

# **Solving the Open Vehicle Routing Problem with Fuzzy Restrictions on Time Windows**

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## **Abstract**

Nowadays, with the urbanization and globalization of trade, companies are facing fundamental challenges. This is why companies seek to stabilize and increase their market share while minimizing costs and maximizing profits. The distribution problem as a structure constructed by the participants, represents the most important part of logistic expenses. Vehicle routing problem has been a growing interest as a distribution problem, it concerns to create the routes of a set of vehicles in order to deliver a list of customers. In the goal to minimize the cost of delivering goods. Open vehicle routing problem (OVRP) is an extension of vehicle routing problem where Each route could be a Hamiltonian path over the route's subset of customers. In this paper we provide an extension of the open vehicle routing problem, called Open vehicle routing problem with fuzzy time windows, we adopt a mathematical model for this type of problem, and by using alpha cut method through a decomposition of this original problem in their original fuzzy form into a set of crisp models (OVRPFTW) we propose a solution of this problem. Computational results indicate that this model can fulfill a significant cost-saving while securing a high level of service.

## **Keywords**

Distribution problem, Open vehicle routing problem with fuzzy time windows, alpha cut method.

## **Introduction**

With the increase of globalization and urbanization. Supply chains have evolved very rapidly over the last few years. These new configurations have created new problems for companies, which have had to adapt. They must now turn to commercially available techniques and tools to better control and manage their supply chains. A lot of emphasis has been put on planning the different activities of the supply chains. Furthermore, logistics has invaded all areas: the transport of goods and people, industry, and storage (raw materials, stocks, and finished products), ... Indeed, transport has an influence on the performance of the logistics system through the procurement process, distribution production (product delivery and return (collection)). In the aim to keep up challenges of modern society, the organization of logistics processes requires processes the application of innovative technologies, and optimization effects of physical processes. The focus is on continuous process optimization for the aim of providing the end-user with faster, better quality, and more improved performance of the cost services.

In the literature, three decision-making levels are distinguished: strategic, tactical, and operational. This classification differs from one company to another because it is made according to several criteria. In this regard, we can cite the temporal scope of the application of the decision as an example. The strategic level concerns the decisions to be taken to implement the long-term vision of the company (one year and more). These decisions mainly include the design of the distribution network, which involves the choice of location, size, and number of warehouses. The tactical level corresponds to a set of medium-term decisions (a few months), which allow the company to achieve the strategic level (for example, deciding on the modes of transport and the purchase of new vehicles). The operational level consists of the decisions taken every day to meet the company "s short-term needs. The classical approach established for the optimization of logistic systems is the decomposition of the latter into sub-problems according to the above-mentioned decision levels.

In general, distribution system planning is a fundamental part of any logistics system. it represents the most important part of the logistics expenses. Indeed, Distribution logistics is a framework built by the participants in a competitive exchange process to make goods and services available to customers, users, intermediaries, or buyers. The objectives of downstream logistics, or distribution logistics, are multiple. Among the main objectives, it is possible to mention the satisfaction of final customers, the search for profitability, or the maximization of the quality of products and services. Logistics is the source of many costs that are usually called logistics costs. Transporting large quantities of products over large and medium distances generates transport costs that can be transport costs that can represent a significant percentage of the product's cost price. of the product's cost price. Any increase or decrease in transport costs will immediately improve or immediately improve or degrade the margin, hence the importance of reducing these costs. Therefore, the optimization of transportation costs has become a key factor in the success of any company's success.

Vehicle routing problem (VRP) is a main problem of merchandise conveyance because it is a link in the logistic chain that aims to give a plan of routes from a warehouse to a set of geographically scattered customers, The first authors who have presented this problem are Dantzig and Ramser(1959) In this study, the authors examine the issue of ideal conveyance of a fleet of gasoline conveyance trucks from a bulk terminal that supplies a huge number of benefit stations. The shortest courses between any two focuses within the framework are studied to limit the entire mileage traveled by the fleet whereas fulfilling the requests of each station. The authors propose a formulation for a linear programming method to get a nearly ideal solution.

