# An Efficient Simulated Annealing Algorithm for the Joint Order Batching and Picker Routing Problem in Manual Order Picking Systems

**Felicia Nathania and Komarudin**
Department of Industrial Engineering
Universitas Indonesia
Depok, West Java 16424, Indonesia
felicianath10@gmail.com, komarudin01@gmail.com

## Abstract

Order picking is the process of collecting products from the warehouse to fulfill specific customer orders, which is known to be labor- and time-intensive. Order picking significantly determines the warehouse's performance, contributing 50% to 75% of operational costs. Therefore, the efficiency of the order-picking process also affects the supply chain performance. This study investigates two of five main order-picking problems jointly, which are classified in its day-to-day operational problem: order batching and picker routing. The algorithm to solve the problem is developed using metaheuristics, the simulated annealing algorithm. The algorithm avoids local optimization to find better solutions. The algorithm is validated through preliminary testing against problem instances of online grocery shopping cases with 1560 products where each order can consist of many items. The proposed algorithm has proven to be more efficient than the previous approach, which gives quality solutions using less run-time, especially for larger problem instances.

## Keywords
Order Batching, Picker Routing, Order Picking, and Simulated Annealing Algorithm.

## 1. Introduction
Competition in the marketplace has increased in intensity, extending from competition between companies to supply chains (Grosse and Glock 2015). In addition, there are increasing customer expectations towards received products. Thus, companies must maintain or even increase their competitive advantage by increasing operational efficiency through fast and accurate product delivery (Ho and Lin 2017; Grosse and Glock 2015). On the other hand, the COVID-19 pandemic has stimulated the growth of the digital economy due to restrictions on physical activity on a large scale. In 2021, Bain and Company reported that 80% of internet users engage in online transactions. This is supported by the growth of e-commerce services along with their users. As there is enormous potential in the digital economy, this boosts the growth of the logistics and warehousing sector due to increasing demand for product delivery. Thus, companies are starting to focus on building competitive advantages in the logistics and warehousing area as this area determines operational efficiency and effectiveness, leading to an increase in the whole supply chain performance.

Specifically, warehouses have enormous improvement potential as they have a significant role in the supply chain (Amorim-Lopes et al. 2021) and contribute to the performance and cost of the supply chain (Chen et al. 2015). Order picking is one of the essential activities in every warehouse, which can be defined as collecting products from the warehouse to fulfill specific customer orders (de Koster et al. 2007; Chen et al. 2015; Diefenbach and Glock 2019). Order-picking systems are dominated by manual systems consisting of up to 80% of warehouses (de Koster et al. 2007; Grosse and Glock 2015; Diefenbach and Glock 2019). Manual order picking systems are labor- and time-intensive, thus also cost-intensive (Diefenbach and Glock 2019) with a contribution of 50% to 75% of the warehouse's total operational cost (Chen et al. 2015; Diefenbach and Glock 2019). Therefore, the efficiency of order picking directly and significantly determines the warehouse and, thus, the supply chain performance. As a result, order picking is often the prioritized area for increasing productivity (de Koster et al. 2007). In literature, there are five orders picking problems that can be classified into the strategic level, namely layout design; tactical level, namely storage assignment, and zoning; also, operational level consisting of routing and batching (Chen et al. 2015; Grosse and Glock 2015; van Gils et al. 2018).

Specifically, routing and batching decisions highly determine order-picking distance and performance. As both decisions are highly related to the other, they need to be considered together. Past literature has given much attention to the topic of routing and batching. Unfortunately, they still have limitations that this study will tackle. Considering the significance of order batching and picker routing, along with limitations from previous literature, there is a need for a more efficient algorithm to solve the problem. This study focuses on optimizing a Joint Order Batching and Picker Routing Problem (JOBPRP), which includes determining customer order combinations in a pick round and the visit sequence for each storage location in each round. Order batching will allow orders to be grouped in picking, and picker routing will allow picking to be done at a shorter distance, increasing order-picking performance and efficiency. Thus, reducing the overall order-picking costs.

