The Development of a Dual-channel Warehouse and Inventory Management Model with Stochastic Demand and Return Products in Sports Retailer

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Abstract

Every industry is competing, especially retail, to transition or increase its sales channels not only through physical stores, but also through online channels. Likewise, warehouses are required to be able to participate in the order fulfillment process both through stores and online. This research develops an inventory model for dual channel warehouses with limited warehouse space and product returns. With the warehouse divided into two parts, namely for offline order fulfillment and for online order fulfillment. The development of this model also considers demand uncertainty and delivery lead time. The warehouse space constraint is convex nonlinear programming and uses Karush-Kuhn-Tucker analysis in solving it. A closed-form solution was developed using normal distribution in the case of no limited warehouse space constraints, and an iterative algorithm using bisection method for the case with limited warehouse space constraints. Numerical analysis was conducted directly on a warehouse in a sports retail company to get a real-world view of the proposed model. The results show that warehouse space greatly affects the size of the order quantity. The sensitivity analysis conducted also shows that the proposed model offers an optimal solution for the size of the order quantity and reorder point and can also minimize the total warehouse cost.

Keywords
dual-channel warehouse, uncertain demand, product returns, inventory management, total cost minimization, warehouse space constraints.

1. Introduction
The rapid development of the Internet and information technology in recent decades has shifted the focus from traditional to online sales. The popularity of online marketing motivates manufacturers and retailers to adopt multichannel distribution systems to sell products (Zhu et al., 2020; Modak, 2017). Online marketing is one way to increase company revenue (Asl-Najafi et al., 2021). Although online marketing drives sales development compared with traditional retail, many issues still need to be studied in online supply chain systems and traditional (offline) retail, one of which is related to the sales model (Wang et al., 2022). The online and offline sales model is also known as a dual-channel sales model. The advantages of the dual-channel sales model are that it makes consumers feel a more comfortable shopping experience, helps companies reduce costs and promote their products, and can improve traditional industries (Zhu et al. 2020). This is because some consumers reason that they do not like shopping in retail stores because of their busy schedules, stores having long queues, weather conditions, and others. On the other hand, some consumers also dislike online shopping because they prefer to check the items themselves, ask for advice and assistance from the store, and immediately take home the purchased items rather than paying shipping fees and waiting for delivery (Modak, 2017).

Companies that introduce online sales face many challenges, in addition to the usual challenges of conventional businesses in terms of logistics and delivery processes. Such challenges include large order volumes with very small quantities, short delivery lead times, flexible delivery (e.g., night delivery and even 24-hour delivery), and picking and packing processes for single-unit orders (offline only). These challenges require warehouses or distribution centers to be prepared for orders from both offline and online stores (Alawneh and Zhang, 2018; Master, 2015). In addition, the challenge in running a dual-channel warehouse is how to organize the warehouse and manage inventory to fulfill both online and offline (store) orders. The two important differences are the order size and timing. Online orders are small and random, whereas offline orders are usually large and scheduled (Alawneh and Zhang, 2018; Agatz et al., 2008). Warehouses operating in today's digital e-commerce era must have versatile infrastructure. Infrastructure can share information, connect with each other, and handle different orders from different consumer segments with different features, such as diverse order sizes and delivery lead times (Alawneh and Zhang, 2018).

Many companies with dual-channel distribution systems have experienced difficulties in developing effective inventory policies to achieve optimal channel performance. One of the main issues they face is deciding the optimal order quantity and reorder point when a new sales channel is introduced. In addition, they must consider space constraints and uncertain demand (from both offline and online channels) (Alawneh and Zhang 2018; Agatz et al., 2008). According to Feng et al. (2017), warehouse space is an important consideration when making optimal decisions in the industry. Increased demand, such as seasonality, can strain the supply chain space. In general, retail stores inevitably face severe seasonality, which makes it impossible for warehouses to respond to all demand without stockpiling products (Bartholdi and Hackman, 2019). Typically, for heavy or bulky items, such as refrigerators and large furniture, a dedicated e-fulfillment warehouse is the right choice because it has low cost efficiency in moving such items frequently across different areas of the warehouse. However, for most electronics, department stores, and grocery stores, a dual-channel warehouse is the right choice. The choice of a dual-channel warehouse is made because the added dedicated e-fulfilment area can be designed to provide an efficient and flexible solution for small orders in bulk, such as a warehouse with low density, low inventory, specialized equipment or structure, and long operation time (Alawneh and Zhang, 2018).

In addition, many warehouses have to handle product returns that run at around 5% in retail. Product returns are a key function in any e-commerce-enabled warehouse, with returns reaching 25-30%, proportional to the magnitude of sales (Bartholdi and Hackman, 2019). A warehouse can be divided into two areas to optimize warehouse operations. One of these areas has a low density for order picking and the other has a high density for storing goods and replenishing the central picking area using inventory control policies. The demand that occurs is classified as stochastic demand by assuming deterministic lead times (Alawneh and Zhang, 2018). The e-commerce industry has been using dual-channel warehouses for several years; however, only a few studies have discussed dual-channel warehouses. Therefore, the proposed model considers warehouse structure and space, online fulfillment operations, order costs, storage costs, and backorder costs. In addition, it considers stochastic demand and lead-time uncertainty. This study provides additional variables in the form of product returns from stores and online consumers when designing a dual-channel warehouse.