Since 1959, different authors have paid considerable attention to the problem using whole-number programming approaches and combinatorial optimization to assign a set of vehicles to serve several customers. The main concern of this approach was to can be applied in the areas of transport, logistics, and distribution. According to De Jaegere et al. (2014), the VRP literature can be divided into twelve sub-areas of research: vehicle routing problem with capacity constraints; vehicle routing problem with time windows; Heterogeneous Vehicle Routing Problem; Vehicle Routing Problem with Multiple Depots; Vehicle Routing with Returns (Backhauls); Vehicle Routing Problem with Delivery Sharing; Dynamic Vehicle Routing Problem; Problem of periodic vehicle rounds; problems of vehicle rounds with stochastic demands; problems of vehicle rounds with simultaneous collection and delivery; time-dependent vehicle routing problem; multi-compartment vehicle routing problem; Cumulative vehicle routing problem, and many other variants with different target types, constraints or both. Open vehicle routing problem is considered among the main variants of this problem and it has itself several extensions among them there is open vehicle routing problem with time windows(OVRPTW).

Furthermore, a large proportion of real-world optimization problems are concerned with uncertainties that must be accounted for, (Yang et al. 2022). Despite the importance of optimization under uncertainty, little work has been devoted to optimization under uncertainty. Furthermore, existing studies, while generally applicable to the combined case, have only conducted experiments on continuous problems or test functions. However, it is not obvious that the performances are identical for combinatorial problems such as vehicle routing problem.

As introduced previously, another important aspect for solving open vehicle tour problems is the consideration of time windows (VRPTW) (Yunyun et al. 2018), which allows to reflect the reality in terms of customers' availability. Time

windows are supposed crisp in standard versions of OVRPTW, nevertheless, time windows are not always strictly obeyed in real life, consequently, fuzzy time windows appear and some flexibility is allowed (Diao et al. 2021).

## **1.1 Objectives**

there are two objectives for this contribution: the first one is to propose and solve a open vehicle routing problem with time window (OVRPTW), the second, is to provide an extension of this problem, called Open vehicle routing problem with fuzzy time windows where the flexibility of time windows is allowed, and to show the correlation between the flexibility of time windows and the distribution costs applying a transformation of the open vehicle routing problem into sub-problems of crisp models.

The paper is organized as follows: Section 2 presents a literature review of the open vehicle routing problem Section 3 shows the problem statement and formulation. in Sections 4 and 5 the Towards fuzzy model and solution approach are presented, in Section 6 experimentation results are presented are discussed. finally, the last section summarizes and draws inferences from the findings of this study.

## **2. Literature Review**

Vehicle routing problem (VRP) is defined as a problem of determination set of paths for a fleet of vehicles based on deposits to serve the request for a set of clients geologically scattered, at the minimum cost of total travel, The open vehicle routing problem (OVRP) is a subset of the well-known vehicle routing problem. (VRP) in which vehicles service a geographically scattered group of customers, In this type of problem Vehicles are not needed to return to the depot after serving the last customer on the route They must, however, return by the same path if they are required. As a result, the fundamental distinction between the OVRP and the VRP is that the VRP's solution is to find a set of Hamiltonian cycles. instead, the solution of the OVRP is to find a set of Hamiltonian paths.

Open vehicle routing problem (OVRP) has obtained sparse interest withinside the literature as compared to the VRP.OVRP was first addressed by Schrage in 1981 in a paper that addresses the issue to classify the main characteristic of VRPs in reel life. Since then, Several publications have appeared documenting OVRP, and several studies have been conducted into its modeling and solutions, Aksen and Özyurt ( 2007) have proposed a heuristic belonging to Cluster first, route second (CFRS) heuristics that form clusters of customers in the first step, Then, by solving a minimum spanning tree problem(MSTP), it produces open routes. in the second step. Sariklis and Susan( 2000) have proposed a traditional heuristic to discover a solution for a symmetric OVRP with no maximum route length limits. The paper presents one of the first examples of using a metaheuristic method known as list-based threshold-accepting (LBTA) to solve a real-life fresh meat distribution problem that is expressed as an OVRP multi-depot is presented in Tarantilis and Kiranoudis (2002). that's centered on the concept of adaptive memory presented in first time by Rochat and Taillard (1995) within the point to illuminate the VRP. Another solution is described by David et al. (2007) to address such issues, they provide a unifying adaptive large neighborhood search (ALNS) heuristic.

