A New Approach for Solving Simple Plant Location Problem (SPLP)

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Abstract

The Simple Plant Location Problem (SPLP), an NP-hard combinatorial problem, is used extensively to decide the minimum number of plants with unlimited production capacity to serve a certain set of markets demand. The objective function is to minimize the sum of fixed costs of locating the plants and variable cost of transportation of moving goods from plants to markets. Strong formulation of SPLP gives better bounds than weak formulations of SPLP. Hence in a branch and bound method, we find that strong formulation processes lesser number of nodes. But the number of strong constraints are quadratic (number of weak constraints are linear); hence at each node we take more time to process the node (Sharma and Verma (2012)) and weak formulation does better in terms of CPU time to give optimal solution. We add few most promising constraints to weak formulations and see that weak + few promising strong constraint formulation of SPLP performs the best.

Keywords
Simple Plant Location Problem, NP Hard, Integer Programming, Valid Inequality, Strong Formulation.

1. Introduction

A "Simple Plant Location Problem," or SPLP, is a problem in which unrestricted-size facilities are distributed among 'm' alternative sites or locations with the goal of reducing the overall cost of providing fixed demands defined at 'n' independent locations. "The SPLP is one of the simplest mixed integer problems," Guignard and Spielberg(1979) write, "exhibiting all of the normal combinatorial difficulties of mixed (0-1) programming while also having a structure that begs the application of numerous specialized techniques." This assertion implies that SPLP is a difficult problem to solve, or, to put it another way, that an exact polynomial time bounded algorithm for its solution will never be discovered. The decision issue associated with SPLP is NP complete, while SPLP itself is NP hard. The formal theory of NP completeness is built on the concept of deterministic and non-deterministic Turing machines.

For an excellent treatise on SPLP reader is referred to Jakobs and Pruzan (1983). Sharma and Muralidhar (2009) added a new valid inequality (linking constraints that link location and distribution variables) which was discovered to be a new weak constraint.

For more complex location problems (multistage, multi-period, inventory and shortage variables) reader is referred to Sharma (2018, 2018, 2018 and 2019) and Sharma (2019, 2019, 2020, 2021 and 2022).

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In this paper we give six different formulations of SPLP using two weak and one strong formulation of SPLP given in literature and find that it is best to add most promising strong constraints to either of the two weak formulations available in literature. This is useful contribution we make and in this paper we give extensive numerical results.

2. Standard Formulation of SPLP

Consider a set i = 1..m candidate sites for facility location, and a set k = 1..n of demand locations. Each facility i has a fixed cost f_i. Every demand point k has a demand D_k, and C_{ik} is the unit transportation cost from facility i to k, which can be given in terms of per unit based on the type of transport and the travel distance between the two points. If a location 'i' is selected to supply market 'k' with quantity X_{ik}, then it means plant is established at location i, and hence binary variable associated with location i, 'y_i' will take value 1 and 0 otherwise. We can normalize each demand by dividing by total demand of the markets, i.e. $d_k = D_k / \sum_1^n D_k$. Similarly x_{ik} can be defined as quantity shipped from plant 'i' to market 'k' as a fraction of total market demand i.e. $x_{ik} = X_{ik} / \sum_1^n D_k$.

Mathematically, the earliest SPLP is formulated in the following way (Krarup & Pruzan 1983)

Minimize $\sum i \sum k \, (x_{ik} * C_{ij}) + \sum i \, (y_i * f_i)$ (1)

s.t.

$\sum i \, x_{ik} >= d_k$ for all 'k' (2)

$\sum i \, x_{ik} <= y_i$ for all 'i' (3)

$x_{ik} \geq 0$ for all i,k (4)

$y_i \in \{0,1\}$ all i. (5)

Generally the integer condition and the binary condition is relaxed to find the fair enough solution. But again that does not guarantee the accuracy.

$0 \leq y_i \leq 0$ for all i (6)

The above formulation (Eq.1 to 6) is called weak formulation (Let us call it Weak1). If in place of constraint (3) we use constraint (7), then it is called as strong formulation (SPLP becomes Eq. 1, 2, 4, 5, &7).

$x_{ik} <= d_k * y_i$ for all i, k (7)

Sharma and Muralidhar (2009) gave a new valid inequality (8) that improved the formulation.

$y_i - (1/n) \sum k (x_{ik}/d_k) \geq 0, \forall i = 1, \ldots, m$ (8)

This is another form of weak formulation (Let us call Weak formulation 2: Eq. 1,2,4,5, & 8)

3. A New Formulation of SPLP

We propose few new strong constraints to improve the relaxed solution. Here in place of total strong constraint (7), we are using only 10% of them (that is we are using only 5 constraint, for 50 potential locations and 50 demand points problem i.e. for 50X50 problem) that are most promising, here for us the most promising constraints are those whose market demands are lowest (Say dm1, dm2, dm3, dm4 and dm5). The lowest market demand among total 50 is dm1. Similarly dm1< dm2< dm3< dm4 < dm5 and these all 5 demand are lesser than other 45 markets for a 50 market problem. Therefore the most promising 5 constraints are:

$x_{i\,m1} \leq y_i * d_{m1}$ (9)

$x_{i\,m2} \leq y_i * d_{m2}$ (10)

$x_{i\,m3} \leq y_i * d_{m3}$ (11)

$x_{i\,m4} \leq y_i * d_{m4}$ (12)

$x_{i\,m5} \leq y_i * d_{m5}$ (13)

By using these weak and strong constraints we can define the following 6 models as A1, A2, A3, B1, B2 and B3 -

3.1 Model A1: (A1=Weak1 + Strong constraints)

Eq. 1, 2, 3, 4, 5, &7

3.2 Model A2: (A1=Weak2 + Strong constraint)

Eq. 1, 2, 4, 5, 7, & 8

Eq. (8) is recognized as new weak (Weak 2) constraint in place of Eq. (3) as given by Sharma and Muralidhar (2009).

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3.3 Model A3: (A3=Weak1 + Weak2 + Strong constraint)

Eq. 1, 2, 3, 4, 5, 7, & 8

Eq. (3) is recognized as weak (Weak 1) constraint and Eq. (8) is recognized as new weak (Weak 2) constraint given by Sharma and Muralidhar (2009).

3.4 Model B1: (B1=Weak1 + Most promising strong constraints)

Here in place of strong constraint (7), we use only the most promising constraints Eq. 9 to Eq. 13 (and they are the 10% of total)

Eq. 1, 2, 3, 4, 5, 9,10,11,12, &13.

3.5 Model B2: (B2=Weak2 + Most promising strong constraints)

We replace weak constraint Eq. (3) by Eq. (8) in above formulation

Eq. 1, 2, 4, 5, 8, 9,10,11,12, &13.

3.6 Model B3: (B3= Weak1 + Weak2 + Most promising strong constraints)

We add weak constraint Eq. (3) in above formulation

Eq. 1, 2, 3, 4, 5, 8, 9,10,11,12, &13.

Here we give sample 5 most promising constraints with d_{m1} to d_{m5} being the smallest values of demand of any market.

4. Comparison of Different SPLP Models

We generate the random data for this problem and compare the above models in terms of performance both in terms of objective function, computational efficiency in terms of number nodes used in branch and bound algorithm to solve SPLP. We designed this problem for 50 markets and 50 plants (those having incapacitated production capability). So for solving the problems we generated the data set randomly with three variables normalized demand, fixed cost for opening plant at location i and cost of transportation between node i and k (d_k, f_i, c_{ik}) and we solved 30 of this type problems so we generated 30 different dataset. GAMS code is used to find the solution of these problems. Because the problem formulation is MIP, each model is solved in GAMS using branch and bound, and the solver used is CPLEX. AMD Ryzen 5 3500U with Radeon Vega Mobile Gfx 2.10 GHz is used to run the programs, and CPU time was recorded.

The number of iterations, number of nodes, cpu solution time (root relaxation solution time), elapsed real time and objective function value were chosen as comparison criteria. The comparisons are made in nine levels, on above six models, i.e. between A1 & A2, A1 & A3, A2 & A3, B1 & B2, B1& B3, B2& B3, A1 & B1, A2 & B2 and A3 & B3 (Table 1-Table 3).

Table 1. Comparing model A1 and A2 results

SrN	Iterations		Number of Nodes		Elapsed real time (in sec)		Root relaxation solution time (in sec)		Objective function value	
	A1	A2	A1	A2	A1	A2	A1	A2	A1	A2
1	2408	2801	3	7	0.36	0.47	0.19	0.11	3499.5507	3468.3891
2	2156	2354	5	6	0.48	0.36	0.28	0.11	4869.7924	4909.79
3	4545	3892	9	9	0.78	0.52	0.3	0.13	4785.2337	4785.2337
4	3114	2533	7	7	0.64	0.41	0.31	0.13	4508.3098	4508.3098
5	2835	2382	7	7	0.58	0.38	0.22	0.08	4560.4523	4560.4523
6	1183	1240	3	3	0.19	0.11	0.13	0.06	4228.582	4228.582
7	1126	1068	3	3	0.2	0.13	0.13	0.06	4017.2566	4017.2566