1.1. Objectives

The objective of this study is to develop a dual-channel warehouse model by considering stochastic demand and product returns from stores and online consumers. Then, the optimal reorder point and order quantity are determined by considering the warehouse space.

2. Literature Review

Inventory is the number of raw materials or goods that are available for use at any time in the future. According to Slack et al. (2007), inventory is the accumulation of resources that flows through processes, operations, or supply networks. Inventory is any organizational resource that is stored in anticipation of meeting the demand. An inventory is a component, material, or finished product that is available on hand, waiting to be used or sold (Baroto, 2002). Various forms of inventory management have been studied in the dual-channel supply chain literature (Alawneh & Zhang, 2018). Chiang and Monahan (2005) proposed a model that can be described as one of the first models to examine the inventory policy of a two-echelon dual-channel supply chain that receives demand from different customer segments. They assumed that inventory is held in the manufacturer's warehouse to fulfill online demand and in retail stores to fulfill offline demand. They developed an inventory-based inventory management strategy to minimize the operating cost of the system by accounting for inventory holding and lost sales costs.

Swaminathan and Tayur (2003) described the major adjustments required for a conventional supply chain to cope with the e-commerce fulfillment process. They conclude that channel integration in a dual-channel supply chain increases profits, reduces inventory, and improves customer service. However, the model studied in this study mainly focuses on e-commerce. Therefore, dual-channel operations and their interdependencies were not discussed. Supply chain management in an e-commerce environment was conducted (Agatz et al., 2008), focusing on distribution network design, warehouse layout, inventory, and space management topics. The dual-channel fulfillment process is divided into integrated fulfillment (using one warehouse to meet the demand of different sales channels) and dedicated fulfillment (using dedicated warehouses for different channels). Zhao et al. (2016) suggested a new inventory strategy called online-to-offline strategy. They consider a dual-channel supply chain with one manufacturer and one retailer. They also proposed centralized and decentralized inventory models with and without lateral transshipment. The decision variables in their model are the inventory level for the store and transshipment price. However, no ordering or storage costs were considered. Sarkara et al. (2015) revealed that the latest inventory management systems and cost reduction of radio-frequency identification technology have made the continuous review inventory control policy (Q, R) a very attractive approach. Alawneh and Zhang (2018) used the annual ordering cost, annual storage cost, annual reorder cost, and backorder cost, considering some constraints, which is usually the replenishment rate. In general, it is difficult to obtain a closed-form solution, and well-known iterative algorithms are used to obtain the optimal order quantity. This has led to the use of many heuristics or approximation approaches to solve the model.

According to Taha, H (2017) Operations Research (OR) is a quantitative and interdisciplinary field that uses analytical methods and advanced mathematics to help decision makers solve complex problems and improve their decision-making process. OR uses a scientific approach to identify, model, and solve problems by developing and analyzing mathematical models, algorithms, and simulations. OR is widely used in industries such as manufacturing, transportation, logistics, finance, and healthcare to help organizations make informed decisions, allocate resources efficiently, and improve their overall performance. All OR models consist of three basic components: the decision variable to be determined, the goal that needs to be optimized (maximized or minimized), and the constraints that the solution must meet. Problems that can be solved through operational research include linear and nonlinear programming. A linear programming problem can be expressed as a system of linear equations or inequalities, which can be solved using mathematical algorithms to determine the optimal values for the decision variables. These decision variables represent the amount of goods or services to be produced or consumed or the amount of resources to be allocated to different activities (Hillier & Lieberman, 2010). Linear programming has many applications in operational research, including production planning, resource allocation, transportation and logistics, scheduling, and financial planning. In general, nonlinear programming problems are more difficult to solve than linear programming problems because the search for an optimal solution may involve many local optima, or the problem may be nonconvex. Nonlinear programming problems have one or more nonlinear constraints, and may have unique optimal solutions, multiple optimal solutions, or no optimal solutions. In addition, solution methods for nonlinear programming problems tend to be more computationally intensive than those for linear programming problems (Hillier & Lieberman, 2010).

According to Nocedal and Wright (2006), the Karush-Kuhn-Tucker (KKT) method is an important concept in the mathematical optimization theory used to solve optimization problems with constraints. KKT provides the conditions that need to be met by a point to be the optimal point in an optimization problem with constraints. KKT produces a system of nonlinear equations and inequalities that relate the gradient of the objective function to that of the constraint function. KKT equations and inequalities produce sufficient conditions for optimality in convex optimization problems, so they can be used to solve optimization problems efficiently (Beck, 2017). However, (Beck, A., 2017) the KKT equations are not always sufficient to solve more general optimization problems. In more complex optimization problems, such as nonlinear optimization with nonlinear constraints, the KKT equations only provide necessary but insufficient conditions to determine optimality. The KKT equations are commonly used in optimization problems involving equality and inequality constraints. This problem can be solved by constructing the Lagrange function of the problem, which can then be used to determine KKT conditions (Nocedal and Wright, 2006). In the KKT equation, there are several conditions that must be met for the point to be considered an optimal point (Boyd and Vandenberghe, 2004): the gradient of the objective function must be equal to a linear combination of gradient bounds normalized by lambda coefficients (i.e., steady state), all constraints must be satisfied or normalized by lambda factors (i.e., duality constraints), and lambda coefficients cannot be negative (i.e., non-negative condition). The three conditions are necessary for the optimal point in a nonlinear optimization problem with constraints. The KKT conditions can also be expressed as a nonlinear system of equations, and if the system can be solved, then the point that satisfies all KKT conditions can be considered as the optimal point.

