

# **A Robust Hybrid ANFIS-Wavelet Approach for Pattern Recognition**

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## **Abstract**

Lately, accurate recognition of Process Control Chart Patterns (CCPs) has been considered one of the significant tactics for supervising manufacturing processes in order to achieve better control and to improve products quality. In addition to the single patterns, a huge work has been done to recognize concurrent ones that are usually due to the presence of two or more single patterns. Feature-based approaches are more efficient in pattern recognition especially in the case of concurrent patterns. Furthermore, a selection of an optimal set of features can significantly reduce the diagnostic search process. In this paper, a new approach based on a combination of Adaptive neuro-fuzzy inference system (ANFIS), Wavelet analysis (WA) and Principal components analysis (PCA) is used to recognize concurrent patterns. ANFIS has proven to show high accuracy in pattern recognition. WA is used in this paper to improve the characteristics of the patterns to facilitate the recognition process by adding frequency features to the original pattern. Then, thirteen statistical features are extracted and PCA reduces their number to the most important three. ANFIS is used for training and testing the data based on the three extracted features as inputs and the patterns as target outputs. Extensive performance evaluation was carried out under normally and various non-normally distributed data. The non-normality of the inputs is based on two Gamma and Beta distributions. Results indicate that the proposed approach performs with high accuracy even in the case of non-normally distributed patterns.

## **Keywords**

Pattern recognition, Adaptive neuro-fuzzy inference system, concurrent control charts, principal components analysis, Wavelet Analysis, shape feature, statistical feature

## **1. Introduction**

Statistical process control charts play an essential role in monitoring and controlling the quality of a manufacturing process. The process is considered to be out of control if a point falls outside of the control limits, or if a sequence of points exhibits an unusual pattern. It's known that a specific unnatural pattern on a statistical control chart can usually be caused by a specific set of transferable causes (Western Electric 1956). Analysis and diagnosis of unnatural patterns is an essential aspect of statistical process control charting. Realization and recognition of unnatural patterns can be used essentially to narrow the possible causes that should be investigated and, therefore, reduce the diagnostic time (H. Cheng and Cheng 2009). The reduced variation in the process represents an essential role in the success of operations in a globally competitive market. Control charts also known as Shewhart control charts were first proposed by Walter Andrew Shewhart in 1931 (Shewhart 1931). Since then these charts have been broadly used to monitor and control manufacturing processes. Once an out-of-control state has been identified, steps can be taken to determine root causes of variation then the necessary corrective action. Deming (Deming 1982) showed that it is not valuable to determine the causes of the variations when the process exhibits common variations. On the other hand, when the process displays uncommon variations, it is beneficial to detect and try to eliminate them. Yet, Shewhart charts lack the ability to provide information about the trend of the process over time or to detect small shifts in the process mean or standard deviation. Statistical process control chart, on the other hand, is able to determine unnatural patterns and trends regardless of the magnitude of the shift in the process mean or standard deviation. These unnatural patterns are the illustration of the long term behavior of the manufacturing processes alerting of needed actions to be taken to bring the process back to its normal state.

## **2. Literature Review**

Several research papers have proposed approaches to identify all known statistical control chart patterns. These approaches can be summarized under four categories; signal based approaches (C.-S. Cheng 1997; C.-S. Cheng and HUBELE 1996; H. Cheng and Cheng 2009; R.-S. Guh 1999; Al-ghanim and Ludeman 1997; Perry, Spoorre, and Velasco 2001), statistical features extraction based approaches (Pham and Wani 1997; Ranaee and Ebrahimzadeh 2011; Ranaee, Ebrahimzadeh, and Ghaderi 2010; H. Cheng and Cheng 2009), shape features extraction based approaches (Gauri and Chakraborty 2009; Gauri 2010; Pham and Wani 1997), and a combination of statistical and shape features extraction based approaches (Gauri and Chakraborty 2006; Ranaee, Ebrahimzadeh, and Ghaderi 2010). The shape-based features have shown a higher classification rates than statistical based features while, the combination of statistical and shape features approaches have even a greater accuracy rates. But first the patterns must be identified. Western Electric Company (Western Electric 1956) identified several patterns including cyclic, systematic, increasing and decreasing trends, shift up and down, and stratification patterns. Wang et al. (J. Wang, Kochhar, and Hannam 1998) have systematized 30 possible unnatural patterns which includes the Western Electric single patterns and another set of mixed patterns known as concurrent patterns. The concurrent patterns are a mixture of 2 or more single patterns (R.-S. Guh 1999).