### 1.1 Objectives

This study aims to propose a metaheuristics algorithm to solve the Joint Order Batching and Picker Routing Problem (JOBPRP) efficiently, especially for large problems. The simulated annealing algorithm will be used to develop solutions for the problem. The main contribution of this study is to solve large JOBPRP instances with quality solutions in a shorter time compared to past studies through the implementation of the developed algorithm. The proposed algorithm is validated using publicly available problem instances of real-world data from Valle et al. (2017) containing 1560 products with up to 30 customer orders. Preliminary testing is performed to compare the developed algorithm with the existing method for the same instances, showing the proposed method's advantage, practicality, and efficiency in terms of run-time and solution quality.

## 2. Literature Review

Manual order picking time is spent on five activities: 50% on travel through the storage space, 20% on searching the order, 15% on picking and moving the items to pick equipment, 10% on setups, and 5% on other activities (Tompkins et al. 2003). Since traveling is the component with the most significant time proportion of at least 50% of the order-picking time (Chen et al. 2015; Amorim - Lopes et al. 2021), it becomes clear that there is a significant improvement opportunity present for improving order-picking efficiency, especially on traveling. JOBPRP is a joint problem consisting of two crucial operational-level decisions in order picking – order batching and picker routing – which determines the efficiency of the picking operations (Valle et al. 2017; van Gils et al. 2018). Order batching refers to combining several customer orders into a batch that will be picked in one pick round to save time and effort. Picker routing is the process of deciding the order in which picking locations are visited. These operational decisions significantly determine order-picking distance, and considering the strong relationship between them, they need to be considered jointly to increase order-picking efficiency (Won and Olafsson 2005; Briant et al. 2020; Xie et al. 2023). Solving the routing problem will decrease transportation distance while solving the batching decision will group orders of reasonable distances to be picked together. A joint approach for both problems will eventually reduce picking distance and thus the time and resources used in the activity, leading to higher efficiency and lower costs (Chen et al. 2015; Grosse and Glock 2015). Therefore, the solutions of JOBPRP will significantly increase order picking performance in fulfilling customers' orders.

Literature has focused on improving the order-picking process, and many are available on order batching and picker routing. Won and Olafsson (2005) show it is more efficient to solve order batching and picker routing problems jointly instead of sequentially. A few papers have directed focus on this area. Lin et al. (2016) have investigated joint order batching and picker routing for the Manhattan routing problem using particle swarm optimization. Valle et al. (2020) used partial integer optimization to solve batching and indirect routing problems.

Some have investigated batching and routing together with other problems, namely assignment (Kübler et al. 2020; Cao et al. 2020) and sequencing (Cao et al. 2020), and proposed heuristics algorithms for the formulated problems. Unfortunately, this literature has higher complexity in terms of problem formulations. Others have proposed an exact algorithm to solve the joint problem. Valle et al. (2017) proposed a branch and cut algorithm for JOBPRP and managed to optimally solve instances with up to 20 orders. Briant et al. (2020) have proposed a column generation heuristic for JOBPRP. Zhang and Gao (2022) also developed a branch and cut algorithm to solve order batching and picker routing. Ardjmand et al. (2020) tried solving the problem with an addition of assignment decision using a hybrid column generation, genetic algorithm, and artificial neural network algorithm. However, the developed exact algorithms are not feasible for larger instances as they result in high computational times.

Thus, heuristics have mainly been used for solving JOBPRP and some with additional problems. Attari et al. (2021) solved order batching and picker routing for small-sized sample problems using a genetic algorithm, particle swarm optimization, and honey artificial bee colony. Aerts et al. (2021) proposed a 2-level variable neighborhood search algorithm for the problem. Others would include additional problems, such as Ardjmand et al. (2018), which used Lagrangian decomposition heuristic and particle swarm optimization, along with a hybrid simulated annealing and ant colony optimization to solve order batching, picker routing, and assignment problems. However, these algorithms only apply in some warehouses using a single block warehouse or small-sized sample problems (Attari et al. 2021).