3. Methods

The modified model differs from that proposed by Alawneh and Zhang (2018). The first difference is the addition of returned products from consumers to the warehouse, which causes an additional component of the cost of returning goods. The second difference is the absence of backorder costs because the model will be implemented for the needs of warehouses in the retail industry that do not use backorders if goods are not available but immediately experience lost orders or lost sales with a dual-channel warehouse model developed as follows in Figure 1:

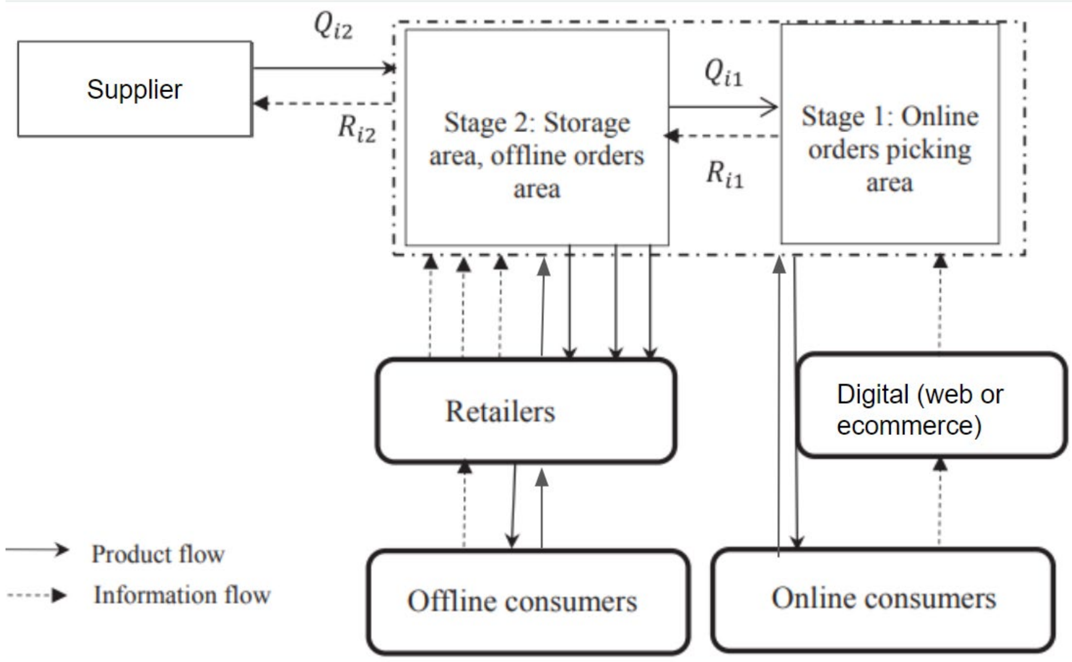

Figure 1. Dual-channel warehouse model

3.1 Notations and Assumptions

3.1.1 Notations

The notations used in developing the mathematical model are given as follows (Table 1):

Table 1. Mathematical model

Notations	Description
i	item index
j	index area ; j=1 for online demand and j=2 for offline + online demand
L_{ij}	lead time for item i in area j
D_{ij}	Annual demand expectation for item i in area j
h_{ij}	holding cost per unit for item i in area j
b_{ij}	backorder cost per unit for item i in area j
A_{ij}	order cost for item i in area j
E_{ij}	annual total expectation return per item for item i in stage j
X_{ij}	Demand during lead time for item i in area j
$f(X_{ij})$	Probability density function from lead time demand

Notations	Description
μ	Average
μ_{Lij}	Average from lead time
μ_{dij}	Average from demand per week
μ_{xij}	Average from demand during lead time
γ_{ij}	Box dimension
α	probability of the minimum total order requirement that can be stored in the warehouse
σ	standard deviation
σ_{Lij}	standard deviation from lead time
σ_{dij}	standard deviation from demand per week
σ_{xij}	standard deviation from demand during lead time
S	warehouse space
Y	Box dimensions in cubic meters
μ_{Yij}	average of box dimensions in cubic meters
σ_{Yij}	standard deviation of box dimensions in cubic meters
θ	Lagrange multiplier for warehouse space constraints

with the decision variable as follow in Table 2.