A review of the existing research reveals that several papers proposed several approaches to CCP recognition. Wu, Liu and Zhu (Wu, Liu, and Zhu 2014) and Zhang and Cheng (Zhang and Cheng 2015) used support vector machine (SVM) with statistical and shape features as inputs for single patterns. (Wu, Liu, and Zhu 2014; J. Wang, Kochhar, and Hannam 1998; Nelson 1984) used statistical techniques, others used rule based expert systems (Yang and Yang 2005) and, (Hassan et al. 2003; C.-S. Cheng 1997; Hwang and Hubele 1993; Pham and OZTEMEL 1992; Pham and Chan, n.d.; Al-Assaf 2004; R.-S. Guh 1999; Karaoglan 2011; JANG, YANG, and KANG 2003) opted for artificial neural network (ANN) techniques.

Yang & Yang (Yang and Yang 2005) used a control chart pattern recognition system using a statistical correlation coefficient in order to recognize both single and concurrent patterns with an acceptable performance. The advantage of this approach is that the recognition process doesn't require a long training process. Al-ghanim and Ludeman (Al-ghanim and Ludeman 1997) proposed a technique based on statistical correlation analysis where a set of optimal detection devices called the matched filters was developed. Pham and Wani (Pham and Wani 1997) used a rule based system for classifying unnatural patterns. Hassan and Shaharoun (Hassan et al. 2003) combined an ANN-based approach with statistical approaches. Guh and Tannock (R.-S. Guh 1999) proposed a backpropagation neural network approach to characterize pattern parameters in different conditions however, the approach showed higher efficiency for single patterns only. Cheng and Cheng (H. Cheng and Cheng 2009) proposed a self-organizing Map (SOM) neural network technique assuming no preliminary information about the signals. Jalil et. al (Addeh, Ebrahimzadeh, and Nazaryan 2013) used (ANFIS) for classification and Cuckoo optimization algorithm (COA) to improve the generalization performance of the recognizer with 49 extracted shape and statistical features.

Other researchers used wavelet analysis with statistical features as inputs. Others used WT and multiclass support vector machines (Du, Huang, and Lv 2013), Xie et al. (Xie et al. 2013) and Gu et al. (Gu et al. 2013) used singular spectrum analysis. Some others (C.-H. Wang, Dong, and Kuo 2009) applied independent component analysis (ICA) to first separate the individual signals in the case of concurrent signals.

Most of these researches used many extracted features to achieve higher recognition accuracy however, the higher the number of features used the higher is the size of search space, and the higher the risk of overfitting. Thus, researchers such as Gauri and Chakraborty (Bag, Kumar Gauri, and Chakraborty 2012) used classification and regression tree (CART) systematic approach to select 7 most useful features out of 30 shape features. Chang and Ho (Chang and Ho 1999), Cheng (C.-S. Cheng 1995), and Guh (Ruey-Shiang Guh 2002) tested the robustness of ANN with non-normally distributed raw inputs with single patterns. Pelegrina et. al (Pelegrina, Duarte, and Jutten 2016) showed that in the presence of different no-normally distributed CCPs, the accuracy of the classifier may be influenced by the signal separation and the features extraction methods.

The purpose of this research is to provide a robust approach based on WT and ANFSI to the recognition of statistical process CCPs under normality and various non-normalities of data.

## **3. Methodology**

The classification framework in Figure 1 is composed of two stages. The first one is related to the training stage which is divided into four steps. The CCPs are generated in the first step then in the second step, using WT the patterns are

de-noised and important pattern coefficients and features are extracted furthermore, the wavelet energy of the extracted input pattern coefficients is calculated. Then, the statistical and shape features of the signal are extracted. Using PCA in the third step, only the most influencing features are selected for recognition. In the fourth step, an ANFIS classifier model is generated based on the selected features.

The second stage is the classification process. During this stage, a random signal containing two mixed patterns is generated. Then, the signal is transformed using wavelet transform and only statistical features selected in step two from the training stage are extracted and used with the wavelet coefficients to recognize the pattern.