8	3014	2871	5	5	0.38	0.28	0.16	0.06	4631.9184	4631.9184
9	3045	2272	5	5	0.83	0.33	0.48	0.13	4733.4272	4733.4272
10	2536	2041	5	5	0.66	0.34	0.33	0.09	4657.791	4657.791
11	5988	5305	15	15	0.72	0.31	0.27	0.08	4786.6477	4786.6477
12	2374	1805	5	7	0.42	0.22	0.2	0.05	4755.8074	4755.8074
13	101	86	0	0	0.05	0.05	0.02	0	2561.7659	2561.7659
14	2627	2336	5	5	0.36	0.25	0.19	0.08	4785.8278	4785.8278
15	1895	1500	0	0	0.86	0.28	0.47	0.14	4714.5194	4714.5194
16	3035	2518	5	5	0.67	0.27	0.33	0.06	4624.9854	4624.9854
17	2333	1723	5	5	0.7	0.41	0.3	0.13	4728.5929	4728.5929
18	4947	4103	13	13	0.66	0.45	0.28	0.13	4658.2496	4658.2496
19	2476	1917	5	5	0.64	0.3	0.34	0.08	4697.2881	4697.2881
20	3146	2464	5	5	0.78	0.42	0.28	0.14	4625.0869	4625.0869
21	2480	2153	5	3	0.38	0.23	0.16	0.06	4720.4251	4720.4251
22	3412	2739	9	9	0.58	0.17	0.17	0.06	4682.1751	4682.1751
23	2282	2056	3	3	0.38	0.27	0.17	0.08	4687.4422	4687.4422
24	3086	2413	5	5	0.44	0.28	0.2	0.06	4690.0325	4690.0325
25	4850	4394	13	13	0.86	0.53	0.34	0.13	4766.9392	4766.9392
26	1675	1508	0	0	0.41	0.16	0.2	0.06	4462.6847	4462.6847
27	3982	3209	7	7	0.56	0.53	0.19	0.13	4785.9239	4785.9239
28	1920	1773	3	3	0.3	0.19	0.19	0.11	4525.9369	4525.9369
29	4264	3545	9	9	0.55	0.25	0.19	0.06	4755.0864	4755.0864
30	2480	1996	3	3	0.44	0.27	0.22	0.08	4672.3296	4672.3296

Table 2. Comparison of Model A1 and A2 for Elapsed Time, Root Mean Time, Iterations and Number of Nodes

Criterion	μA1	μA2	|t-value|	Null Hypothesis
Elapsed Time	0.5287	0.3057	7.916	A2 is sig. better than A1
Iterations	2843.83	2433.23	7.231	A2 is sig. better than A1
Number of Nodes	5.57	5.73	1.000	A2 is as good as A1

From Table 2 we can interpret that model A2 is showing significant change over model A1 in terms of elapsed time, iterations and its values are less for model A2. So A2 is significantly better than model A1 where elapsed time and iterations are prime concerns. For number of nodes both models are showing no significant difference.

Table 3. Comparison of Model A1 And A2 for Objective Function Value

Criterion	μA1	μA2	|t-value|	Null Hypothesis
Objective function value	4539.3353	4539.62	0.171	A2 is as good as A1
Root Mean Time	0.2413	0.0893	10.733	A2 is sig. better than A1

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From Table 3 and Table 4, we can also interpret that there is no significant change in objective function value for both models A1 and A2. However, there is a significant change in root mean time and its value is less for model A2 so A2 is significantly better than A1 where root mean time (cpu time) is a prime concern. So concluding from above discussion we can say that model A2 is better than model A1.

Table 4. Comparing model A1 and A3 results

SrN	Iterations		Number of Nodes		Elapsed real time (in sec)		Root relaxation solution time (in sec)		Objective function value	
	A1	A3	A1	A3	A1	A3	A1	A3	A1	A3
1	2408	1989	3	3	0.36	0.44	0.19	0.25	3499.5507	3439.4497
2	2156	2735	5	5	0.48	0.58	0.28	0.3	4869.7924	4890.6099
3	4545	4672	9	9	0.78	0.91	0.3	0.39	4785.2337	4785.2337
4	3114	3373	7	7	0.64	0.72	0.31	0.33	4508.3098	4508.3096
5	2835	3104	7	7	0.58	0.67	0.22	0.27	4560.4523	4560.4523
6	1183	1252	3	3	0.19	0.19	0.13	0.11	4228.582	4228.582
7	1126	1350	3	3	0.2	0.33	0.13	0.2	4017.2566	4017.2566
8	3014	3244	5	5	0.38	0.45	0.16	0.19	4631.9184	4631.9184
9	3045	2498	5	5	0.83	0.98	0.48	0.45	4733.4272	4733.4272
10	2536	2639	5	5	0.66	0.69	0.33	0.38	4657.791	4657.791
11	5988	6278	15	15	0.72	0.56	0.27	0.25	4786.6477	4786.6477
12	2374	2293	5	7	0.42	0.48	0.2	0.23	4755.8074	4755.8074
13	101	101	0	0	0.05	0.05	0.02	0	2561.7659	2561.7659
14	2627	2854	5	5	0.36	0.42	0.19	0.22	4785.8278	4785.8278
15	1895	2036	0	0	0.86	0.94	0.47	0.41	4714.5194	4714.5194
16	3035	2977	5	5	0.67	0.44	0.33	0.19	4624.9854	4624.9854
17	2333	2345	5	5	0.7	0.67	0.3	0.33	4728.5929	4728.5929
18	4947	4872	13	13	0.66	0.7	0.28	0.28	4658.2496	4658.2496
19	2476	2576	5	5	0.64	0.67	0.34	0.38	4697.2881	4697.2881
20	3146	3513	5	5	0.78	0.78	0.28	0.34	4625.0869	4625.0869
21	2480	2747	5	5	0.38	0.33	0.16	0.19	4720.4251	4720.4251
22	3412	3438	9	5	0.58	0.45	0.17	0.2	4682.1751	4682.1751
23	2282	2449	3	3	0.38	0.42	0.17	0.22	4687.4422	4687.4422
24	3086	2949	5	5	0.44	0.45	0.2	0.2	4690.0325	4690.0325
25	4850	5391	13	13	0.86	0.94	0.34	0.36	4766.9392	4766.9392
26	1675	1728	0	0	0.41	0.47	0.2	0.27	4462.6847	4462.6847
27	3982	4052	7	7	0.56	0.67	0.19	0.27	4785.9239	4785.9239
28	1920	2001	3	3	0.3	0.31	0.19	0.22	4525.9369	4525.9369
29	4264	4643	9	9	0.55	0.52	0.19	0.2	4755.0864	4755.0857
30	2480	2516	3	3	0.44	0.44	0.22	0.23	4672.3296	4672.3296

