minimization of the total rest differences throughout the season and they show that optimization of each round separately achieves this objective (see Çavdaroğlu and Atan 2020). This second paper has another common feature with our approach to the problem which is the round-by-round treatment of the entire season. The main difference with our study is the sole focus on performance related concerns in matchday determination whereas we also stress the impact of the economic dimension that brings the need for a balanced distribution of days of each round among the participating teams.

This paper is an extension to Göçgün and Bakır (2022). Authors provide a significant improvement in terms of the matchday distribution among the Turkish Super League teams when compared to the original schedule determined by the local federation. The current extension is to actively weigh in the benefits of a balanced schedule in point of rest durations. The specifics are provided in the next section where we describe our problem and its mathematical formulation.

3. Problem description and mathematical formulation

We consider the problem of scheduling Turkish Super League (TSL) fairly matches to days. The objective is to determine which day each game will be scheduled to, given a finalized fixture, taking into account performance metrics related to ideal matchday distribution as well as rest duration. A finalized fixture implies that which team will play against which team in each round is known.

TFF organizers have been considering the following restrictions for the abovementioned matchday scheduling problem:

- Matches should be scheduled on any of the following days for each round: Friday to Monday.
- Matchdays should be determined by accounting for the fact that certain teams have extra games in other tournaments such as the Champions League and Turkish Cup. The rest of the duration between two consecutive matches for each team should be at least two days. To illustrate, a team having an extra match on Wednesday is not allowed to have a Super League match in the same week on Friday.
- There is an ideal matchday distribution for any round. While TFF does not publicly announce what it is, we take that distribution to be 1 - 3 - 4 – 1 based on our rough-cut analysis. That is, ideally, there will be one match on Friday, three matches on Saturday, four matches on Sunday, and one match on Monday.

TFF organizers manually determine the matchday schedule for a particular round (i.e., usually called a week in Turkish) in advance. Determining the schedule manually for, say, round n, typically results in a match distribution that significantly deviates from the 1 - 3 - 4 - 1 distribution, especially when there are European Cup and Turkish Cup games between rounds n-1 and n, and/or rounds n and n+1. Further, in manual scheduling, the rest durations of teams also show variation; some teams are exposed to unfairness as they rest much less than other teams.

We model the aforementioned matchday scheduling problem using nonlinear binary integer programming. The details of our mathematical formulation are discussed next.

3.1 Mathematical Formulation

As indicated earlier, our mathematical model is an extension of the model described in Göçgün and Bakır (2022). The authors propose an integer programming formulation for a class of matchday scheduling problems faced by TFF, adapting the Monden heuristic. This method, which is widely applied in the field of mixed model assembly line scheduling, is used to minimize the variation of component parts in final part manufacturing (see Göçgün and Bakır 2022 for the details). Using the idea behind the Monden heuristic, their formulation aims to obtain a fair matchday distribution throughout the season and ``ideal assignment of games to different days in each round of the tournament".

We extend the mathematical model in Göçgün and Bakır (2022) by collectively minimizing the total deviation from ideal matchday distribution, and the total deviation from ideal rest durations. This model is solved optimally for each of the 34 rounds throughout the entire season. The components of our model are explained next.

3.1.1 Parameters

Parameters used in our study are defined in this section.

$$mt_{irj} = \begin{cases} 1, \text{if team i plays against team j in round r} \\ 0, \text{otherwise.} \end{cases} \tag{1}$$

Here i = 1, ..., I, j = 1, ..., I, $i \neq j$, and r = 1, ..., R. Note that I = 18 and R = 34. Note also that the value of this parameter is known, as the seasonal fixture is known in advance.

$$m_{o_{irk}} = \begin{cases} 1, \text{if team i has an extra game between} \\ \quad \text{rounds r} - 1 \text{ and r on day k} \\ 0, \text{otherwise} \end{cases} \tag{2}$$

The ranges of parameter subscripts here are i = 1, ..., I, r = 2, ..., R, k ∈ K = {Tue, Wed, Thu}. Another parameter is $d\_index_d \in \{1, 2, ..., 7\}$ where the days of the week are indexed in order from Friday to Thursday. Next, we define the rest duration parameters.