This approach be simulated and the results will be compared for different random distributions, namely normal, gamma and beta distributions to check the robustness of the ANFIS classifier. The gamma and beta distributions are parameterized based on the values of  $\alpha$  and  $\beta$ . Beta and gamma distributions approximate to normal as  $\alpha$  and  $\beta$  values become large. Furthermore, the asymptotic Normality holds only when the distribution is symmetric that when  $\alpha=\beta$  in the case of beta and gamma distributions. Thus to simulate completely different randomness from normal distribution,  $\alpha$  and  $\beta$  values of the CCP concurrent patterns must be small. In this research two gamma distributions ( $\alpha=0.5$ ,  $\beta=1$  and  $\alpha=1$ ,  $\beta=1$ ), and two beta distributions ( $\alpha=0.5$ ,  $\beta=0.5$  and  $\alpha=5$ ,  $\beta=1$ ) are selected.

Wavelet transform are best used with non-stationary signals where the mean and variance change over time such as in trends, cycles and sudden shifts in the signal. Additionally, non-stationarity of the signal can be identified based on to the behavior of the frequency contents of the signal with respect to time. The non-stationary behavior of the signal increases even more in the existence of concurrent patterns hence, converting a signal from time domain to frequency domain may result in better identification of the different concurrent patterns.

In the next sections, the theoretical aspects behind each step are described in more details.

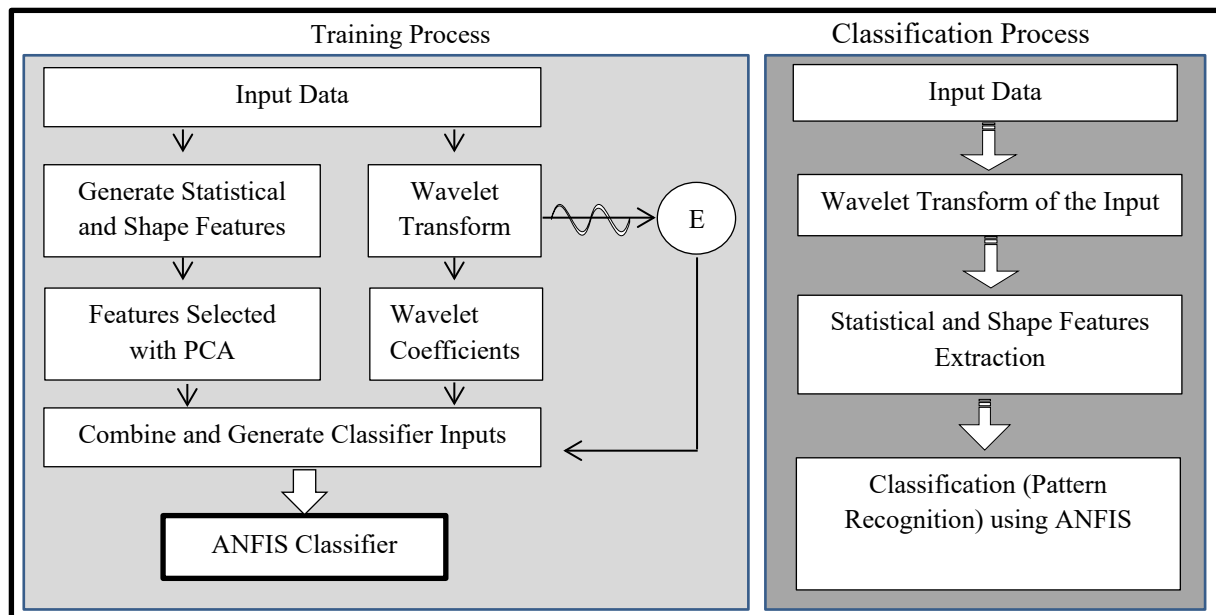


Figure 1. The classification process based on the wavelet coefficients and energy

In the next sections, the theoretical aspects behind each step are described in more details.

### 3.1 Patterns Generation

In this research, one normal pattern and six non-normal patterns were considered: Normal pattern (Norm), upward shift (UShift), downward shift (DShift), cyclical pattern (Cycl), systematic pattern (Syst), upward trend (UTrend), and downward trend (DTrend). Monte Carlo simulation was used to generate training samples based on predefined mathematical models and parameters proposed by Cheng (C. S. Cheng 1989) and Swift (Swift 1987). Figure 2 gives an example for each pattern class.