The most interesting approach to this issue has been proposed by Ropke and Pisinger (2006) that use a heuristic focus on the record-to-record travel deterministic simulated annealing (record-to-record travel) to solve an OVRP problem for the instance in size from 200 to 480 customers. Aksen and Özyurt (2007) Developed an extension of the open vehicle routing trouble wherein cars leave from the depot, visit a group of customers, and then terminate their trips at specific nodes known as driving nodes. The concept of "Soft Time windows" has been presented to deal with the issues surrounding the abuse of time windows. when a Time window is violated in the vehicle routing problem with soft time windows (vrpstw), an expense cost is provided, and the penalty price is frequently believed with the degree of violation to be linear. The vrpstw Efficiently models many situations where time window constraints do not preserve strictly. Choi and Sexton Sexton and Choi (1986) have been The first to don't forget a smooth time windows inside the context of a single routing problem concerning deliveries and pickups. They used a bender's decomposition method to solve this problem. Ferland and Fortin (1989) used a heuristic approach to solve a comparable sliding time window problem in which the time windows of two customers are adjusted to get a reduced pricing solution. Balakrishnan (1993)utilized simple course creation heuristics to extend quick arrangements to Vrpstw issues. Chiang and Russell (2004) advanced a tabu search technique for vrpstw. Calvete et al. (2007) purpose programming methodologies, they modeled and solved the vrpstw.

### **3.Methods**

Vehicle routing problem is generally treated in the literature either exactly or approximately methods. Exact methods are used to solve relatively small problems to determine the most optimal solution by exhaustively exploring the set of possible solutions we can cite as examples: the Linear Integer Programming (Branch & Cut) Padberg (1987) and the Tree Search (Branch & Bound). These methods can be used as well to develop bounds to evaluate the solutions' quality obtained by the approximate methods. The latter are methods used for larger problems, they seek to obtain good-quality solutions in a reasonable time but they do not guarantee their optimality. Approximate methods are divided into two groups: heuristics and meta-heuristics. Among the most used heuristics for VRP, we can mention constructive methods such as the gain algorithm two-phase methods such as the Cluster first-Route second algorithm, and improvement methods such as the local search algorithm.

Meta-heuristics are more advanced methods that can be adapted to solve several optimization problems. These methods are classified in the literature according to the number of solutions they handle: the single solution meta-heuristics such as simulated annealing and General Variable Neighborhood Search El Raoui et al. (2021) and tabu search meta-heuristics with a population of solutions such as genetic algorithms and ant colonies. For more details on the resolution techniques used for VRP, the reader can refer to the state of the art by Gendreau and Potvin (2008) for limited-capacity vehicle tours, who review a set of meta-heuristics proposed in the literature for the VRP and its extension

### **Problem Statement and Formulation**

#### **Problem Statement**

Based on the approach presented in Repoussis and Tarantilis (2007), This paper's purpose is to present a variant of the open vehicle routing problem with time windows (OVRPTW), whose goal is to find a set of optimized Hamiltonian paths for a heterogeneous fleet of vehicles with finite capacity, domiciled in the same depot, as well as visiting a group of clients with known demand. Visits are allowed only at predetermined time intervals (time windows). the OVRPTW covers three main sub problems:

- Delivery: The delivery vehicles are assigned specific routes and are not obligated to return to the depot.
- Pick-up: Vehicles are deployed topickup routes that begin at the depot and finish at clients on the other end of the route.
- Both delivery and pick-up: vehicles return to the depot After finishing all deliveries, visiting costumers in the opposite order and selecting up items which have bedispatched to the distribution center, or they go back to the depot by using following routes for pick-up and deliver items to costumers in the opposite order.

in our problem we will be interested in studying open vehicle routing problem with delivery where the vehicles are not obligated to return to the distribution center.