Table 5. Comparison Table of Model A1 and A3 For Elapsed Time, Root Mean Time, Iterations and Number of Nodes

Criterion	μA1	μA3	|t-value|	Null Hypothesis

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Elapsed Time	0.5287	0.5557	1.734	A1 is sig. better than A3
Root Mean Time	0.2413	0.2620	2.472	A1 is sig. better than A3
Iterations	2843.83	2953.83	2.559	A1 is sig. better than A3
Number of Nodes	5.57	5.50	0.441	A1 is as good as A3

From Table 5 we can interpret that model A3 is showing significant change over model A1 in terms of elapsed time, root mean time and iterations and its values are less for model A1. A1 is suggested to use over model A3 where elapsed time, root mean time and iterations are prime concerns. For number of nodes both models are showing no significant difference.

Table 6. Comparison of Model A1 And A3 for Objective Function Value

Criterion	µA1	µA3	|t-value|	Null Hypothesis
Objective function value	4539.3353	4538.0258	0.611	A1 is as good as A3

From Table 6 we can also interpret that there is no significant change in objective function value for both models A1 and A3. So from above discussion we can conclude that model A1 is better than model A3.

Table 7. Comparing model A2 and A3 results

SrN	Iterations		Number of Nodes		Elapsed real time (in sec)		Root relaxation solution time (in sec)		Objective function value	
	A2	A3	A2	A3	A2	A3	A2	A3	A2	A3
1	2801	1989	7	3	0.47	0.44	0.11	0.25	3468.3891	3439.4497
2	2354	2735	6	5	0.36	0.58	0.11	0.3	4909.79	4890.6099
3	3892	4672	9	9	0.52	0.91	0.13	0.39	4785.2337	4785.2337
4	2533	3373	7	7	0.41	0.72	0.13	0.33	4508.3098	4508.3096
5	2382	3104	7	7	0.38	0.67	0.08	0.27	4560.4523	4560.4523
6	1240	1252	3	3	0.11	0.19	0.06	0.11	4228.582	4228.582
7	1068	1350	3	3	0.13	0.33	0.06	0.2	4017.2566	4017.2566
8	2871	3244	5	5	0.28	0.45	0.06	0.19	4631.9184	4631.9184
9	2272	2498	5	5	0.33	0.98	0.13	0.45	4733.4272	4733.4272
10	2041	2639	5	5	0.34	0.69	0.09	0.38	4657.791	4657.791
11	5305	6278	15	15	0.31	0.56	0.08	0.25	4786.6477	4786.6477
12	1805	2293	7	7	0.22	0.48	0.05	0.23	4755.8074	4755.8074
13	86	101	0	0	0.05	0.05	0	0	2561.7659	2561.7659
14	2336	2854	5	5	0.25	0.42	0.08	0.22	4785.8278	4785.8278
15	1500	2036	0	0	0.28	0.94	0.14	0.41	4714.5194	4714.5194
16	2518	2977	5	5	0.27	0.44	0.06	0.19	4624.9854	4624.9854
17	1723	2345	5	5	0.41	0.67	0.13	0.33	4728.5929	4728.5929
18	4103	4872	13	13	0.45	0.7	0.13	0.28	4658.2496	4658.2496

19	1917	2576	5	5	0.3	0.67	0.08	0.38	4697.2881	4697.2881
20	2464	3513	5	5	0.42	0.78	0.14	0.34	4625.0869	4625.0869
21	2153	2747	3	5	0.23	0.33	0.06	0.19	4720.4251	4720.4251
22	2739	3438	9	5	0.17	0.45	0.06	0.2	4682.1751	4682.1751
23	2056	2449	3	3	0.27	0.42	0.08	0.22	4687.4422	4687.4422
24	2413	2949	5	5	0.28	0.45	0.06	0.2	4690.0325	4690.0325
25	4394	5391	13	13	0.53	0.94	0.13	0.36	4766.9392	4766.9392
26	1508	1728	0	0	0.16	0.47	0.06	0.27	4462.6847	4462.6847
27	3209	4052	7	7	0.53	0.67	0.13	0.27	4785.9239	4785.9239
28	1773	2001	3	3	0.19	0.31	0.11	0.22	4525.9369	4525.9369
29	3545	4643	9	9	0.25	0.52	0.06	0.2	4755.0864	4755.0857
30	1996	2516	3	3	0.27	0.44	0.08	0.23	4672.3296	4672.3296