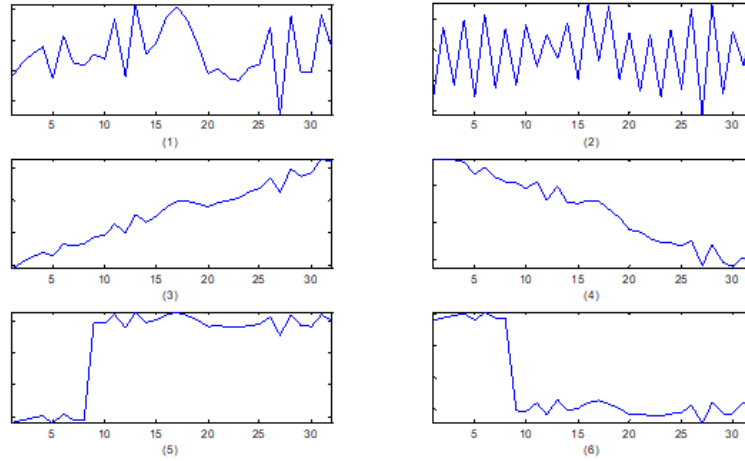


Figure 2. Unnatural patterns (1) Normal+Cyclical (2) Normal+Systematic (3) Normal+ Increase.trend (4) Normal+Decrease.trend (5) Normal+Up Shift (6) Normal+Down Shift.

Concurrent patterns based on the mixture of two simultaneous single patterns were generated. Each of these CCP is a weighted sum of normal pattern and an abnormal statistical process CCP. The weights  $a$  and  $b$  are the mixing parameters which depend on the fidelity of the sensors used to record the signals. These concurrent patterns were simulated using Equation 1.

$$1) \quad y(i, j) = a(\mu + (R(j) * \sigma)) + b(\text{Unnatural pattern})$$

### 3.2 Wavelet Transform (WT)

WT is an extension to the classic Fourier transform. While Fourier transform works on a single scale (time or frequency) basis, WT works on multi-scale basis. WT is based on a mathematical function used to divide a given continuous-time signal into different scale components. In this research, wavelet coefficients extracted from the CCPs, using Haar Transform (HT), are added to the input vector before the extraction of statistical and shape features. The strength of the WT lies in its ability to compute the spectral analysis of a signal over a time domain by cutting up small waves of the signal at different time scales.

WT of a function  $f(t)$  is the decomposition of  $f(t)$  into a set of basis wavelets:

$$2) \quad W(a, b) = \int_{-\infty}^{+\infty} f(t) \psi_{a,b}^*(t) dt$$

The wavelets are derived from a single wavelet  $\Psi(t)$ , called the mother wavelet, by dilation and translation:

$$3) \quad \Psi_{a,b}^*(t) = \frac{1}{\sqrt{a}} \Psi\left(\frac{t-b}{a}\right),$$

Where  $a$  is the scale parameter,  $b$  is the translation parameter, and  $\Psi(t)$  is the mother wavelet.

The Haar transform (HT) is one of basic transformations from space to local frequency domain. Using HT, a signal can be decomposed into approximation (or trend) and detail (or fluctuation). A precise equation of the approximate and detail coefficients are in Equation 4 and 5 respectively:

$$4) \quad a_n = \frac{f_{2n-1} + f_{2n}}{\sqrt{2}}, n = 1, 2, 3, \dots, N/2$$

$$5) \quad d_n = \frac{f_{2n-1} - f_{2n}}{\sqrt{2}}, n = 1, 2, 3, \dots, N/2$$

#### 3.2.1 Wavelet Energy

Vibrating signals with different pulse durations generate different energy values. The total energy of a discrete time signal is defined in the frequency domain of the HT coefficients:

$$6) \quad E_m = \sum_{n=0}^{2^{M-m}-1} (D_{m,n})^2$$

Where  $E_m$  is the wavelet energy of  $D_{m,n}$  wavelet coefficient at the decomposition level index  $m$  and time location  $n$ , and  $M$  is the total number of levels.  $2^{M-m}$  represents the total number of Wavelet coefficients in decomposition level  $m$ .