#### **Problem Formulation**

Let  $G = (N, A)$  be the graph of the road network.,

$V = \{1, \dots, |V|\}$  is the set of vehicles,

$N = \{1, \dots, n\}$  is the set of nodes,

$C = \{2, \dots, n\}$  is a set of customers, the Distribution Center is represented by node 1. .

Let  $A = \cup_{(i,j) \in N^2} A_{(i,j)}$  be the set of arcs, who  $A_{(i,j)} = \{(i,j)^p; p = 1, \dots, |A_{(i,j)}|\}$  is the set of alternative paths between node  $i$  and  $j$ . We associate with each  $(i,j)^p$ , a travel cost  $w_k$  and a travel time  $t_{ij}$ . and we associate to each customer a time window  $[e_i, l_i]$ .

Parameters:

$c_{ij}$ : The cost of transportation per unit distance from  $i$  to  $j$

$t_{ij}$ : Travel time between customers  $i$  and  $j$

$w_k$ : reflects the cost of hiring of a vehicle  $k$

$C$ : The vehicle capacity

$q_i$ : The demand of customer  $i$

$e_i$  : earliest time windows that customer  $i$  can be serviced by any vehicle  
 $l_i$ :latest time windows that customer  $i$  can be serviced by any vehicle  
 $s_i$ : sevice time for each customer  $i$   
 $M$  : is a large number

Decision variables

$$x_{ij}^k = \begin{cases} 1 & \text{if customer } i \text{ precedes customer } j \\ 0 & \text{otherwise} \end{cases}$$

$$z_{ij}^k = \begin{cases} 1 & \text{if vehicle } k \text{ is active} \\ 0 & \text{otherwise} \end{cases}$$

$a_i$  :is the customer's arrival time  
 $p_i$  :is the customer's departure time

Furthermore, when a vehicle it services at least one customer is considered as active .Given the variables and parameters listed above, the problem can be formulated as follows:

OVRPTW Problem:

$$\min \sum_{k=1}^{|V|} \sum_{i=1}^n \sum_{j=1}^n c_{ij} x_{ij}^k + \sum_{k=1}^{|V|} w_k z_k$$

This objective function simulates the trade-off between vehicle expenses and route. The purpose of this function is related to the operation of vehicles to their acquisition or initial preparation. The cost of the routes that all vehicles follow once they leave the warehouse plus the cost of the first segment of each route are presented in the first term of this function. The overall vehicle acquisition or set-up cost is shown in the second term.

Subject to:

$$\sum_{k=1}^{|V|} \sum_{i=1}^n x_{ij}^k = 1, \quad \forall j = 2,3, \dots, n \quad (1)$$

$$\sum_{k=1}^{|V|} \sum_{j=1}^n x_{ij}^k = 1, \quad \forall i = 2,3, \dots, n \quad (2)$$

Constraints(1)and(2) guarantee that precisely oneleavesand enters from each client and from the depot.

$$x_{ij}^k \leq z_k \quad \forall i, j = 2,3, \dots, n \quad (3)$$

Constraint (3) ensures that all customers are served by active vehicles (vehicles with  $z_k = 0$ ) by connecting the  $x$  and  $z$  variables.

$$\sum_{i=1}^n x_{iu}^k - \sum_{j=1}^n x_{uj}^k = 0, \quad \forall k = 2,3, \dots, |V|, \forall u = 1,2, \dots, n \quad (4)$$

Constraint set (4) is a common flow conservation equation that ensures that each vehicle path remains continuous.