Table 2. Decision variable

Variabel	Parameter
Q_{i1}	Order quantity for item i in stage 1
R_{i1}	Reorder point when new order is placed for item i in stage 1
Q_{i2}	Order quantity for item i in stage 2
R_{i2}	Reorder point when new order is placed for item i in stage 2

3.1.2 Assumption

Alawneh and Zhang (2018) made some assumptions and made a preliminary analysis of their model, which will also be applied to the development of this model because it is very relevant to obtain optimal results and in accordance with the sports retail industry, as follows:

- If the demand during lead time for item i in stage j is in a situation where demand and lead time are normally distributed and statistically independent, then the mean and standard deviation of demand during lead time are :

$$\mu_{xij} = \mu_{Lij} \times \mu_{dij} \text{ dan } \sigma_{xij} = \sqrt{\mu_{Lij} \times \sigma_{dij}^2 + \mu_{dij}^2 \times \sigma_{Lij}^2} \quad (1)$$

  If the situation where the lead time is fixed, then the average and standard deviation of demand during lead time are :

$$\mu_{xij} = L_{ij} \times \mu_{dij} \text{ dan } \sigma_{xij} = \sqrt{L_{ij} \times \sigma_{dij}^2} \quad (2)$$

- Customer loyalty in each channel (j) was between 0 and 100%. This means that if one channel has 100% customer loyalty, then sales will be considered lost in the other channels. Demand is then calculated as follows:
  a. If in a dual-channel warehouse condition, the demand on the offline channel will aggregate from the online and offline demand $D_2 = D_r + D_d$. While the demand in the online channel is only for online demand, only $D_1 = D_d$.
  b. If in a single-channel warehouse condition, in this case only retail or offline, then the demand in the offline channel will be added to the percentage of the possibility of consumers moving from the online channel $D_2 = D_r + \beta_1 D_d$. The demand for the online channel is zero (0).
  c. If, in a single-channel warehouse condition, in this case only online, the demand on the offline channel will be the aggregate of the online demand added to the percentage of the possibility of consumers moving from the offline channel $D_2 = D_d + \beta_2 D_r$. While the demand on the online channel will be $D_1 = D_d + \beta_2 D_r$.
- A normal distribution was used because historical data of the mean and standard deviation were available.
- Demand for each channel is treated as independent random variable
- Using inventory policy (Q,R) → Quantity and reorder points.
- Each channel cannot cross-order, and each channel has its own reorder point connected to other channels.
  - Online channels get goods from offline channels
  - Offline channels get goods from suppliers
- Goods that include returns are the result of defects and overstock in each store.
- Goods for the return process are always available.
- Return transportation costs to and from the warehouse are borne by stores and consumers.

3.2 Mathematical Model

The objective function of this model is to minimize the total annual expected cost in a dual-channel warehouse, which consists of the order cost, storage cost, shortage or backorder cost, and product return cost. The backorder cost will still be included in the developed model; it will only be considered zero at the time of the calculation. To provide the inventory policy (Qij, Rij), the average inventory level for stages 1 and 2 is the cycle average for one year of inventory plus safety stock (Alawneh and Zhang, 2018). The average inventory level is approximately expressed as $Q_{ij}/2 + R_{ij} - \mu_{xij}$, where $R_{ij} - \mu_{xij}$ is the safety stock.

Then, the objective function equation is:

$$Z(Q_{i2},R_{i2},Q_{i1},R_{i1}) = \sum_i \frac{A_{i2}D_{i2}}{Q_{i2}} + \sum_i \frac{A_{i1}D_{i1}}{Q_{i1}} + \sum_i h_{i2}\left[\left(\frac{Q_{i2}}{2} + (R_{i2} - \mu_{xi2}\right)\right] + \sum_i h_{i1}\left[\left(\frac{Q_{i1}}{2} + (R_{i1} - \mu_{xi1}\right)\right] + \sum_i \frac{b_{i2}D_{i2}}{Q_{i2}}\left[\int_{R_{i2}}^{\infty} (x_{i2} - R_{i2})f(x_{i2})dx_{i2}\right] + \sum_i \frac{b_{i1}D_{i1}}{Q_{i1}}\left[\int_{R_{i1}}^{\infty} (x_{i1} - R_{i1})f(x_{i1})dx_{i1}\right] + \sum_i h_{i2}E_{i2} + \sum_i h_{i1}E_{i1} \quad (3)$$

where the first equation is the annual ordering cost, which is the ordering cost multiplied by the number of ordering cycles in one year. The third and fourth equations represent the estimated annual storage cost. The fifth and sixth equations are the annual backorder costs, which are equal to the backorder cost multiplied by the expected number of shortages per cycle for one year. The seventh and eighth equations are the cost of product returns for one year, which is the storage cost multiplied by the number of product returns for one year.

Considering warehouse space constraints and uncertain demand, it sets the probability that the total warehouse stock at the time the order is received is not less than α. The equation for the warehouse space constraint problem is as follows (Alawneh and Zhang, 2018):

$$P\left[\left(\sum_i \gamma_{i2}(Q_{i2}+R_{i2}-x_{i2})+\sum_i \gamma_{i1}(Q_{i1}+R_{i1}-x_{i1})\right)\leq S\right]\geq \alpha \quad (4)$$

$$Q_{ij}, R_{ij} \geq 0 \quad \forall\, i, j \quad (5)$$

The equation can also be written as follows:

$$P\left[\sum_i \gamma_{i2}x_{i2}+\gamma_{i1}x_{i1}\geq \sum_i \left(\gamma_{i2}(Q_{i2}+R_{i2})+\gamma_{i1}(Q_{i1}+R_{i1})\right)-S\right]\geq \alpha \quad (6)$$

$$Y=\sum_i \sum_j \gamma_{ij}x_{ij}, \mu_Y=\sum_i \sum_j \gamma_{ij}\mu_{ij}, \sigma_Y^2=\sum_i \sum_j \gamma_{ij}^2\sigma_{ij} \quad (7)$$

and z1-α is the cumulative probability distribution value of demand at point 1-α (Alawneh and Zhang, 2018; Ghalebsaz-Jeddi et al., 2004), the equation for the warehouse space constraint problem is obtained:

$$\sum_i \left(\gamma_{i2}(Q_{i2}+R_{i2})+\gamma_{i1}(Q_{i1}+R_{i1})\right)\leq S+\mu_Y+z_{1-\alpha}\sigma_Y \quad (8)$$