### 3.3 Control Charts Features Extraction

In this paper we consider 13 statistical and shape features. Mean ( $\mu$ ), Mean Square Error (MSE), Slope (Sp), Kurtosis (Ku), Skewness (Sk), Slope of the least square line (SLP-LS), Sum of least square regression error (SE-LS), Area between the mean line and the pattern (APM), Area between the least square line and the pattern (APS), The number of crossings of the pattern with the mean line (nc1), The number of least-square line crossings (nc2), Ratio between variance of data points and the mean sum of square errors of the least square line (RVE), and Mean absolute deviation (MAD).

These features were carefully selected in order to identify all the CCPs as proved in the literature. Each feature has more or less discriminative power to separate various types of CCPs.

### 3.4 Principal Components Analysis

To reduce the complexity of the classifier and to prevent it from over fitting, the number of features used in the training and classification process must be reduced. One of the methods used to reduce the dimension of data is principal components analysis (PCA). PCA is a multivariate statistical analysis method used to reduce data dimensionality by eliminating redundancy and focusing on the most important variables (principal components PCs) while retaining most of the information. The reduction from a  $d$ -dimensional dataset to an  $m$ -dimensional subspace (where  $k < d$ ) is performed if there is a correlation between variables. The PCA steps in data dimensionality reduction are described as following:

**Step 1:** Data standardization

$$7) \quad y_{ij} = \frac{x_{ij} - \bar{x}_j}{c_j} \in Y$$

**Step 2:** Generate the covariance matrix  $C$ , and calculate the principal components (eigenvectors  $e_i$ ) and eigenvalues( $\lambda_i$ ).

$$8) \quad C = \frac{1}{N} [Y - \bar{Y}l][Y - \bar{Y}l]^T$$

**Step 3:** Solve for the characteristic value of  $C$  and corresponding  $V$  value according to:

$$9) \quad (\lambda I - C)V = 0$$

The number  $m$  of PCs will be selected if the cumulative contribution rate is more than 85%. This transformation will lower the data dimension without losing too much information.

The thirteen features are extracted from the different control chart patterns and using PCA analysis the list of features is reduced to three most influencing ones (Mean, MAD and APM). The cumulative contribution rate of the three features is more than 92%. In fact, using Mean and MAD can theoretically achieve more than 85% accuracy in classifying CCPs. It can also be noticed from the plot that there is a small confusion between normal+normal, normal+cyclic and normal+systematic patterns due to the similarity of the signals of these patterns in the time domain.

### 3.5 Adaptive network based fuzzy inference system structure (ANFIS)

The ANFIS is a combination of fuzzy systems, developed by Zadeh (Zadeh 1987) and an ANN. the adaptive network-based fuzzy inference system (ANFIS) was first introduced by Jang in 1993 (Jang 1993). The inputs and outputs are mapped through input and output member functions (MFs) and set of fuzzy if-then rules. In an iteration process, ANFIS can refine the fuzzy if-then rules and membership functions to describe the input-output behavior of the complex system. Fuzziness of the data can be mathematically formulated to deal with imprecision and vagueness in different situations.

By definition a fuzzy set  $B$  is characterized by a member set function (MF),  $\mu_B$ , mapping the elements of  $B$  to the unit interval  $[0, 1]$ .

$$10) \quad \mu_B: X \rightarrow [0,1],$$

$$11) \quad B = \{(x, \mu_B(x)) | x \in X\} \quad \text{Where } X \text{ is the universe of discourse}$$

The member set function (MF) classifies the element in the set, whether it is discrete or continuous. It can have different shapes like triangle, trapezoidal, Gaussian, etc. This research is confined to triangular fuzzy numbers. It is defined by the membership function

$$12) \quad \mu_B(x) = \begin{cases} \frac{x-b_1}{a_0-b_1}, & \text{for } b_1 \leq x \leq a_0 \\ \frac{x-b_2}{a_0-b_2}, & \text{for } a_0 \leq x \leq b_2, \end{cases}$$

Where  $[b_1, b_2]$  is the supporting interval and the point  $(a_0, 1)$  is the peak.  
A-cut of a fuzzy number A can be written as:

$$13) \quad \text{Cut } \alpha(A) = \{x \mid \mu_A(x) \geq \alpha\}$$

### 3.6 Non-Normally Distributed Patterns

The non-normal distributions considered in this research are Gamma and Beta distributions. The impact of these distributions may be similar to a variation in the standard deviation but with extreme degrees of non-randomness levels, the classifier model can be tested for its true robustness.