$$\sum_{(i,j) \in S \times S} x_{ij}^k \leq |S| - 1, \forall S \subseteq N: 2 \leq |S| \leq n, \forall k \in V \quad (5)$$

this constraint can be formulated by Miller-Tucker-Zemlin formulation (MTZ) as the following form:

$$u_i - u_j + (n - 1)x_{ij}^k \leq n - 2 \quad \forall k = 2,3, \dots, |V|, \quad \forall i, j = 1,2, \dots, n$$

with  $u$  is a float decision variable  $2 \leq u_i \leq n, \forall i = 2,3, \dots, n$

$$\sum_{i=1}^n q_i \left( \sum_{j=1}^n x_{ij}^k \right) \leq C, \quad \forall k = 1,2, \dots, |V| \quad (6)$$

Constraint (5) eliminates sub-tours and constraint (6) it is a capacity constraint requires that the total customer demand should not exceed to the vehicle capacity C.

$$a_j \geq (p_i + t_{ij}) - M(1 - x_{ij}^k), \quad \forall k = 1, 2, \dots, |V|, \forall i, j = 1, 2, \dots, n \quad (7)$$

$$a_j \leq (p_i + t_{ij}) + M(1 - x_{ij}^k), \quad \forall k = 1, 2, \dots, |V|, \forall i, j = 1, 2, \dots, n \quad (8)$$

Constraints (7)-(8) are linked to time windows and ensure that the schedule is feasible for each vehicle. To clarify, the arrival time at customer j is equal to the travel time between two customers i and j plus the departure time from customer i, if customers i and j are on the same vehicle k route.

$$a_i \leq p_i - s_i, \quad \forall i = 2, 3, \dots, n \quad (9)$$

$$e_i \leq p_i \leq l_i, \quad \forall i = 2, 3, \dots, n \quad (10)$$

constraints (9)-(10) Confirm that the links between the arrival, departure, and service times of the customer are compatible with the customer's time window.

$$p_1 = 0 \quad (11)$$

Constraint (11) Sets the departing time from the warehouse to zero, assuming that all routes begin at the depot.

$$\sum_{j=2}^n x_{1j}^k \leq 1, \quad \forall k = 1, 2, \dots, |V| \quad (12)$$

$$\sum_{i=2}^n x_{i1}^k = 0, \quad \forall k = 1, 2, \dots, |V| \quad (13)$$

constraints (12)-(13) Ensure that just one vehicle goes from the depot to serve a chain of customers, and that none returns to the depot, accordingly.

### **Towards fuzzy model**

Clearly, Time windows constraints are regularly violated in practice in real-life applications, they can be violated for a financial and operational reasons. we will suppose that service may start outside the time window and we accept the violations of the values. Thus, we define the Endurable Lateness Time (ELT) and the Endurable Earliness Time (EET) as:

$$EET = e - \Delta^e$$

$$ELT = l + \Delta^l$$

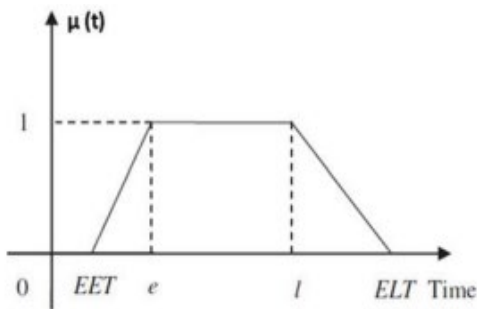


Figure 1. The Degree of Satisfaction with Fuzzy Time Windows

A fuzzy constraint can be used to model the customer's flexibility. In this model, we take into account the following fuzzy constraint:

$$e_i \leq t_i \leq l_i, \quad \forall i = 2, 3, \dots, n$$

Where  $\leq$  indicated that the time constraint could be partially fulfilled.

