Impact of Base Beam Length on Piezoelectric Cantilever Energy Harvesters: A Numerical Investigation

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Abstract

The cantilever beam serves as a prominent choice for harnessing strain-based energy through piezoelectric materials. Ongoing research is focused on enhancing the output power density from cantilever energy harvesters. Most of the prior investigations have predominantly employed fully coated cantilever beams. However, due to the high cost associated with piezoelectric materials, it becomes necessary to explore the impact of altering the base beam's length while keeping the piezoelectric material's length constant. This investigation aims to ascertain the potential for increasing the output power density using the same length of piezoelectric material. In this study, a comprehensive analysis was conducted, maintaining uniform thickness and width for both the base beam and piezoelectric material. The positioning of the piezoelectric material on the base beam was selected to maximize power output for a specific configuration. Recognizing that the power output of a cantilever energy harvester is dependent upon the resonant frequency and optimal electrical load, this study also presents an analysis of these factors. It is found that an increase in the length of the base beam causes a corresponding rise in output power. Particularly, tripling the length of the base beam yields a sevenfold increase in output power density.

Keywords
Energy Harvesting, Cantilever Beam, Piezoelectricity.

1. Introduction
In the search for sustainable energy sources and the reduction of dependence on traditional power generation methods, the field of energy harvesting has attracted attention. Energy harvesting involves converting ambient energy (e.g., mechanical vibrations, temperature differences, and sunlight) into usable electrical energy. Among the various techniques, piezoelectric energy harvesting has drawn considerable attention as a potential approach for converting ambient energy into usable electrical power. It utilizes the piezoelectric effect to convert mechanical vibrations into electrical energy. The piezoelectric effect refers to the phenomenon where piezoelectric materials produce an electric charge when the material is subjected to deflection. On the other hand, Cantilever beams have emerged as a preferred choice of vibration energy harvesting due to their higher average strain compared to other configurations (Roundy, Wright and Rabaey 2003).

The output power density in piezoelectric energy harvesting is greatly influenced by the beam's geometry. Research has shown that trapezoidal beam geometry offers superior performance over rectangular geometry (Zhang *et al.*, 2017). A comprehensive analysis by Pradeesh and Udhayakumar (2019b) studied the impact of taper in different cantilever beam geometries, including rectangular, trapezoidal, and inverted trapezoidal shapes. An innovative T-shaped piezoelectric energy harvester design was proposed by Alameh et al. (2018) aiming to enhance the overall performance of the system. Sunithamani et al. (2017) investigated unimorph piezoelectric energy harvesters by varying the shape of the proof mass. Their findings demonstrated that a disc-shaped proof mass outperformed a ring-shaped proof mass. Salem et al. (2021) delved into the relationship between resonance frequency, output power, and working range by varying the length of piezoelectric material segments. Wang et al. (2020) analyzed the impact of elastic model ratios and thickness ratios of piezoelectric and base beam layers on the performance of unimorph cantilever energy harvesters.

Pradeesh and Udhayakumar (2019a) explored the positioning of piezoelectric materials along a cantilever beam and the effect of proof mass shape on output power. Their study revealed that positioning the piezoelectric material at the beam's fixed end results in optimal output power. Tang and Wang's research (2017) explored how proof mass size affects energy harvester performance, revealing that even minor changes in proof mass geometry impact resonant frequency, strain distribution, and subsequently, output power. In a study by Zhou et al. (2018), the performance of a piezoelectric beam energy harvester was investigated through variations in the length of the piezoelectric material along the beam. The study demonstrated that optimizing the length of the piezoelectric layer can lead to improved energy harvesting performance.

Izadgoshasb et al. (2018) enhanced the efficiency of the piezoelectric cantilever energy harvester through the optimization of its orientation, utilizing vibrations derived from human motion. Gao et al. (2013) investigated the utilization of a piezoelectric cantilever with a cylindrical extension for energy harvesting, utilizing vortex-induced vibration as the source of vibration.

Though a good number of researches have been conducted on optimizing the geometry of cantilever energy harvesters, it was found that a detailed analysis of the variation of the base beam length is still necessary. The main focus of the current study is to explore the impact of varying the length of the base beam. In order to reduce the cost of material, a smaller length of piezoelectric material is chosen, while keeping its length constant. The thickness and width of both the base beam and piezoelectric material, as well as the proof mass are chosen to be constant for all geometries. To establish a fair basis for comparison, this study measures the output as power density, indicating the power output in relation to the volume of piezoelectric material. Aside from examining the power density output of the beams, the study also explored how the resonant frequency and optimal electric load change with different lengths of base beam. This is because the maximum power density output of piezoelectric cantilever energy harvesters is achieved at their resonant frequency and optimal electric load.