Equations (4), (6), and (8), denoted as problem (P), are nonlinear programming constraints where it is difficult to find a closed-form solution. Problem (P), which is a nonlinear programming constraint, it is necessary to first know that the equation is included in convex or concave functions. Subsequently, the derivative function of the objective function equation with non-negative parameters $Q_{i1}, R_{i1}, Q_{i2}, R_{i2}$ is performed. From the calculation results, all second derivatives are greater than 0 for all non-negative $Q_{i1}, R_{i1}, Q_{i2}, R_{i2}$. Thus, objective function Z is convex.

Because the problem is a convex nonlinear program, which means that the solution to the problem is unique and satisfies the necessary conditions of Karush-Kuhn-Tucker (KKT), a Lagrange function is considered. After adding the Lagrange function and warehouse space constraint, the objective function becomes:

$$\begin{aligned} L(Q_{i2},R_{i2},Q_{i1},R_{i1}) = {} & \sum_i \frac{A_{i2}D_{i2}}{Q_{i2}}+\sum_i \frac{A_{i1}D_{i1}}{Q_{i1}}+\sum_i h_{i2}\left[\left(\frac{Q_{i2}}{2}+(R_{i2}-\mu_{xi2})\right)\right]+ \\ & \sum_i h_{i1}\left[\left(\frac{Q_{i1}}{2}+(R_{i1}-\mu_{xi1})\right)\right]+\sum_i \frac{b_{i2}D_{i2}}{Q_{i2}}\left[\int_{R_{i2}}^{\infty}(x_{i2}-R_{i2})f(x_{i2})dx_{i2}\right]+ \\ & \sum_i \frac{b_{i1}D_{i1}}{Q_{i1}}\left[\int_{R_{i1}}^{\infty}(x_{i1}-R_{i1})f(x_{i1})dx_{i1}\right]+\sum_i h_{i2}E_{i2}+\sum_i h_{i1}E_{i1}+ \\ & \theta\left[\sum_i \left(\gamma_{i2}(Q_{i2}+R_{i2})+\gamma_{i1}(Q_{i1}+R_{i1})\right)-S-\mu_Y-Z_{1-\alpha}\sigma_Y\right] \end{aligned} \quad (9)$$

where is the Lagrange multiplier for the warehouse space constraint. For the Karush-Kuhn-Tucker minimization problem, the decision variables must be greater than 0 ($Q_{ij}, R_{ij} \geq 0 \quad \forall\, i, j$), and the objective function must be convex.

Then we can find the optimal solution following the first-order KKT condition as follows:

from $\frac{\partial L}{\partial Q_{ij}}=0$ we get

$$-\frac{A_{ij}D_{ij}}{Q_{ij}^2}+\frac{h_{ij}}{2}-\frac{b_{i1}D_{i1}}{Q_{ij}^2}\left[\int_{R_{i1}}^{\infty}(x_{i1}-R_{i1})f(x_{i1})dx_{i1}\right]+\gamma_{ij}\theta = 0 \quad (10)$$

or if there is no backorder cost, we get :

$$-\frac{A_{ij}D_{ij}}{Q_{ij}^2}+\frac{h_{ij}}{2}-\frac{b_{i1}D_{i1}}{Q_{ij}^2}+\gamma_{ij}\theta = 0 \quad (11)$$

Then we can get the equation to get Q_{ij} as follows:

$$Q_{ij} = \sqrt{\frac{2D_{ij}A_{ij}}{h_{ij}+2\gamma_{ij}\theta}} \quad (12)$$

Then from $\frac{\partial L}{\partial R_{ij}} = 0$ is obtained

$$h_{ij}+\frac{b_{ij}D_{ij}}{Q_{ij}}\left[f(x_{ij})dx_{ij}\right]+\gamma_{ij}\theta = 0$$

(13)

Then from $\frac{\partial L}{\partial \theta} \leq 0$ is obtained

$$\sum_i \quad \sum_j \quad \left(\gamma_{ij}(Q_{ij}+R_{ij})\right) - S - \mu_Y - Z_{1-\alpha} \leq 0$$

(14)

Next, we discuss the solution approach using normal distribution demand because it is to obtain the demand during the lead time needed to calculate the reorder point. For a normal distribution, two situations were studied: with and without warehouse space space constraints. Discuss the problem without warehouse space constraints because it aims to develop a solution for situations that may occur in practice.

3.2.1 Normal Distribution and Deterministic Lead Time without Space Constraints

In situations where sufficient historical data are available, the normal distribution probabilities for the demand and lead time can generally be estimated. Where it is known that $R_{ij} = \mu_{xij} + k\sigma_{xij}$ as presented by Alawneh and Zhang (2018). k is a safety factor that can be obtained from formulation $z_{1-\alpha}\sqrt{L_{ij}}$. If all of these formulations are substituted into the reorder point formulation, the following equation is obtained:

$$R_{ij} = (L_{ij}\times \mu_{dij}) + (z_{1-\alpha}\sqrt{L_{ij}})(\sqrt{L_{ij}\times\sigma_{dij}^2}) \quad (15)$$

Using (12), Q_{ij} can be obtained. In addition, because the warehouse space constraint is inactive, θ = 0.