In this paper Gamma and Beta distribution random numbers are generated using Marsaglia and Tsang's method (Marsaglia and Tsang 2000), and the distributions we consider are  $(\alpha=0.5$  and  $\beta=1)$  and  $(\alpha=1$  and  $\beta=1)$ . For the Beta distributions we consider  $(\alpha=0.5$  and  $\beta=0.5)$  and  $(\alpha=5$  and  $\beta=1)$  as shown in Figure 3.

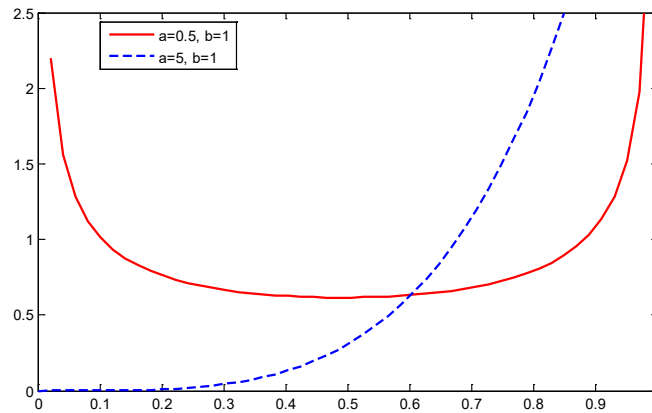


Figure 3. Probability Density Functions of Beta Distributions

With different type of randomness, the shape and the location of the patterns are different from the normally distribute patterns which increases the confusion of the classifier.

## 4. Experiments and results

In order to test the performance of the approach, a model without the wavelet transform was generated. 2100 control chart samples were generated, 300 of each type of patterns. 1470 samples were used for the training phase, 315 for validation and 315 for testing. The ANFIS model was trained using a hybrid algorithm that combines the back propagation gradient descent method with the least squares method.

A first classifier model is generated based on two features (Mean and MAD). The model is tested with input data generated based on the five previously mentioned distributions;

Figure 4 shows that the classifier performs very well with normally distributed data however, when data are gamma or beta distributed, the accuracy drops down to 43% in the case of gamma (126% decrease) and to 70% in the case of beta distribution (38% decrease in accuracy).

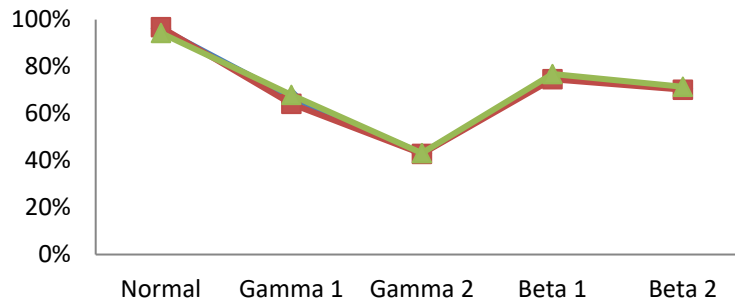


Figure 4. Accuracy values of the ANFIS classifier based on (Mean and MAD)

A second classifier model is generated based on three features (Mean, MAD and APM). The model is tested with input data generated based on the five previously mentioned distributions

### Wavelet Transform Coefficients and Wavelet Energy Approach

In this approach the wavelet energy is calculated for each input data and used as input to the classifier model alongside with the statistical features.

This approach is tested with classifier based on (wavelet coefficients, Mean, MAD and Energy) and classifier 2 based on (wavelet coefficients, Mean, MAD, APM and Energy). Result data of the five above mentioned distributions are shown in Table 1. As it can be seen, classifier 2 is performing much better compared to classifier 1. It can also be noticed that classifier 2 based on decomposition level 4 shows better overall performance compared to other decomposition levels.