2. Governing Equations

Within a piezoelectric, there is a coupling between the strain and the electric field, which is determined by the constitutive relation.

$$\sigma = C_E\varepsilon - e^T E \tag{3.1}$$

$$D = e\varepsilon + \varepsilon_0\varepsilon_{rS}E \tag{3.2}$$

C_E, e, and ε_{rS} are the material properties. For the solution of piezoelectric problem equations of solid mechanics and electrostatic have to be solved along with equation (3.1) and equation (3.2).

Constitutive relationship of electric displacement

$$D = \varepsilon_0 E + P \tag{3.3}$$

Here, ε_0 is the permittivity of vacuum; P is the electric polarization vector.
Now Charge density can be related by

$$\rho_v = \nabla . D \tag{3.4}$$

The equilibrium equation for solid mechanics,

$$\rho\frac{\partial^2 u}{\partial t^2} = \nabla_X P^T + F_V e^{i\varphi} \tag{3.5}$$

Here, F_V is a body force with components and P is the first Piola-Kirchhoff stress tensor.

3. Design and Boundary Conditions

The analysis was conducted entirely using COMSOL Multiphysics, employing solid mechanics, electrostatics, and electric circuit physics. As illustrated in Figure 1, resembling a cantilever beam, one end of the beam was held fixed, while the opposite end remained free. Piezoelectric material (PZT 5A) coating of 0.5 mm thickness was provided at the beam's fixed end as placing the piezoelectric material at the fixed end ensures maximum power output when the beam is usually subjected to partial coating (Pradeesh and Udhayakumar, 2019a). The width of both the piezoelectric material and the base beam remained constant at 10 mm. A proof mass of 0.17 gm was attached to the free end. To ensure comparability, each beam was investigated with its respective optimal electric load, considering that piezoelectric energy harvester performance is load-dependent and optimized load varies with geometry changes. The analysis began with determining the eigenfrequency of each beam to identify their mode-1 resonant frequency. Later on, a frequency domain study was performed. This involved subjecting each beam to various frequencies to find out how the output power density changes around the resonant frequency. The measured energy harvester output was quantified as power density, representing the output power (μW) per unit volume of piezoelectric material (mm^3).

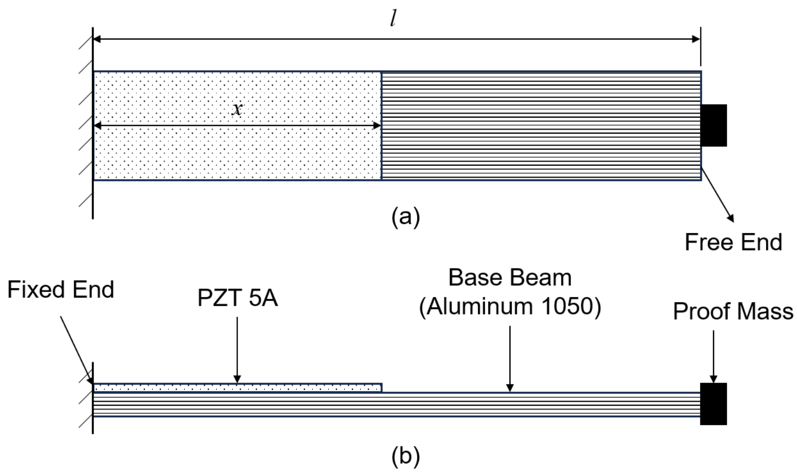

Figure 1. Cantilever beam energy harvester. (a) top view, (b) Side view

Figure 1 shows the configuration of the energy harvester, wherein *'l'* signifies the length of the base beam, and *'x'* denotes the length of the piezoelectric material.