Table 8. Comparison of Model A2 and A3 For Elapsed Time, Root Mean Time, Iterations and Number of Nodes

Criterion	µA2	µA3	|t-value|	Null Hypothesis
Elapsed Time	0.3057	0.5557	8.898	A2 is sig. better than A3
Root Mean Time	0.0893	0.2620	13.602	A2 is sig. better than A3
Iterations	2433.23	2953.83	7.524	A2 is sig. better than A3
Number of Nodes	5.73	5.50	1.157	A2 is as good as A3

From Table 8, we can interpret that model A3 is showing significant change over model A2 in terms of elapsed time, root mean time and iterations and its values are less for model A2. A2 is significantly better than model A3 where elapsed time, root mean time and iterations are prime concerns. For number of nodes both models are showing no significant difference.

Table 9. Comparison of Model A2 and A3 for Objective Function Value

Criterion	µA2	µA3	|t-value|	Null Hypothesis
Objective function value	4539.62	4538.0258	1.409	A2 is as good as A3

From Table 9, we can also interpret that there is no significant change in objective function value for both models A2 and A3. So from above discussion we can conclude that model A2 is better than model A3. So among A1, A2 and A3 we can conclude A2 is the best. Among models A1, A2 and A3 we find that A2 is the best.

Table 10. Comparing model B1 and B2 results

SrN	Iterations	Number of Nodes	Elapsed real time (in sec)	Root relaxation solution time (in sec)	Objective function value

	B1	B2	B1	B2	B1	B2	B1	B2	B1	B2
1	290992	89069	4045	935	31.44	6.69	0.05	0.03	3544.7765	3294.6263
2	94463	69782	1332	1161	21.58	7.64	0.03	0.05	4841.781	4842.3169
3	112255	199875	1414	2531	25.61	18.78	0.02	0.02	4727.1273	4727.1273
4	75368	122217	1062	1621	12.11	8.25	0.02	0.03	4502.1034	4502.1034
5	88970	126505	1210	1733	23.09	20.14	0.02	0.02	4488.4645	4488.4645
6	43198	45599	816	762	6.16	5.2	0.02	0.03	4120.8908	4120.8908
7	41633	49745	661	852	6.84	5.17	0.02	0.05	3957.0832	3957.0832
8	130181	188791	1858	2798	28.91	24.8	0.05	0.02	4621.6985	4621.6985
9	132000	180781	2358	2684	32.23	23.83	0.05	0.05	4643.2034	4643.2034
10	182472	178848	2385	2347	27.81	25.14	0.02	0.05	4576.033	4576.033
11	164296	285601	2158	3330	23.86	27.25	0.03	0.02	4784.3596	4784.3596
12	105482	119359	1399	1611	16.16	6.66	0.02	0.02	4531.0877	4531.0877
13	125	135	0	0	0.05	0.05	0	0	2560.8532	2560.8532
14	137748	150253	2320	2183	24.63	24.31	0.02	0.03	4739.0711	4739.0711
15	125758	162267	2817	2353	24.33	24.55	0.03	0.02	4603.7884	4605.5839
16	142986	128141	2157	1584	15.92	12.63	0.02	0.02	4604.6414	4604.6414
17	106277	136306	1455	1755	25.25	19.19	0.02	0.03	4636.3388	4636.3388
18	125506	207104	1924	2865	20.7	18.42	0.03	0.02	4648.1211	4648.1211
19	106525	192134	1585	2527	27.84	21.05	0.03	0.03	4613.832	4613.832
20	120172	183579	1264	2227	27.89	19.45	0.02	0.03	4624.6722	4624.6722
21	133603	131898	2279	1674	14.64	12.78	0.02	0.02	4696.8817	4696.8817
22	82519	91530	913	1282	13.97	5.81	0.02	0	4680.9568	4680.9568
23	132137	184376	2384	2885	14.55	11.34	0.02	0.02	4545.9055	4545.9055
24	117462	202795	1781	2700	15.06	18.39	0.02	0.03	4601.8459	4601.8459
25	177755	236761	2360	2496	33.89	32.72	0.05	0.03	4746.9133	4746.9133
26	71726	86839	1080	1082	9	4.91	0.03	0	4436.9961	4436.9961
27	140714	222789	1929	3174	24.81	20.44	0.03	0.02	4774.4725	4774.4725
28	75429	106765	1349	1622	6.92	5.48	0.02	0.02	4488.4723	4488.4723
29	133846	203927	1776	2020	15.03	18.91	0.02	0	4753.2123	4753.2123
30	119294	181383	1949	2904	15.03	12.72	0.02	0.02	4651.5085	4651.5085

Table 11. Comparison of Model B1 And B2 for Elapsed Time, Root Mean Time, Iterations and Number af Nodes

Criterion	μB1	μB2	|t-value|	Null Hypothesis
Elapsed Time	19.5103	15.4233	4.024	B2 is sig. better than B1
Root Mean Time	0.0257	0.0243	0.486	B2 is as good as B1
Iterations	117029.73	148838.47	3.060	B2 is sig. better than B1
Number of Nodes	1734.00	1989.93	1.706	B2 is sig. better than B1

From Table 11, we can interpret that model B2 is showing significant change over model B1 in terms of elapsed time, iterations and number of nodes and its values are less for model B1. B1 is suggested to use over model B2 where

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elapsed time, iterations and number of nodes are prime concerns. For root mean time both models are showing no significant difference.