The first rest duration parameter is $id1_d$. It represents the ideal number of days a team that played a game in round r-1 on day d rests without a game. To illustrate, for d = Sat (Sun), $id1_{Sat}$ ($id1_{Sun}$) equals 8 (7) because the respective team intuitively desires to have rest as much as possible, implying that the ideal matchday for that team for round r is Monday. Parameter $id1_d$ is therefore expressed as

$$id1_d = \begin{cases} 9, & \text{if d = Fri,} \\ 8, & \text{if d = Sat,} \\ 7, & \text{if d = Sun,} \\ 6 + m_1, & \text{if d = Mon,} \end{cases} \tag{3}$$

where m_1 is a priority factor and $0 < m_1 < 1$. We explain the rationale for inclusion of this additional parameter later in this section.

The second rest duration parameter is $id2_d$. It represents the ideal rest duration of a team that has an extra game only between rounds r-1 and r and on day d. For example, if d = Tue, then the respective team desires to have a round r-game on Monday to have as many days of rest as possible. Parameter $id2_d$ is given by:

$$id2_d = \begin{cases} 5 + m_2, & \text{if d = Tue,} \\ 4 + m_3, & \text{if d = Wed,} \\ 3 + m_4, & \text{if d = Thu,} \end{cases} \tag{4}$$

where priority factors are $0 < m_i < 1$, i = 2, 3, 4.

The third ideal rest duration parameter is $id3_d$. It is expressed as

$$id3_d = \begin{cases} 3 + m_5, & \text{if d = Tue+,} \\ 4 + m_6, & \text{if d = Wed+,} \\ 5 + m_7, & \text{if d = Thu+,} \end{cases} \tag{5}$$

where $0 < m_i < 1$, $i = 5, 6, 7$, and priority factors satisfy $m_5 = m_4 > m_6 = m_3 > m_7 = m_2 > m_1$. Moreover, d ∈ {Tue+, Wed+, Thur+} indicates that the respective team will have an extra game only on day d between rounds r and r+1 (i.e., within a week following the week for which scheduling is performed), and the days of the midweek that appear in the index is accompanied by the "+" sign to highlight this situation. The fixed component of rest durations stated in the definition of $id3_d$ are determined after taking Friday as the ideal day of play in round r for any team that has a midweek schedule right afterwards.

The reason why we include the priority factors m_i, i = 1, ..., 7 in specifying ideal rest durations can be explained as follows. One of the constraints of our model states that at most, say, 2 matches can be assigned to Monday. Suppose that only team 1 and team 10 have both additional matches between rounds r-1 and r on Thursday, and hence will be better off if they play on Monday in round r. This implies that we should prioritize them over other teams for whom the ideal day is Monday, too. Otherwise, the model will randomly choose two teams for assigning Monday matches. Also, it is reasonable to dictate that the priority factor m_4 for a team that previously has a Thursday game is higher than the priority factor of any other team in a comparable situation; similarly the priority factor for Wednesday, m_3, should be higher than m_2, the priority factor for Tuesday. For the Monday slots, the model should also prioritize a team having a game on Monday in round r-1 and having no additional match over teams having a game on any of Friday, Saturday, and Sunday and having no additional games. Further, a team having an additional game on any of Tuesday, Wednesday, and Thursday should be prioritized for the Monday slots over a team that had a Monday game in round r-1 and does not have any additional game. Hence, for example, $m_2 > m_1$ must hold.

The last ideal rest duration parameter is $id4_{d,d2}$. It represents the ideal rest duration of a team that has an extra game on day d between rounds r-1 and r and has an extra game on day d2 between rounds r and r+1. The value of this term, $id4_{d,d2}$, is determined by

$$id4_{d,d2} = \left\lfloor \frac{d_{index_d} - d_{index_{d2}} + 6}{2} \right\rfloor, \tag{6}$$

where $d \in \{Tue,\ Wed,\ Thur\}$, $d2 \in \{Tue+,\ Wed+,\ Thur+\}$. Determining the ideal rest durations is critical to finding the optimal balance between objectives of scheduling games with fair duration of rest and assignment of games to matchdays based on economic fairness. Four alternative cases arise as we have described in definitions of parameters $id1_d$, $id2_d$, $id3_d$, and $id4_{d,d2}$. To simplify the notation, we denote the ideal rest duration for every team $\mathrm{I}, \mathrm{i} = 1, \dots, \mathrm{I}$ by z_i which is defined as follows:

$$z_i = \begin{cases} id1_d, \text{ d } \in \text{D } = \{\text{F, S, Su, M}\}, \text{if team I played a game in round r} - 1 \text{ on day d and it does not have an extra game,} \\ id2_d, \text{ d } \in \text{K } = \{\text{T, W, Th}\}, \text{if team I has an extra game only between rounds r} - 1 \text{ and r and on day d,} \\ id3_d, \text{ d } \in \text{D1 } = \{\text{T+, W+, Th+}\}, \text{ if team I has an extra game only between rounds r and r} + 1 \text{ and on day d,} \\ id4_{d,d2}, \text{ d } \in \text{D1 } = \{\text{T+, W+, Th+}\}, \text{ if team I has an extra game between rounds r} - 1, \text{ and r on day d and between rounds r and r} + 1 \text{ on day } \text{d}_2, \end{cases} \tag{7}$$

where F, S, Su, and M represent Friday, Saturday, Sunday, and Monday, respectively.

Another set of parameters commonly used in our problem formulation include the objective weights, which are defined below:

w_d: weight assigned to deviation D_d, which is a metric that measures a "matchday count deviation from an ideal and fair distribution" in a given round,

c_1: weight assigned to the summation of the weighted deviations,

c_2: weight assigned to deviation V, which is a metric that measures the deviation from an ideal distribution for each round of play,

w_1: weight for the matchday distribution component of the objective function,

w_2: weight for the rest duration component of the objective function.

The second component of the objective function is constructed to reflect the importance of fair matchday assignment. Fairness is judged first according to the total number of times each team plays on Fridays, Saturdays, Sundays and Mondays throughout the season. Like we had an ideal rest duration for each team, there is also an ideal distribution of matchdays up to a certain round of the season which is calculated using the parameter a_d:

$$a_d = \begin{cases} \dfrac{s^*}{34}, & \text{if d = Fri,} \\ \dfrac{t^*}{34}, & \text{if d = Sat,} \\ \dfrac{u^*}{34}, & \text{if d = Sun,} \\ \dfrac{v^*}{34}, & \text{if d = Mon,} \end{cases} \tag{8}$$

where, the numerators (s^* to v^*) of fractions listed in each case count the number of games that should ideally be played on the associated matchday in the entire season. Since our tournament includes 34 rounds, $s^* + t^* + u^* + v^* = 34$. In the numerical analyses presented in Section 4, we take $(s^*, t^*, u^*, v^*) = (5, 12, 12, 5)$. We also have a round specific objective to schedule a certain number of games from Friday to Monday based on broadcasting company preferences more so than team preferences. The ideal count of scheduled matches in any round is denoted by b_d where

$$b_d = \begin{cases} e^*, & \text{if d = Fri,} \\ f^*, & \text{if d = Sat,} \\ g^*, & \text{if d = Sun,} \\ h^*, & \text{if d = Mon,} \end{cases} \tag{9}$$

where $e^* - f^* - g^* - h^*$ are parameters symbolizing the ideal number of games played in a given round from Friday to Monday. As mentioned earlier, we use the distribution $e^* - f^* - g^* - h^* = 1 - 3 - 4 - 1$ in our numerical study.

3.1.2 Variables and Problem Formulation

The decision variable of the mathematical model that we present for round r is $m_{d_{irjd}}$. We define it below:

$$m_{d_{irjd}} = \begin{cases} 1, & \text{if team i plays against team j in round r on day d} \\ 0, & \text{otherwise.} \end{cases} \tag{10}$$

Here $i = 1 \ldots, I$; $j = 1 \ldots, I$; $i \neq j$; $d \in D = \{\text{Fri}, \text{Sat}, \text{Sun}, \text{Mon}\}$. The mathematical problem is solved for match assignments on a round-by-round basis. Match assignments up to a certain round r are needed to calculate the seasonal matchday deviation metric in the objective function of the model developed for round r. Accordingly, $m_{d_{ixjd}}$, $x = 1, \cdots, r-1$ are parameters for the problem formulated for round r. As a function of the decision variables and model parameters, we also calculate the rest duration $rest_{ir}$ of team i between rounds $\mathrm{r}-1$ and r, $\mathrm{i}=1,\ldots,\mathrm{I}$; $\mathrm{r}=2,\ldots,\mathrm{R}$ based on two conditions as follows. First, if $m_{o_{i(r+1)k}} = 0 \ \forall \mathrm{k}, \mathrm{k} \in \mathrm{K}$:

$$
\begin{aligned}
rest_{ir} = &\left(1 - \sum_{k\in K} m_{o_{irk}}\right) \\
&\times \left(\sum_{j=1, i\neq j}^{I} \sum_{d\in D} m_{d_{irjd}} \times mt_{irj} \times d_{index_d}\right. \\
&\left. - \sum_{j=1, i\neq j}^{I} \sum_{d\in D} m_{d_{i(r-1)jd}} \times mt_{i(r-1)j} \times d_{index_d} + 6\right) \\
&+ \left(\sum_{k\in K} m_{o_{irk}}\right) \times \left(\sum_{j=1, i\neq j}^{I} \sum_{d\in D} m_{d_{irjd}} \times mt_{irj} \times d_{index_d} - \sum_{k\in K} m_{o_{irk}} \times d_{index_k} + 6\right)
\end{aligned}
\tag{11}
$$

Second, if $\exists \mathrm{k}$ such that $m_{o_{i(r+1)k}} = 1$:

$$
rest_{ir} = \sum_{k\in K} m_{o_{i(r+1)k}} \times d_{index_k} - \sum_{j=1, i\neq j}^{I} \sum_{d\in D} m_{d_{irjd}} \times mt_{irj} \times d_{index_d} - 1. \tag{12}
$$

To illustrate, if team i played a game on Saturday in round $\mathrm{r}-1$, is scheduled to have a round-r game on Sunday, and does not have any extra games, then $\mathrm{rest}_{\mathrm{ir}}$ turns out to be 7 (3 - 2 + 6). As another illustration, if team i played a game on Saturday in round $\mathrm{r}-1$, is scheduled to have a round-r game on Sunday and has an extra game between rounds $\mathrm{r}-1$ and r on Wednesday, then $rest_{ir}$ turns out to be 3 (3 - 6 + 6).

Two metrics are used for penalizing deviations from ideal matchday assignments: D_d and V. D_d measures a seasonal matchday count deviation from an ideal up to a certain round r. In order to calculate this deviation, the difference of the total number of previous round assignments and the ideal count up to any given round is computed. The second metric V measures the deviation for each round of the season.

$$
D_d = (\sum_{i=1}^{I} \sum_{j=1, i\neq j}^{I} m\_d_{irjd} + \sum_{k=1}^{r-1} m\_d_{i(r-1)jd} - r \times a_d)^2. \tag{13}
$$

$$
V = \sum_{d\in DS} \left(\frac{\sum_{i=1}^{I} \sum_{j=1, i\neq j}^{I} m_{d_{irjd}}}{2} - b_d\right)^2 \tag{14}
$$

The entire problem formulation for any round r of the season is presented below. This problem is solved for each round given the realizations of the previous rounds 0 to $\mathrm{r}-1$ and other tournaments up to that point.

$$
\min_{m_{d_{irjd}} \in \{0,1\}} w_1 \times \left(c_1 \times \sum_{d\in DS} w_d \times D_d + c_2 \times V\right) + w_2 \times (\sum_{i=1}^{I} \sum_{j=1, i\neq j}^{I} mt_{irj} \times (rest_{ir} - z_i)^2) \tag{15}
$$

subject to:

$$m_{d_{i,r,j,d}} = m_{d_{j,r,i,d}}, i = 1, \dots, I, j = 1, \dots, I, i \neq j, d = 1, \dots, D, \quad (16)$$

$$m_{d_{i,r,j,Fri}} + m_{o_{i,r,Thur}} \leq 1, i = 1, \dots, I, j = 1, \dots, I, i \neq j, \quad (17)$$

$$m_{d_{i,r,j,Fri}} + m_{o_{i,r,Wed}} \leq 1, i = 1, \dots, I, j = 1, \dots, I, i \neq j, \quad (18)$$

$$m_{d_{i,r,j,Sat}} + m_{o_{i,r,Thur}} \leq 1, i = 1, \dots, I, j = 1, \dots, I, i \neq j, \quad (19)$$

$$m_{d_{i,r,j,Sun}} + m_{o_{i,(r+1),Tue}} \leq 1, i = 1, \dots, I, j = 1, \dots, I, i \neq j, \quad (20)$$

$$m_{d_{i,r,j,Mon}} + m_{o_{i,(r+1),Tue}} \leq 1, i = 1, \dots, I, j = 1, \dots, I, i \neq j, \quad (21)$$