On the other hand, other literature have proposed algorithms for case-specific versions of JOBPRP. Tsai et al. (2008) proposed a genetic algorithm for a 3D-warehouse layout. Ardjmand et al. (2019) developed algorithms for a put-wall-based picking system. Xie et al. (2023) developed a variable neighborhood search algorithm for a hybrid system of multi-depot AGV-assisted mixed shelves system. Thus, these algorithms cannot be generalized.

Simulated annealing has also been used to solve JOBPRP. Simulated annealing is a metaheuristic that prevents solutions from being stuck at the local optima. Matusiak et al. (2014) researched order batching and picker routing for precedence-constrained customer orders using A*-algorithm and simulated annealing algorithm and produced savings of 16% travel distance. Ardjmand et al. (2019) have developed a list-based simulated annealing and genetic algorithm for order batching and picker routing in warehouses with put wall picking systems. In previous literature, simulated annealing has been proven to give quality solutions with good scalability (Matusiak et al. 2014; Ardjmand et al. 2019).

The literature shows that due to the complexity, most papers have used heuristics to solve the integrated problem. Unfortunately, there are a few limitations from previous literature. First, previous algorithms proposed have limited implementation scale on large instances. Second, some formulations and algorithms proposed previously are complex. Thus, causing long computational times for solving the problem. Lastly, there are limited practical aspects in previous literature. Thus, developed algorithms cannot be used in solving general practical problems. Therefore, due to the significance of JOBPRP and literature limitations, there is a need to develop an algorithm able to solve JOBPRP efficiently.

On the other hand, previous research has rarely used simulated annealing to solve JOBPRP. Specifically, the literature still needs to develop a simulated annealing algorithm to solve JOBPRP in manual order-picking systems for multi-block warehouses to minimize the travel distance. This forms the research gap filled by this paper. Therefore, we developed a simulated annealing algorithm to solve JOBPRP with a more efficient computing time, especially for larger problems. Additionally, the performance of the developed algorithm is observed and compared with previous literature using the same instances in the preliminary testing stage.

## 3. Methods

Valle et al. (2017) formulated the model for JOBPRP to minimize picking distance, the decision variable to determine trolley use for a specific order, the travel route of the trolley, the use of the trolley in the specific problem, and the vertex visited by the trolley. The model is constrained by capacity, the assignment of an order to a trolley, vertex visited subtour constraints and variable definition constraints.

This paper developed an efficient simulated annealing algorithm to solve JOBPRP in multi-block warehouse with manual order-picking systems. This is done using the proposed methodology described in Figure 1, consisting mainly of initialization and the 2-stage simulated annealing algorithm. The initialization step includes opening and reading data files to provide input and parameters to run the algorithm. The files used in the initialization are the warehouse layout, product list, product location list, and order files.

Subsequently, the main simulated annealing algorithm is done in 2 main stages. The first stage is grouping orders for the batching decision to provide the algorithm's initial solution. This is done using a simple integer programming model to minimize trolley use, the decision variable of trolley used and assignment of orders to trolley, and the constraints of trolley capacity, assignment of precisely one trolley for one order, ensuring that a trolley is used for order only if the trolley is used, and ensuring that when a trolley is used, it at least picks one order (Valle et al. 2017). Next, the inputs are used to generate a distance graph of the warehouse vertices and find the shortest distance between vertices in the warehouse using the Floyd-Warshall algorithm. The second stage is the process of solving the routing for each batch using local search operators chosen at random. Parameters for the current route and distance, number of iterations, and starting temperature are manually inputted to run the program. Simulated annealing uses random probability in accepting a worse solution according to the temperature in every iteration. It is based on the annealing process to prevent solutions from getting stuck in local optima. In the algorithm, local search operators are exchange, insert, and 2-Opt. Exchange is the process of switching two chosen nodes' positions. Insert refers to inserting one chosen node behind the other chosen node without changing the other nodes' positions. Lastly, 2-Opt refers to choosing a segment between 2 nodes and reversing the position sequence of the nodes in the segment without changing the position other than the chosen segment. After each iteration, the result will be compared to the current distance. If it is worse, it will trigger the calculation of acceptance probability for the worse result. This probability will be negligible with each iteration; thus, more results are explored during the beginning of iterations until the best result is obtained. The program will finish at the determined iteration number.