Table 12. Comparison of Model B1 and B2 for Objective Function Value:

Criterion	μB1	μB2	|t-value|	Null Hypothesis
Objective function value	4491.5697	4483.3091	0.990	B2 is as good as B1

From Table 12 and Table 13, we can also interpret that there is no significant change in objective function value for both the modes B1 and B2. So from above discussion we can conclude that model B1 is as good as model B2.

Table 13. Comparing model B1 and B3 results

SrN	Iterations		Number of Nodes		Elapsed real time (in sec)		Root relaxation solution time (in sec)		Objective function value	
	B1	B3	B1	B3	B1	B3	B1	B3	B1	B3
1	290992	264892	4045	2537	31.44	28.8	0.05	0.05	3544.7765	3510.7987
2	94463	158191	1332	1747	21.58	24.91	0.03	0.05	4841.781	4829.5536
3	112255	208961	1414	2436	25.61	33.7	0.02	0.03	4727.1273	4727.1273
4	75368	140106	1062	1321	12.11	23.55	0.02	0.03	4502.1034	4502.1034
5	88970	120027	1210	1354	23.09	25.02	0.02	0.05	4488.4645	4488.4645
6	43198	93609	816	937	6.16	14.45	0.02	0.06	4120.8908	4120.8908
7	41633	51526	661	733	6.84	7.63	0.02	0.02	3957.0832	3957.0832
8	130181	200810	1858	1836	28.91	44.59	0.05	0.05	4621.6985	4621.6985
9	132000	223037	2358	2738	32.23	41.02	0.05	0.06	4643.2034	4643.2034
10	182472	187471	2385	1583	27.81	41.97	0.02	0.08	4576.033	4576.033
11	164296	247375	2158	2116	23.86	30.73	0.03	0.02	4784.3596	4784.3596
12	105482	185381	1399	2320	16.16	21.38	0.02	0.03	4531.0877	4531.0877
13	125	155	0	0	0.05	0.03	0	0	2560.8532	2560.8532
14	137748	170291	2320	1565	24.63	45.45	0.02	0.05	4739.0711	4739.0711
15	125758	168939	2817	2252	24.33	42.33	0.03	0.03	4603.7884	4605.5839
16	142986	160362	2157	1690	15.92	16.72	0.02	0.02	4604.6414	4604.6414
17	106277	144339	1455	1797	25.25	21.14	0.02	0.06	4636.3388	4636.3388
18	125506	208114	1924	1917	20.7	31.09	0.03	0.03	4648.1211	4648.1211
19	106525	199185	1585	2367	27.84	41.81	0.03	0.05	4613.832	4613.832
20	120172	190720	1264	1708	27.89	42.98	0.02	0.03	4624.6722	4624.6722
21	133603	187105	2279	2026	14.64	23.03	0.02	0.02	4696.8817	4696.8817
22	82519	171278	913	1607	13.97	22.09	0.02	0.02	4680.9568	4680.9568
23	132137	164008	2384	2348	14.55	22.61	0.02	0.03	4545.9055	4545.9055
24	117462	193740	1781	2091	15.06	21.86	0.02	0.06	4601.8459	4601.8459
25	177755	225180	2360	2133	33.89	48.58	0.05	0.05	4746.9133	4746.9133

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26	71726	154255	1080	1797	9	15.98	0.03	0.03	4436.9961	4436.9961
27	140714	172242	1929	1411	24.81	26.11	0.03	0.02	4774.4725	4774.4725
28	75429	145021	1349	1406	6.92	15.95	0.02	0.03	4488.4723	4488.4723
29	133846	214271	1776	1969	15.03	23.94	0.02	0.03	4753.2123	4753.2123
30	119294	191626	1949	2142	15.03	22	0.02	0.03	4651.5085	4651.5085

Table 14. Comparison of Model B1 and B3 for Elapsed Time, Root Mean Time, Iterations and Number of Nodes

Criterion	μB1	μB3	|t-value|	Null Hypothesis
Elapsed Time	19.5103	27.3817	7.145	B1 is sig. better than B3
Root Mean Time	0.0257	0.0373	3.843	B1 is sig. better than B3
Iterations	117029.73	171407.23	9.418	B1 is sig. better than B3
Number of Nodes	1734.00	1796.13	0.621	B1 is as good as B3

From Table 14, we can interpret that model B3 is showing significant change over model B1 in terms of elapsed time, root mean time and iterations and its values are less for model B1. B1 is significantly better than model B3 where elapsed time, root mean time and iterations are prime concerns. For number of nodes both models are showing no significant difference.