3.2.2 Normal Distribution and Stochastic Lead Time without Space Constraints

The average and standard deviation for demand requirements during stochastic lead times can be formulated as follows:

$$\mu_{xij} = \mu_{Lij}\times \mu_{dij} \text{ dan } \sigma_{xij} = \sqrt{\mu_{Lij}\times\sigma_{dij}^2+\mu_{dij}^2\times\sigma_{Lij}^2} \quad (16)$$

Where it is known that $R_{ij} = \mu_{xij} + k\sigma_{xij}$ as presented by Alawneh and Zhang (2018). k is a safety factor that can be obtained from the formulation $z_{1-\alpha}\sqrt{L_{ij}}$. If all these formulations are substituted into the reorder point formulation, the equation is obtained:

$$R_{ij} = (\mu_{Lij}\times \mu_{dij}) + (z_{1-\alpha}\sqrt{L_{ij}})(\sqrt{\mu_{Lij}\times\sigma_{dij}^2+\mu_{dij}^2\times\sigma_{Lij}^2})$$

(17)

By using equation (12), Q_{ij} can be obtained. Also, since the warehouse space constraint is not active, θ = 0.

3.2.3 Normal Distribution with Space Constraints

When the warehouse space constraint is on, the optimal solution can be determined by solving the dual-Lagrangian problem given in the objective function. It is possible to solve this problem without considering the warehouse constraints through the existing equation on normal distribution and then check the constraints in equation (8). If the constraints are satisfied, we can conclude that we have found the optimal solution to the original problem. Otherwise, we can use the subgradient or bisection search method to solve the dual-Lagrangian problem. A unique solution exists because the problem is convex. From equation (9), we obtain :

$$\frac{\partial L}{\partial \theta} = \sum_i \quad (\gamma_{i2}(Q_{i2} + R_{i2}) + \gamma_{i1}(Q_{i1} + R_{i1})) - S - \mu_Y - Z_{1-\alpha} \leq 0$$

or can be written as:

$$g(\theta) = \sum_i \quad (\gamma_{i2}(Q_{i2} + R_{i2}) + \gamma_{i1}(Q_{i1} + R_{i1})) - S - \mu_Y - Z_{1-\alpha} \leq 0 \tag{18}$$

Because there is one variable and a unique solution, the bisection method can be used to determine the solution. Therefore, if there are two different values of θ_1 and θ_2 such that $g(\theta_1)$ and $g(\theta_2) < 0$. The following bisection method algorithm was proposed:

1. Let θ_1= 0 and let θ_2 be the smallest number such that $g(\theta_2) < 0$.
2. Let Q_1 , R_1 be the solution when $\theta = \theta_1$, and let Q_2 , R_2 be the solution when $\theta = \theta_2$.
3. Let $\theta = \frac{\theta_1 + \theta_2}{2}$, and solve for Q and R; then, find g(θ).
4. If g(θ) > 0 then $\theta_1 = \theta$, $Q_1 = Q$ dan $R_1 = R$; if If g(θ) < 0 then $\theta_2 = \theta$, $Q_2 = Q$ dan $R_2 = R$.
5. If $(g(\theta_1) - g(\theta_2)) < \varepsilon g$, then stop. Otherwise, return to step 3.

3.2.3 Verification of Model or Sensitivity Analysis

According to Harrell et al. (2003), model verification is the process of determining whether the created model reflects the right conceptual model. Model verification is performed by conducting a sensitivity analysis of the proposed model. To demonstrate the robustness or resilience of the proposed solution, a numerical analysis or sensitivity analysis was conducted that included key model parameters, including demand (total expected demand and average demand over lead time), product returns, and available warehouse space. Several scenarios were conducted in which each input parameter was increased or decreased as follows:

1. Demand will be increased by 10% and also decreased by 10%,
2. Average demand during lead time will be added 10% and also reduced 10%,
3. Total product returns will be added 10% and also subtracted 10%,
4. The warehouse space will be reduced by 10% and 20 %, respectively.

4. Results and Discussion

4.1 Normal distribution demands without space constraint

In the sports retailer company, the fast moving product inside the warehouse is huge with a lot of boxes for every item. The large number of products in the warehouse makes inventory control planning less effective and efficient. It is necessary to select the type of product that will be the object of research for inventory control planning. Selection of products that will be used as objects of study using the pareto method, better known as ABC classification analysis. The products that will be used as research objects are products in class A, because products in class A have the largest investment value, namely 80% of the total inventory investment made. All products in class A consist of 562 items that are the object of research.

The first calculation is done for normally distributed demand and deterministic lead time, but without any warehouse space constraints. The decision variables in the first calculation are Q_{i1} , R_{i1} , Q_{i2} , R_{i2}. The number of products that must be ordered for each product uses equation (12), but because there is no warehouse space constraint, then $\theta = 0$. Next, calculate the reorder point using equation (15). With the overall product details as in Table 1, the list of total demand cost results is normally distributed and has a deterministic lead time with no warehouse space or space constraints.