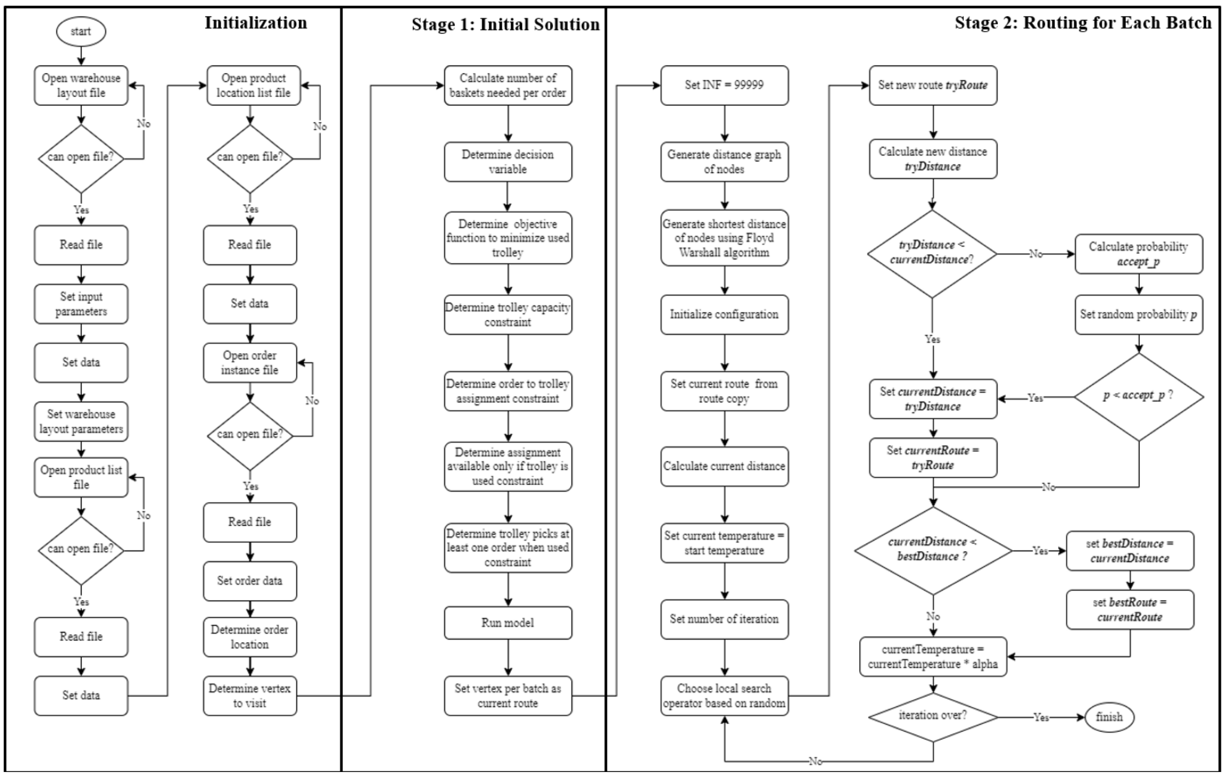


Figure 1. Flowchart of the proposed methodology

## 4. Data Collection

This paper developed an efficient approach for JOBPRP. The algorithm is developed for manual order-picking systems and tested against publicly available problem instances of the Foodmart case used in Valle et al. (2017). A preliminary computational study is done using the developed algorithm to validate the algorithm and measure its performance, along with its comparison with previous literature using the exact instances.