Table 15. Comparison of Model B1 and B3 For Objective Function Value

Criterion	μB1	μB3	|t-value|	Null Hypothesis
Objective function value	4491.5697	4490.0894	1.239	B1 is as good as B3

From Table 15, we can also interpret that there is no significant change in objective function value for both the modes B1 and B3. So from above discussion we can conclude that model B1 is better than model B3.

Table 16. Comparing model B2 and B3 results

SrN	Iterations		Number of Nodes		Elapsed real time (in sec)		Root relaxation solution time (in sec)		Objective function value	
	B2	B3	B2	B3	B2	B3	B2	B3	B2	B3
1	89069	264892	935	2537	6.69	28.8	0.03	0.05	3294.6263	3510.7987
2	69782	158191	1161	1747	7.64	24.91	0.05	0.05	4842.3169	4829.5536
3	199875	208961	2531	2436	18.78	33.7	0.02	0.03	4727.1273	4727.1273
4	122217	140106	1621	1321	8.25	23.55	0.03	0.03	4502.1034	4502.1034
5	126505	120027	1733	1354	20.14	25.02	0.02	0.05	4488.4645	4488.4645
6	45599	93609	762	937	5.2	14.45	0.03	0.06	4120.8908	4120.8908
7	49745	51526	852	733	5.17	7.63	0.05	0.02	3957.0832	3957.0832

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8	188791	200810	2798	1836	24.8	44.59	0.02	0.05	4621.6985	4621.6985
9	180781	223037	2684	2738	23.83	41.02	0.05	0.06	4643.2034	4643.2034
10	178848	187471	2347	1583	25.14	41.97	0.05	0.08	4576.033	4576.033
11	285601	247375	3330	2116	27.25	30.73	0.02	0.02	4784.3596	4784.3596
12	119359	185381	1611	2320	6.66	21.38	0.02	0.03	4531.0877	4531.0877
13	135	155	0	0	0.05	0.03	0	0	2560.8532	2560.8532
14	150253	170291	2183	1565	24.31	45.45	0.03	0.05	4739.0711	4739.0711
15	162267	168939	2353	2252	24.55	42.33	0.02	0.03	4605.5839	4605.5839
16	128141	160362	1584	1690	12.63	16.72	0.02	0.02	4604.6414	4604.6414
17	136306	144339	1755	1797	19.19	21.14	0.03	0.06	4636.3388	4636.3388
18	207104	208114	2865	1917	18.42	31.09	0.02	0.03	4648.1211	4648.1211
19	192134	199185	2527	2367	21.05	41.81	0.03	0.05	4613.832	4613.832
20	183579	190720	2227	1708	19.45	42.98	0.03	0.03	4624.6722	4624.6722
21	131898	187105	1674	2026	12.78	23.03	0.02	0.02	4696.8817	4696.8817
22	91530	171278	1282	1607	5.81	22.09	0	0.02	4680.9568	4680.9568
23	184376	164008	2885	2348	11.34	22.61	0.02	0.03	4545.9055	4545.9055
24	202795	193740	2700	2091	18.39	21.86	0.03	0.06	4601.8459	4601.8459
25	236761	225180	2496	2133	32.72	48.58	0.03	0.05	4746.9133	4746.9133
26	86839	154255	1082	1797	4.91	15.98	0	0.03	4436.9961	4436.9961
27	222789	172242	3174	1411	20.44	26.11	0.02	0.02	4774.4725	4774.4725
28	106765	145021	1622	1406	5.48	15.95	0.02	0.03	4488.4723	4488.4723
29	203927	214271	2020	1969	18.91	23.94	0	0.03	4753.2123	4753.2123
30	181383	191626	2904	2142	12.72	22	0.02	0.03	4651.5085	4651.5085

Table 17. Comparison of Model B2 And B3 for Elapsed Time, Root Mean Time, Iterations and Number of Nodes

Criterion	µB2	µB3	|t-value|	Null Hypothesis
Elapsed Time	15.4233	27.3817	9.659	B2 is sig. better than B3
Root Mean Time	0.0243	0.0373	5.022	B2 is sig. better than B3
Iterations	148838.47	171407.23	2.852	B2 is sig. better than B3
Number of Nodes	1989.93	1796.13	1.622	B2 is as good as B3

From Table 16 and Table 17, we can interpret that model B3 is showing significant change over model B2 in terms of elapsed time, root mean time and iterations and its values are less for model B2. B2 is significantly better than model B3 where elapsed time, root mean time and iterations are prime concerns. For number of nodes both models are showing no significant difference.

Table 18. Comparison of Model B2 and B3 for Objective Function Value

Criterion	µB2	µB3	|t-value|	Null Hypothesis
Objective function value	4483.3091	4490.0894	0.937	B2 is as good as B3

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From Table 18, we can also interpret that there is no significant change in objective function value for both the modes B2 and B3. So from above discussion we can conclude that model B2 is better than model B3. So among B1, B2 and B3 we can conclude B1 and B2 are equally good and both are better than B3.