The second calculation is done for normally distributed demand and stochastic or uncertain lead times, but without any warehouse space or space constraints. The number of products that must be ordered for each product is still the same as the first calculation using equation (12), but without the constraints of warehouse space. Then, the calculation results of Q_{i1} , Q_{i2} will be the same as the first calculation. We can continue to use equation (17) to calculate the reorder point. With the overall product details as in Table 3 and 4, the list of total demand cost results is normally distributed and has a stochastic lead time with no warehouse space or space constraints.

Table 3. The results for total cost of normally distributed demand and deterministic lead time in the absence of warehouse space constraints.

No	Item Code	Item Description	Q_{i1}	Q_{i2}	R_{i1}	R_{i2}	Total Cost (offline + online)
1	2165271	Item A	26	970	4	541	Rp 2.249.728.439,03
2	2560115	Item B	20	764	3	331	
3	2881865	Item C	14	703	2	226	
4	492344	Item D	32	760	5	459	
5	2895865	Item E	12	472	2	103	

Table 4. The results of total cost of normally distributed demand and stochastic lead time with no warehouse space constraints

No	Item Code	Item Description	Q_{i1}	Q_{i2}	R_{i1}	R_{i2}	Total Cost (offline + online)
1	2165271	Item A	26	970	4	842	Rp 3.185.060.398,47
2	2560115	Item B	20	764	3	452	
3	2881865	Item C	14	703	2	356	
4	492344	Item D	32	760	5	710	
5	2895865	Item E	12	472	2	130	

After all the calculations are finished, check whether the new Q_{ij} and R_{ij} do not exceed the available warehouse space, and then we can use the warehouse space constraint equation as in equation (8). By using Q_{ij} and R_{ij} from the results of the calculations carried out for the products that were calculated, it was found that Q_{ij} and R_{ij} were optimal and did not exceed the available warehouse space. As shown in Table 5.

Table 5. Warehouse space calculation

	Warehouse space	Calculation Result Using New Q_{ij} and R_{ij}
First Calculation	$5.381\ m^3$	$6.112{,}202\ m^3$
Second Calculation	$6.207\ m^3$	$6.326{,}183\ m^3$

4.2 Normal distribution demands with space constraint

By using Q_{ij} and R_{ij} from the results of the calculations carried out for class A products, it was found that Q_{ij} and R_{ij} were optimal and did not exceed the available warehouse space. However, because warehouse space may exist in reality, calculations are performed using warehouse space constraints. Because the previous calculations were carried out with two types of lead time, namely deterministic lead time and stochastic lead time, warehouse space constraints were also carried out in the two types of lead time.

In the calculation with the previous deterministic lead time, $S + \mu_Y + z_{1-\alpha}\sigma_Y$ was $6.112{,}202\ m^3$ because if there are warehouse capacity constraints, the initial assumption must be smaller than the equation $\sum_i \left(\gamma_{i2}(Q_{i2} + R_{i2}) + \gamma_{i1}(Q_{i1} + R_{i1})\right)$. It is assumed that $S + \mu_Y + z_{1-\alpha}\sigma_Y$ has a value of 5.000 m^3; therefore, the calculation results of Q_{ij} and R_{ij} do not meet the constraints of limited warehouse capacity. Continue with the calculation using the bisection method to obtain the optimal Q_{ij} and R_{ij} in the presence of limited warehouse capacity constraints. Meanwhile, the stochastic calculation of lead time $S + \mu_Y + z_{1-\alpha}\sigma_Y$ is $6.326{,}183\ m^3$ because if there are warehouse capacity constraints, the initial assumption must be smaller than the equation $\sum_i \left(\gamma_{i2}(Q_{i2} + R_{i2}) + \gamma_{i1}(Q_{i1} + R_{i1})\right)$. It is assumed that $S + \mu_Y + z_{1-\alpha}\sigma_Y$ has a value of 6.000 m^3, so the calculation results of Q_{ij} and R_{ij} do not meet the constraints of limited warehouse capacity. Continued with the calculation using the bisection method to get the optimal Q_{ij} and R_{ij} in the presence of limited warehouse capacity constraints. As shown in Table 6, 7, 8 and 9.

Table 6. Bisection method for deterministic lead time

Iterations	θ_1	θ_2	θ	g (θ_1)	$\sum_i \sum_j \gamma_{ij}(Q_{ij} + R_{ij})$	g(θ)
1	0	15000	7500	5381	5104	104
2	7500	15000	11250	5104	4995	-5

Table 7. Bisection method for stochastic lead time

Iterations	θ_1	θ_2	θ	g (θ_1)	$\sum_i \sum_j \gamma_{ij}(Q_{ij} + R_{ij})$	g(θ)
1	0	10000	5000	6207	6021	21
2	5000	10000	7500	5939	5939	-61

Table 8. Total cost using bisection method for deterministic lead time

No	Item Code	Item Description	Q_{i1}	Q_{i2}	R_{i1}	R_{i2}	Total Cost (offline + online)
1	2165271	Item A	25	924	4	541	Rp 2.249.284.935,81
2	2560115	Item B	19	735	3	331	
3	2881865	Item C	14	683	2	226	
4	492344	Item D	32	732	5	459	
5	2895865	Item E	11	454	2	103	