The warehouse layout used is a rectangular warehouse with a maximum capacity of 1582 products consisting of parallel aisles, several cross-aisles, and divided into several blocks, as described in Figure 2. Every aisle has storage on both sides from which pickers can take needed items through vertices in the middle of the aisle. Picking is done using a trolley with eight baskets with a limited capacity to hold 40 items. Order splitting restrictions apply, meaning items of the same order cannot be split into different trolleys, and items of different orders cannot be placed in the same basket. Instances of orders have been formed from a total of 1560 SKUs with a high variation of items in each order because these instances are based on online grocery shopping orders accumulated for two years, and combined purchases over the first Δ days (Δ∈ {5} are used in the preliminary testing stage) (Valle et al. 2017; Briant et al. 2020).

In order to run the developed algorithm, data files are needed for warehouse layout, product list, product location, and customer orders obtained from the Foodmart case problem instances. The preliminary numerical experiment uses Intel Core i7-7700HQ @ 2.80 GHz processor with 8 GB of RAM and Windows 10 as the operating system. The code was written in Python and ran using Spyder (Anaconda 3).

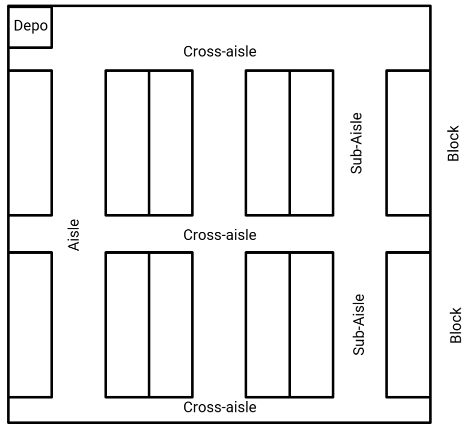


Figure 2. Warehouse layout example

# 5. Results and Discussion
## 5.1 Numerical Results
In this section, the preliminary numerical experiment result is described. In addition, a comparison is made between the 2-stage simulated annealing (2-Stage SA) algorithm developed and the branch-and-cut (BC) algorithm by Valle et al. (2017) for the same instances of the Foodmart benchmark. There is a difference where the BC algorithm was run using a machine with larger RAM. The preliminary testing results are presented in Table 1. The row above shows the algorithm of each result. Δ shows the number of days combined to form the order of the instances. The orders column denotes the number of orders used in the instance. The Time(s) column shows the average computation time used to run the algorithm in seconds. The Distance(OF) column shows the result of the objective function from the algorithm, namely average, best, and worst results from the preliminary testing. The NS shows the number of nodes investigated by Valle et al. (2017). Finally, the number of iterations for each instance using 2-Stage SA is shown in the table.

Table 1. Preliminary testing results incomparison with Valle et al. (2017)

| | BC (Valle et al. 2017) | | | | 2-Stage SA | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| **Δ** | **Orders** | **Time (s)** | **Distance (OF)** | **NS** | **Orders** | **Time (s)** | **Iteration** | **Distance (OF)** | | |
| | | | | | | | | **Average OF** | **Best OF** | **Worst OF** |
| 5 | 5 | 3.9 | 348.6 | 36 | 5 | 25.2 | 100000 | 350.7 | 348.6 | 352.8 |
| 5 | 6 | 2.9 | 364.8 | 1 | 6 | 28.6 | 100000 | 374.6 | 370.7 | 378.6 |
| 5 | 7 | 8.9 | 374.8 | 141 | 7 | 28.6 | 100000 | 383.3 | 377.1 | 388.8 |
| 5 | 8 | 95.2 | 503.8 | 146 | 8 | 56.9 | 100000 | 752.9 | 745.6 | 765.5 |
| 5 | 9 | 151.8 | 539.6 | 268 | 9 | 88.6 | 100000 | 575.5 | 571.5 | 587.5 |
| 5 | 10 | 111 | 581.4 | 88 | 10 | 106.2 | 100000 | 636.8 | 621.0 | 665.1 |
| 5 | 11 | 97.1 | 613.5 | 1009 | 11 | 107.0 | 100000 | 662.3 | 657.9 | 666.0 |
| 5 | 12 | 256.7 | 621.4 | 2261 | 12 | 96.7 | 100000 | 688.6 | 681.5 | 697.6 |
| 5 | 13 | 168.9 | 623.4 | 395 | 13 | 106.9 | 100000 | 735.5 | 721.5 | 753.4 |
| 5 | 14 | 263.8 | 639.3 | 995 | 14 | 108.4 | 100000 | 735.3 | 727.7 | 741.7 |
| 5 | 15 | 348.9 | 653.4 | 975 | 15 | 72.8 | 100000 | 752.2 | 741.7 | 761.7 |
| 5 | 20 | 2990.9 | 870.4 | 28931 | 20 | 161.9 | 100000 | 1031.9 | 1011.1 | 1040.8 |
| 5 | 25 | 21600 | 1127.4 | 123900 | 25 | 126.7 | 100000 | 1327.6 | 1318.1 | 1344.0 |
| 5 | 30 | 21600 | 1221.9 | 70400 | 30 | 74.1 | 100000 | 1482.0 | 1456.4 | 1509.6 |