4. Validation of the Model

The model validation involves the use of a rectangular piezoelectric cantilever energy harvester with dimensions of 100 mm in length, 10 mm in width, and 1 mm in thickness. A piece of piezoelectric material, 10 mm in length and width and 0.5 mm in thickness, is attached to the fixed end of the beam. The frequency domain analysis is conducted by varying the frequency within the range of 70 Hz to 110 Hz. The study reveals a peak output of 0.37 mW at a frequency of 94.3 Hz. A comparison with the findings of Pradesh and Udhayakumar (2019a) from published literature demonstrates an agreement with the result of the work.

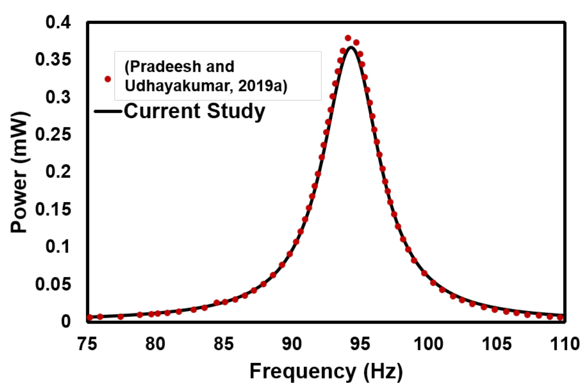

Figure 2. The fluctuation of output power in mW with frequency for the model validation of the computational procedure.

5. Results and Discussion

To investigate the impact of the base beam length on output power density, the piezoelectric material's length was maintained at a constant value of (*'x'*=) 50 mm. Meanwhile, the length of the base beam was adjusted in relation to the ratio *'l/x,'* ranging from 1 to 3, with increments of 0.25. Thus, the study began with the base beam length same as the piezoelectric material length of 50 mm, and this length gradually increased up to 150 mm as the ratio reached 3. Figures 3 and 4 indicate a direct linear relation between the base beam length and the output power density. The power density increased from 1.58 $\mu W/mm^3$ at 340 Hz for a ratio of 1 to 11.44 $\mu W/mm^3$ at 50.3 Hz for a ratio of 3. This shows that a threefold increase in the base beam length corresponds to approximately a sevenfold increase in power density.

As the piezoelectric crystal is located at the beam's fixed end, the deflection pattern of the fixed end plays a vital role in the output from the energy harvester. The deflection pattern at this fixed end significantly influences the energy output. A greater deflection at the fixed end corresponds to a higher amount of strain energy. Consequently, this results in an increased energy output. When the base beam's length is extended while keeping the piezoelectric material's length constant, it leads to an increase in bending moment near the fixed end of the beam. This effect arises due to the elongation of the base beam, which in turn increases the applied load and shifts the beam's center of mass away from the fixed end. Since the bending moment is directly proportional to both the load's magnitude and the distance from it, an increase in the bending moment becomes obvious. The amplified bending moment results in greater deflection of the piezoelectric material situated at the fixed end. This deflection generates higher levels of strain energy accumulation within the piezoelectric material, consequently yielding a higher density of output power.

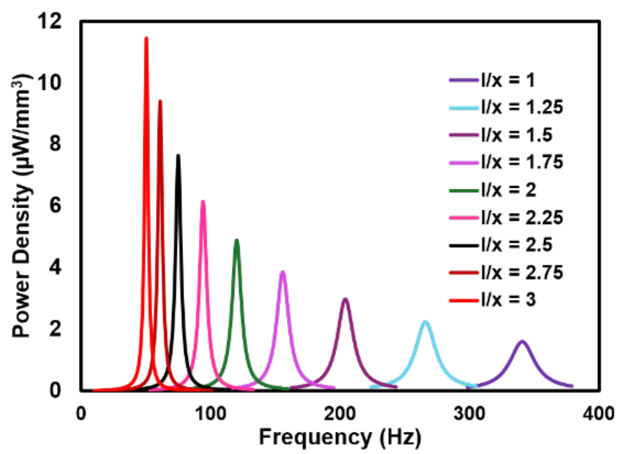

Figure 3. Variation of output power density with frequency for different *'l/x'* ratios.

This finding holds great importance in the designing of energy harvesters employing cantilever beams. The outcomes indicate that by elongating the base beam, one can achieve greater power output density using an equivalent volume of piezoelectric material. This result has the potential to significantly reduce the expenses of energy harvesting. While the expense of the energy harvester would rise with the elongation of the base beam, it's important to note that aluminum, mostly used as a base beam, in comparison to the cost of piezoelectric material, remains significantly cheaper.