Table 9. Total cost using bisection method for stochastic lead time

No	Item Code	Item Description	Q_{i1}	Q_{i2}	R_{i1}	R_{i2}	Total Cost (offline + online)
1	2165271	Item A	25	938	4	842	Rp 3.184.025.822,45
2	2560115	Item B	19	744	3	452	
3	2881865	Item C	14	689	2	356	
4	492344	Item D	32	740	5	710	
5	2895865	Item E	11	460	2	130	

4.3 Sensitivity Analysis

Sensitivity analysis was performed on the calculation of the normal distribution with deterministic lead time without warehouse capacity because it has the lowest total cost. The results of several scenarios conducted in which each input parameter was increased or decreased are as follows in Table 10:

Table 10. Comparison of total cost of sensitivity analysis results

No	Scenario	Total Cost Warehouse
1	D	Rp 2.249.728.439,03
2	D + 10%	Rp 2.329.731.749,50
3	D - 10%	Rp 2.165.369.754,55
4	μ + 10%	Rp 2.249.728.439,03
5	μ - 10%	Rp 2.246.136.458,50
6	E + 10%	Rp 2.260.255.343,02
7	E - 10%	Rp 2.239.201.535,04
8	S - 10%	Rp 2.249.728.439,03
9	S - 20%	Rp 2.248.947.741,87

Table 10 describes the differences in the total costs obtained by calculating the different changes in the parameters of the proposed model. Increasing the expected annual demand (D) by 10% increased the number of order sizes, reorder points, and total warehouse costs by an average of 3.87%, 7.55%, and 3.57%, respectively. Meanwhile, if the expected annual demand decreases by 10%, the number of order sizes, reorder points, and total warehouse costs by an average of -4.13%, -7.56%, and -3.74%. Then, the number of order sizes and reorder points does not increase if the demand during lead time (μ) increases by 10% because the results of rounding up per box per pcs have the same value; it only increases slightly if it is itemized or not rounded up. Consequently, the total cost did not change.

However, if the demand during lead time (μ) is decreased by 10%, the reorder point and total warehouse costs can be decreased by an average of -14.6% and -0.16%, respectively, whereas the number of order sizes does not change because the rounding results are the same. By increasing the total expected annual product return by 10%, the total warehouse cost increases by 0.47%, whereas if it is decreased by 10%, it can also decrease the total warehouse cost by -0.47% with a decrease in the reorder point of -7.56%. Lowering the warehouse capacity by 10% will not make a difference because it is still within the safe limit of the warehouse capacity. However, lowering the warehouse capacity by 20% reduced the number of order sizes by an average of 3.94%. It is noteworthy that warehouse capacity has a significant effect on the number of orders. This is because the system reduces the safety stock if it encounters space issues from the number of order sizes. By comparing the results, it can be proven that the proposed model is optimal.

5. Conclusions and Future Research

This research has developed a model to determine the number of orders and reorder points for dual-channel warehouses, namely offline and online. The model was developed based on Alawneh and Zhang's (2018) dual-channel warehouse inventory model by applying the warehouse capacity. The development of the model lies in the presence of offline and online product returns and eliminates backorder costs because it does not fit the situation in sports retail companies.

The developed model considers the fulfillment of storage on offline and online shelves or channels, warehouse capacity, order costs, storage costs, and product return costs. It also considers the minimum total warehouse cost. The model also considers demand uncertainty and delivery lead-time. A closed-form solution is developed using the normal distribution in the absence of limited warehouse capacity constraints and an iterative algorithm using the bisection method for cases with limited warehouse capacity constraints. The products used as research objects were products in class A, with 562 items in the warehouse.

Under the condition of a normally distributed demand and deterministic lead time, a lower total warehouse cost of Rp. 2,249,728,439.03 is obtained when compared to when the demand conditions are normally distributed with stochastic lead time or uncertainty, namely Rp. 3,185,060,398.47. Under the conditions of normal distributed demand and deterministic lead time, for class A items, the total number of order sizes (Q) on offline channels is 107,582 pcs with a reorder point (R) of 98,604 pcs. On the online channel, there are 21,904 pcs for the total number of order sizes (Q) and 1,510 pcs for the reorder point (R). Meanwhile, under conditions of normally distributed demand and stochastic lead time, for class A items, the total number of order sizes (Q) on the offline channel is 107,582 pcs with a reorder point (R) of 188,954 pcs. On the online channel, 21,904 pcs for the total number of order sizes (Q) and 1,510 pcs for the reorder point (R). The results of the calculation of all class A items for normally distributed demand with deterministic or stochastic lead times did not exceed the capacity of the warehouse provided.

In addition to determining the optimal inventory policy for a dual-channel warehouse, the sensitivity analysis also shows that the proposed model offers an optimal solution and provides a tool for decision support for companies operating in the context of dual-channel warehouses. Future research could develop models using conditions that do not have historical sales, can compare the results between single- and dual-channel accompanied by an analysis from an economic point of view, so that it can be a more accurate input to help companies make decisions and can add other constraints, such as limited warehouse capacity, the cost of returning goods, backorder transportation, or even a dual-channel warehouse that is sustainable and environmentally friendly.

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