From the results of the preliminary numerical testing, the 2-Stage SA is shown to be able to provide significantly more efficient computation times for the same problem, especially in the cases of larger problems, as seen from results for more than 11 orders. Specifically, the more the orders are processed using the algorithm, the larger the time is reduced compared to the CG algorithm, with improvement of up to 99.6% for 30 orders. Unfortunately, this faster computation time comes at the cost of increased objective function results of distances. The increased objective function comes in the average of 13% for all tested instances, with the highest being 48% for the instance of 8 orders and the smallest of 0% for the instance of 5 orders. On the other hand, note that these increases are less significant than the time reduced to obtain the results. These results prove that the 2-Stage SA algorithm can provide quality solutions in efficient computation times. Thus, the developed algorithm can be a reasonable choice for practical industrial operations cases.

### 5.2 Proposed Improvements

The developed 2-Stage SA algorithm is a starting point for a more efficient algorithm in solving JOBPRP with better quality solutions and reasonable computation times, especially for large instances, as seen in real-life warehouse cases. Thus, the future improvements of this preliminary work are to improve the source code of the algorithm in order to provide better results in problem solutions with the significant amount of computation time reduced necessary for increasing the performance and reducing the cost of warehouse operations, leading to improvement for the whole supply chain. Thus, time reduced in computation does not have to come at the cost of solution quality reduced.

## 6. Conclusion

This paper developed an efficient 2-stage simulated annealing algorithm to solve the JOBPRP, especially for large problems, in manual order picking systems for multi-blocks warehouses. Order picking, specifically order batching and picker routing contributes significantly to warehouse and supply chain costs and performance. Even though the order batching and picker routing problems in order picking have received much attention from researchers, some limitations exist, providing the gap to be filled by this paper. The algorithm was developed using simulated annealing metaheuristics in 2 stages, first for providing initial solution from the batching decision, continued with routing for each batch. Preliminary testing results show good algorithm scalability in solving large instances of up to 30 orders with a great variety in each order. In addition, comparison with the previous branch-and-cut algorithm (Valle et al., 2017) shows a significant improvement in computational time reduced, unfortunately at the cost of reduced solution quality of an average of 13%. This tradeoff is still reasonable, considering the significant reduction in computation times. The main contribution of this study is developing an efficient algorithm that can provide quality solutions with significantly short computational times compared to the previous algorithm. The study's main limitation lies in the increased objective function compared to previous literature. Thus, future research can be done in order to enhance the algorithm by incorporating methods to improve the computational results.

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## Biographies

**Felicia Nathania** graduated from Universitas Indonesia with a Bachelor's Degree in Industrial Engineering in 2022 and is currently pursuing a Master's Degree in the Department of Industrial Engineering Universitas Indonesia. Her research interests include production systems, logistics, optimization, and operations research.

**Komarudin** graduated from Vrije Universiteit Brussel in 2015. He is a lecturer in the Department of Industrial Engineering, Universitas Indonesia. His research interests include system modeling, advanced operations research, advanced optimization, game theory, linear and stochastic programming, queuing theory, and data science.