$$m_{d_{i,r,j,Mon}} + m_{o_{i,(r+1),Wed}} \leq 1, i = 1, \dots, I, j = 1, \dots, I, i \neq j, \quad (22)$$

$$\sum_{i=1}^{I} \sum_{j=1, j\neq i}^{I} m_{d_{i,r,j,Sat}} \geq 4 \quad (23)$$

$$\sum_{i=1}^{I} \sum_{j=1, j\neq i}^{I} m_{d_{i,r,j,Sun}} \geq 4 \quad (24)$$

$$\sum_{i=1}^{I} \sum_{j=1, j\neq i}^{I} m_{d_{i,r,j,Mon}} \leq 4 \quad (25)$$

$$\sum_{i=1}^{I} \sum_{j=1, j\neq i}^{I} m_{d_{i,r,j,Fri}} \leq 4 \quad (26)$$

$$\sum_{i=1}^{I} \sum_{j=1, j\neq i}^{I} m_{d_{i,r,j,Sat}} \leq 8 \quad (27)$$

$$\sum_{i=1}^{I} \sum_{j=1, j\neq i}^{I} m_{d_{i,r,j,Sun}} \leq 8 \quad (28)$$

$$m_{d_{i,r,j,d}} \epsilon \{0, 1\}, i = 1, \dots, I, j = 1, \dots, I, r = 1, \dots, R, \qquad d = 1, \dots, D,$$

An explanation of the use of constraints is in order. Constraint (16) ensures that the match assignment is made correctly (i.e., two teams that are scheduled to meet in round r are assigned to the same day of play). If a team has an extra game on either Wednesday or Thursday (between rounds r-1 and r), then it cannot have a game on Friday. Constraints (17) and (18) are imposed to model these restrictions. The following constraint labeled (19) states that if a team has an extra game on Thursday (between rounds r-1 and r), then it cannot have a game on Saturday. Constraints (20) to (22) are added for similar reasons that arise because of extra games that may be scheduled between rounds r and r+1. For instance, Constraints (20) and (21) state that if a team has an extra game on Tuesday (between rounds r and r+1), then it cannot have a game either on Sunday or on Monday. Constraint (22) is there to avoid any Monday schedule for teams that have an extra game next Wednesday (between rounds r and r+1). The remaining constraints from (23) to (28) express lower and upper bounds on the number of matches to be scheduled Friday to Monday in any round. In summary, the maximum number of games on Monday or Friday cannot exceed two whereas the lower and the upper bounds on Saturday and Sunday are two and four, respectively.

4. Results

We compare the performance of the optimal schedule for each specific case against the manual schedule constructed by TFF. Those cases correspond to sensitivity analysis scenarios where we aim to investigate how different values of weights affect the solution. We symbolize each scenario with a pair (i, j) where i is the value of w_1 and j is the value of w_2. The following scenarios are considered: (1, 1), (1, 0.5), (1, 0.25), (1, 0), and (0, 1). The respective optimal schedules obtained by AMPL, CPLEX are named as OPT-1, …, OPT-5. Further, in line with the previous literature, c_1 and c_2 are set to 1; w_{Fri}, w_{Sat}, w_{Sun}, and w_{Mon} are set to 0.11, 0.33, 0.44, and 0.11, respectively.

With regard to rest and matchday deviations for each round, we first compare the manual (TFF) schedule against *OPT-1*. As results reported in Table 1 indicate, *OPT-1* significantly outperforms the TFF schedule in total rest deviations and performs better in total matchday deviations. For all practical reasons, no performance evaluation is made for the very first round of the season. In Round 18, the rest deviation metric value is immaterial because of the relatively long break planned halfway through the tournament right before this round. This leaves 32 rounds for our comparative analysis from the perspective of rest deviations. In 28 of these rounds, *OPT-1* generates matchday assignments that bring more rest time to the participants (as demonstrated by lower optimal values in the rest deviation metric). In most of these rounds, differences suggest very significant improvements for the rest duration.

Table 1. The values of rest deviation and day deviation for each round that is obtained by the TFF and *OPT-1* optimal schedules.