However, Fig. 3 highlights a crucial observation, while increases in the ratios lead to an increase in output power density, there is a trade-off with a reduction in bandwidth. At higher ratios, the output power remains high primarily at the resonant frequency. Even a slight deviation from this frequency results in a sharp decrease in output power density, nearly approaching zero. On the contrary, lower ratios display more widely spread bell-shaped curves, signifying their capability to sustain comparatively high output power density across a relatively broader range of frequencies.

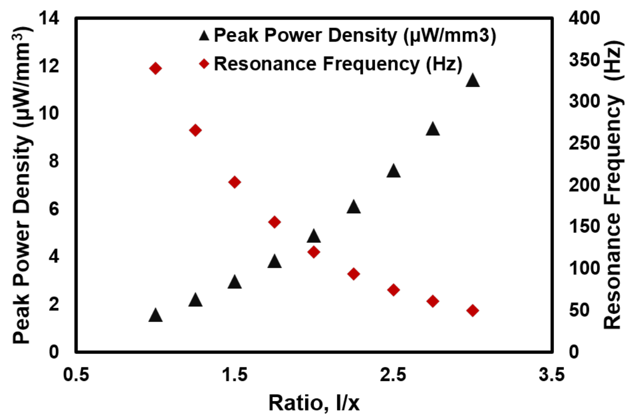

Figure 4. Maximum output power density for different *'l/x'* ratios corresponding to their resonant frequencies.

Figures 3 and 4 illustrate a trend that the resonant frequency of the energy harvester tends to decrease as the *'l/x'* ratio increases. In the context of cantilever beams, the resonant frequency is dependent on the beam's stiffness. As the stiffness of the beam grows, so does its resonant frequency. However, an increase in the *'l/x'* ratio contributes to a higher overall weight of the energy harvester. This, in turn, results in a greater deflection in the beam. Enhanced deflection signifies a reduction in stiffness. Consequently, the resonant frequency experiences a corresponding decrease.

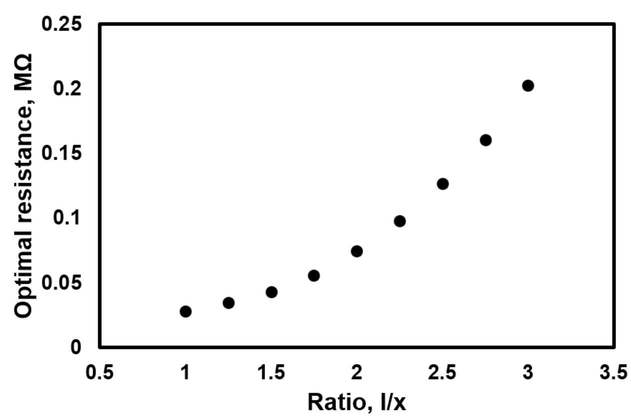

Figure 5. Variation of optimal resistance with *'l/x'* ratios.

Following the principle of maximum power transfer, the peak output power is attained when the external load resistance aligns with the internal impedance of the piezoelectric cantilever beam. Internal impedance is typically governed by the capacitance and the frequency to which the beam is exposed, displaying an inverse relationship. Since the dimensions of the piezoelectric material remained constant throughout the study, variations in internal impedance were due to changes in the excitation frequency of the beams. Fig. 5 shows that the optimal resistance for the beams rises in correlation with the ratios. This is because of a decrease in the resonant frequency as the ratio increases (see Fig. 4). Given that the energy harvester achieves its highest output power density at its resonant frequency, the increase in optimal electric load or resistance is a result of the reduced resonant frequency.

6. Conclusion

This paper presents the impact of altering the base beam length on the output power density of piezoelectric cantilever energy harvesters, while maintaining a constant length of the piezoelectric material. The findings reveal a direct relation between the base beam length and the output power density, i.e., the output power density increases almost linearly with the increase of base beam length. A threefold increase in the base beam length leads to a sevenfold increase in output power density. Considering the significantly higher cost of piezoelectric material than that of the base beam material, the increase in base beam length can significantly reduce the cost of energy harvesting by improving the output power density. While higher base beam ratios can provide a higher power density, there is a trade-off with reduced bandwidth. Moreover, the added mass due to the elongation of the base beam contributes to more deflection and this leads to a decrease in the resonant frequency. This reduction in the resonant frequency results in a rise in the optimal resistance.

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