We present the membership function, which indicates the degree to which the time window constraint is satisfied as shown in Figure 1 :

$$\mu(t) = \begin{cases} 0 & ; \quad t < EET \\ f(t) & ; \quad EET \leq t < e \\ 1 & ; \quad e \leq t < l \\ g(t) & ; \quad l \leq t < ELT \\ 0 & ; \quad ELT \leq t \end{cases}$$

Where  $g(t)$  is a decreasing function and  $f(t)$  is a non-decreasing function defined as follows:

$$f(t) = 1 - \frac{e - t}{\Delta^e}$$

$$g(t) = 1 - \frac{t - l}{\Delta^l}$$

The fuzzy set theory is used in this technique to indicate precision or uncertainty in the parameter . Uncertain parameters are seen as fuzzy integers with membership functions. Figure 2 depicts a parameter of time as a triangular fuzzy integer with a support. The more extensive membership function's support, the greater uncertainty. The  $\alpha$ -cut of the membership function is the fuzzy set that contains all elements with a membership of an  $\alpha \in [0,1]$  or above. It will feature a support at a resolution level of  $\alpha$ . The greater the value of  $\alpha$ , is the greater level of confidence in the parameter.

The method is founded on the main extension , which states that functional connections can be extended to include fuzzy arguments and utilized to use a fuzzy set to represent the dependent variable. This approach can be applied analytically in simple arithmetic operations. However, differential equations and other complex structures are used in the majority of real-world modeling applications, making analytical application of the idea problematic. As a result, interval arithmetic is used to do the analysis.

The membership function is divided horizontally into a limited number of  $\alpha$ -levels ranging from 0 to 1. The model is run for each  $\alpha$ -level of the parameter to identify the minimum and maximum feasible output values. This data is then utilized to directly generate the equivalent fuzziness (membership function) of the output, which is employed as a degree of uncertainty. If the output is monotone in relation to the dependent fuzzy variables, the method is quite straightforward because only two simulations are required for each  $\alpha$ -level (one for each boundary). Instead, optimization methods must be run to find the maximum and minimum output values for each  $\alpha$ -level.

## **Solution Approach**

To the finest of our information, no approach can clear up promptly the problem in its fuzzy form. But, possible rework the authentic fuzzy hassle into a hard and fast as the parametric method for crisp problems does.

Using the concept of  $\alpha$  cuts, this method transforms the fuzzy problem into an equivalent "crisp" problem. The parametric problem is then solved using specific value of  $\alpha$  (where  $\alpha \in [0,1]$  ), the use of specific or approach optimization strategies. In routing problems, the parametric technique has been widely used, like Brito and Moreno (2015) has been additionally implemented in other fuzzy optimization issues.

To solve our fuzzy problem, we will use this parametric approach.

- **$\alpha$ -OVRPTW-P**

The earliest and latest service times that the customer  $i$  can accept, based on a previously denoted fuzzy membership function and a given respectively,  $\hat{e}_i, \hat{l}_i$  (Figure 2) can be defined as follows:

$$\hat{e}_i = f_i^{-1}(\alpha) = e_i - \Delta^e(1 - \alpha)$$

$$\hat{l}_i = g_i^{-1}(\alpha) = l_i + \Delta^l(1 - \alpha)$$

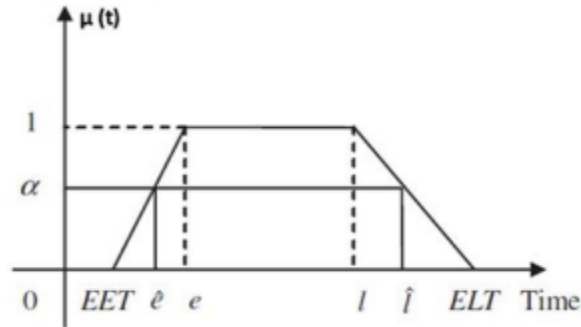


Figure 2. Time Window Bounds Function related to Alpha

For each  $\alpha$  -OVRPTW-P, Time window constraint is become:

$$e_i - \Delta^e(1 - \alpha) \leq p_i \leq l_i + \Delta^l(1 - \alpha)$$

### Computational Experiments

To illustrate the result, we have used the IBM OPL modeling language (IBM CPLEX optimization studio version 12.6.0) for solving the proposed model.