Round #	Rest deviation TFF schedule	Rest deviation *OPT-1* schedule	Day deviation TFF schedule	Day deviation *OPT-1* schedule
1				
2	50.4	32.1	7.9	7.5
3	48	32.1	11.4	6.6
4	58.1	58.1	15.6	5.9
5	41	29.2	19	9.1
6	54.4	26.5	25.8	11.4
7	33.3	23.3	29.5	15.4
8	58.7	58.7	29.7	14.6
9	41.1	23.2	35.1	20.9
10	25.2	25.4	31.4	19.9
11	33.7	23.7	25.6	25.6
12	58.6	58.5	28.7	23.6
13	41.1	23.2	36.6	32.1
14	54.2	24.3	46.3	28
15	34.8	25	47.4	31.2
16	50.9	32.5	49.6	35.5
17	33.1	33.1	53.8	36.3
18	0	0	60.6	35.5
19	37.5	30.1	72.2	41.7
20	37.6	29.6	78.7	43.9
21	47.3	31.7	71.4	44.8
22	49.1	31.1	64.7	45.7
23	33.4	21.5	63.9	47.9
24	48.3	32.5	65.2	49.1
25	50.1	32.1	59.5	52.1
26	64.1	58.1	58.1	51.5

27	50.3	32.1	54.4	57
28	50.1	32.1	52	62.5
29	47.2	23.3	53.1	67.8
30	48.2	32.4	53.1	70.5
31	48.1	32.1	54.3	77.8
32	44.3	26.3	55	84.3
33	42.2	32.2	57.2	88.4
34	88.2	58.1	75.4	89.9
Total	1502	1064	1542	1334

As far as the matchday deviations are concerned, a similar picture emerges in Table 1 except several final rounds of the season. As indicated previously, matchday deviations are calculated from the ideal (5, 12, 12, 5) seasonal distribution.

The values of the total rest deviation and matchday deviation for each scenario are given in Table 2. *OPT-4* and *OPT-5* present the two extremes where we minimize only the matchday deviation metric or only the rest deviation metric, respectively. Intuitively minimum for those metrics are obtained in *OPT-4* for the seasonal matchday distribution and in *OPT-5* for the rest deviation. *OPT-3*, which prioritizes the minimization of matchday deviation over the minimization of rest deviation, yields the minimal total deviation. It is also worth noting that *OPT-4* and *OPT-5* result in almost the same objective function value. Our formulation of the matchday determination problem generates solutions under all scenarios from *OPT-1* to *OPT-5* that significantly outperform the TFF solution in terms of total deviation.

Table 2. The values of total rest deviation and total day deviation.

Solution	Rest deviation	Day deviation	Total
TFF	1502	1542	3044
OPT-1	1064	1334	2398
OPT-2	1100	1234	2334
OPT-3	1236	826	2062
OPT-4	1577	486	2063
OPT-5	1056	3018	2063

5. Conclusions

Fair assignment of matchdays throughout the soccer season in Turkey has been an active point of debate for more than a decade. Participating teams of the top Turkish soccer league have expressed their demand for an improved matchday schedule in each round of the season raising economic as well as performance concerns caused by the current match assignments. This paper seeks to offer a solution by addressing both concerns. The proxy used to measure the economic balance between the teams is deviation from an ideal seasonal matchday distribution that is equally applied to all participants of the league. Regarding performance, we use a proxy metric that is employed to compute the deviation for each team from some ideal rest time since the last match. Round-by-round minimization of the weighted average of those metrics in a dynamic sequence of calculations by making use of all historic matchday schedule information is the method we propose in this study.

The results indicate that the dynamic round-by-round solution approach improves upon the TFF schedule. The best outcome from our analysis is obtained when seasonal matchday distribution metric gets a higher weight in minimization of the deviations. This suggests that maintaining a balance in terms of rest time at the expense of fair assignment of matchdays may challenge efforts to succeed in the latter objective. The trade-off between the two objectives measured similarly by their associated deviation metrics should be well communicated to the stakeholders to reduce potential objections.

We also recognize that further refinements could be made to our solution approach in the future to enhance its ability to generate better schedules. Instead of a strict round-by-round solution approach, one may also solve for multiple rounds if information flows in a way to remove uncertainty for several rounds. This should bring not only fairer schedules, but also offer better visibility to the participating teams. Another extension could be the determination of the entire fixture under the set of objectives. Although fixture determination problem has been studied extensively in the literature, matchday determination has been disregarded. We believe future work should fill this gap as well.

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