Table 1. Accuracy rates of Wavelet-ANFIS classifier using wavelet coefficients and Energy

		Classifier 1 (Mean, MAD & E <sub>n</sub> )	Classifier 2 (Mean, MAD, APM & E <sub>n</sub> )
Decomposition	Distribution	Accuracy Rate	Accuracy Rate
L = 2	Normal	80%	95%
	Gamma 1	65%	84%
	Gamma 2	72%	88%
	Beta 1	94%	100%
	Beta 2	99.23%	99.25%
L = 3	Normal	81%	89%
	Gamma 1	65%	95%
	Gamma 2	72%	82%
	Beta 1	93%	98%
	Beta 2	100.00%	99.52%
L = 4	Normal	80%	88%
	Gamma 1	64%	93%
	Gamma 2	71%	100%
	Beta 1	88%	99%
	Beta 2	98.82%	100.00%

Using three inputs in addition to the wavelet energy improved the classifier accuracy by up to 46% over Classifier 1 (Mean, MAD & E<sub>n</sub>). Results also showed that most of the classifiers' confusion was between normal+normal, normal+cyclic and normal+systematic patterns which prove PCA analysis results. It also shows that this approach is very robust and able to classify patterns with high accuracy no matter how data is distributed. Obtained results showed

that classifier accuracy increases with the number of decomposition levels of the wavelet transform, the higher is the level the lowest are the number of wavelet coefficients and so is the classifier confusion. Figure 5 is a parallel coordinates plot where features are laid out horizontally and each pattern is represented by the median and quartiles (25% and 75% points). It shows the importance of the four features including the wavelet energy in classifying some of the patterns. The energy value is calculated based on level 4 wavelet decomposition.

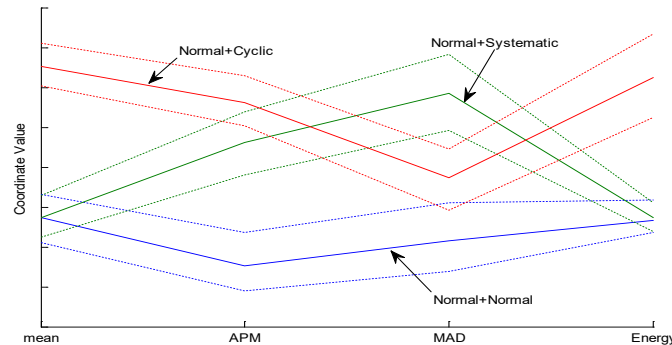


Figure 5. Correlation analysis between 4 inputs for Nm+Nm, Nm+Cycl & Nm+Syst patterns

A comparison between non-wavelet based classifiers and classifier 2 shows a significant improvement of the proposed method that varies between 3% and 57% improvement as clearly shown in Table 2. It also shows that accuracy rates of classifier 4 are more consistent over all the distributions.

Table 2. comparison between non-wavelet and wavelet based classifiers

	Non Wavelet based classifier		Wavelet based classifier
	3 inputs	4 inputs	Classifier 4 L=4
Normal	97%	96%	88%
Gamma 1	64%	93%	93%
Gamma 2	43%	73%	100%
Beta 1	75%	84%	99%
Beta 2	70.2%	76.8%	100.00%

## 5. Conclusion

Due to the importance of statistical process control charts and their essential role in monitoring and controlling the quality of a manufacturing process, it is necessary to develop better methods to recognize accurately abnormalities and irregular patterns in the process. In this research we focused mainly on concurrent patterns due to the difficulty in detecting them by many approaches. This difficulty lies in non-stationarity behavior of these patterns which increases the difficulties of detection. In that respect we proposed to use Haar wavelet transform and use signal frequency characteristics with some selected statistical and shape features along with wavelet coefficients energy as input to the ANFIS classifier. In this approach, statistical and shape features of the input data were selected using PCA and their robustness was tested for different random distributions.

Based on the results obtained, we can conclude that this classifier provides better results with concurrent CCPs. The combination of statistical, shape and frequency features with wavelet energy assures high accuracy and consistency in detecting concurrent patterns over different random distributions. The use of PCA to reduce the number of features not only reduced ANFIS complexity, it also improved accuracy of the classifier.

In future work, we will verify the performance of this approach with real manufacturing processes. In addition, further work will take place to evaluate the effectiveness of the neuro-fuzzy approach to statistical control chart pattern recognition for concurrent patterns containing more than one unnatural pattern, such as stratification and increasing trend.



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