Table 19. Comparison of Model A1 and B1 for Elapsed Time, Iterations and Number of Nodes

Criterion	µA1	µB1	|t-value|	Null Hypothesis
Elapsed Time	0.5287	19.5103	12.146	A1 is sig. better than B1
Iterations	2843.83	117029.73	12.302	A1 is sig. better than B1
Number of Nodes	5.57	1734.00	12.388	A1 is sig. better than B1

From Table 19, we can interpret that model B1 is showing significant change over model A1 in terms of elapsed time, iterations and number of nodes and its values are less for model A1. A1 is significantly better than model B1 where elapsed time, iterations and number of nodes are prime concerns.

Table 20. Comparison of Model A1 and B1 for Objective Function Value

Criterion	µA1	µB1	|t-value|	Null Hypothesis
Objective function value	4539.3353	4491.5697	4.819	B1 is sig. better than A1
Root Mean Time	0.2413	0.0257	12.633	B1 is sig. better than A1

From Table 20, we can also interpret that there is also significant change in objective function value and root mean time (that is cpu time) and its values are lesser for model B1. So B1 is significantly better than model A1 where objective function value and root mean time are prime concern. So from above discussion we can conclude that model B1 is better than model A1.

Table 21. Comparison of Model A2 and B2 for Elapsed Time, Iterations and Number of Nodes

Criterion	µA2	µB2	|t-value|	Null Hypothesis
Elapsed Time	0.3057	15.4233	9.914	A2 is sig. better than B2
Iterations	2433.23	148838.47	12.774	A2 is sig. better than B2
Number of Nodes	5.73	1989.93	13.410	A2 is sig. better than B2

From Table 21, we can interpret that model B2 is showing significant change over model A2 in terms of elapsed time, iterations and number of nodes and its values are less for model A2. A2 is suggested to use over model B2 where elapsed time, iterations and number of nodes are prime concerns.

Table 22. Comparison of Model A2 and B2 for Objective Function Value and Root Mean Time

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Criterion	µA2	µB2	|t-value|	Null Hypothesis
Objective function value	4539.6298	4483.3091	5.529	B2 is sig. better than A2
Root Mean Time	0.0893	0.0243	11.078	B2 is sig. better than A2

From Table 22 we can also interpret that there is also significant change in objective function value and root mean time and its values are lesser for model B2. So B2 is suggested to use over model A2 where objective function value and root mean time are a prime concern. So from above discussion we can conclude that model B2 is better than model A2.

Table 23. Comparison of Model A3 And B3 for Elapsed Time, Iterations and Number of Nodes

Criterion	µA3	µB3	|t-value|	Null Hypothesis
Elapsed Time	0.5557	27.3817	12.469	A3 is sig. better than B3
Iterations	2953.83	171407.23	17.444	A3 is sig. better than B3
Number of Nodes	5.50	1796.13	17.209	A3 is sig. better than B3

From Table 23, we can interpret that model B3 is showing significant change over model A3 in terms of elapsed time, iterations and number of nodes and its values are less for model A3. A3 is significantly better than model B3 where elapsed time, iterations and number of nodes are prime concerns.

Table 24. Comparison of Model A3 and B3 for Objective Function Value and Root Mean Time

Criterion	µA3	µB3	|t-value|	Null Hypothesis
Objective function value	4538.0258	4490.0894	4.698	B3 is sig. better than A3
Root Mean Time	0.2620	0.0373	13.923	B3 is sig. better than A3

From Table 24, we can also interpret that there is also significant change in objective function value and root mean time and its values are lesser for model B3. So B3 is significantly better than model A3 where objective function value and root mean time are prime concern. So from above discussion we can conclude that model B3 is better than model **A3.**

Table 25. Comparison of Model B1 and A2 for Elapsed Time, Iterations and Number of Nodes

Criterion	µB1	µA2	|t-value|	Null Hypothesis
Elapsed Time	19.5103	0.3057	12.217	A2 is sig. better than B1
Iterations	117029.73	2433.23	12.345	A2 is sig. better than B1

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Number of Nodes	1734.00	5.73	12.392	A2 is sig. better than B1

From table 25, we can interpret that model A2 is showing significant change over model B1 in terms of elapsed time, iterations and number of nodes and its values are less for model A2. A2 is significantly better than model B1 where elapsed time, iterations and number of nodes are prime concerns.

Table 26. Comparison Table of Model B1 and A2 for Objective Function Value and Root Mean Time

Criterion	μB1	μA2	|t-value|	Null Hypothesis
Objective function value	4491.5697	4539.6298	4.667	B1 is sig. better than A2
Root Mean Time	0.0257	0.0893	10.934	B1 is sig. better than A2

From Table 26, we can also interpret that there is a significant change in objective function value and root mean time for both models B1 and A2 and its values are lesser for model B1. So from above discussion we can conclude that model B1 is better than model A2.

5. Conclusion

After analysing the above discussion and tables we can conclude that the model B_1 is the best model among models A_1, A_2, A_3, B1 and B_3. . Model B2 is as good as B1. So the contribution in form of most promising constraints holds good in solving SPLP.

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