We have use two instances A and B, for each instance we have choice a number of vehicles  $V$ , Time windows width  $T_w$ , number of customers  $N$   $C$ , and vehicle capacities  $C$ , Instance A has  $N$   $C=4$ ,  $V=2$ ,  $C=2500$  and  $T_w=2h00min$ , and  $\Delta^e = \Delta^l = 15min$

instance B has  $N$   $C=15$ ,  $V=13$ ,  $C=90$  and  $T_w=4$  h. In instance B, we accept that all the clients have the same time window and  $\Delta^e = \Delta^l = 30$

In both instances, We supposed that we are single depot and Demands of customers aqual 1000. And For distances for each instance we have created eucliden distances using python.

We solving for instances A and B the problem for each  $\alpha \in [0, 0.1, \dots, 1]$ . The simulation results are presented in Table 1.

Table 1. Experimental Results

Value	Instance A		Instance B	
	TW bounds	Objective function	TW bounds	Objective function
$\alpha = 0$	[0,2.5]	8272	[0,5]	11732
$\alpha = 0.1$	[0.025,2.475]	8272	[0.05,4.95]	11832
$\alpha = 0.2$	[0.03,2.45]	8307	[0.1,4.9]	11834
$\alpha = 0.3$	[0.075,2.425]	8315	[0.15,4.85]	11855
$\alpha = 0.4$	[0.1,2.4]	8317	[0.2,4.8]	11855
$\alpha = 0.5$	[0.125,2.375]	8317	[0.25,4.75]	11855
$\alpha = 0.6$	[0.15,2.35]	8317	[0.3,4.7]	11855
$\alpha = 0.7$	[0.191,2.325]	8324	[0.35,4.65]	11855
$\alpha = 0.8$	[0.2,2.3]	8330	[0.4,4.6]	11855
$\alpha = 0.9$	[0.225,2.275]	8332	[0.45,4.55]	11855
$\alpha = 1$	[0.25,2.25]	8332	[0.5,4.5]	11855

The variation of experiment results is described in figure 3:



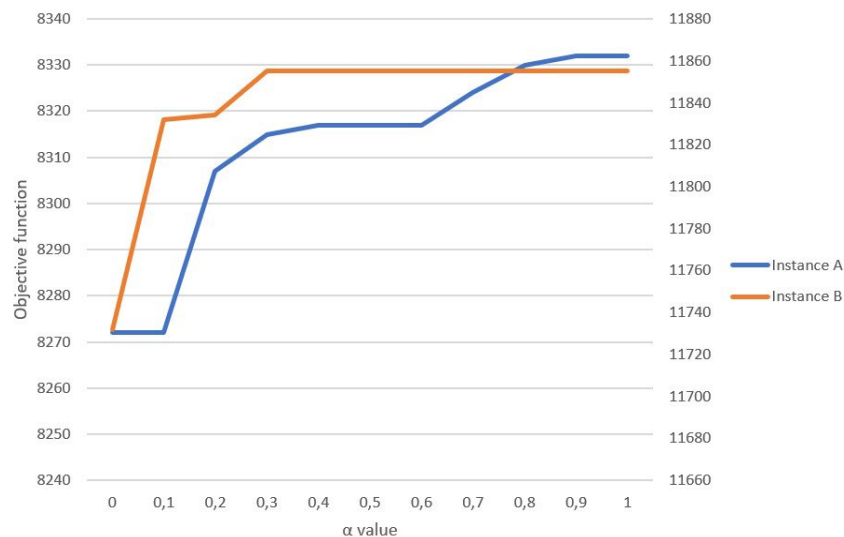


Figure 3. Variation in Distribution Costs in terms of Alpha Cuts

The variation of distribution costs is in terms of  $\alpha$ .  $\alpha=0$  corresponds to the fully relaxed time window,  $\alpha=1$  corresponds to the absolutely limited time window constraint. As it's far anticipated, as result the more secure problem is  $\alpha=0$  where decrease the distribution value is.

Thought the outcomes, In instance A, that once no flexibility is permitted by way of customers  $\alpha=1$ , The most expensive distribution costs are incurred. However, in the other case if the customer a permits a flexibility of 20% ( $\alpha=0.3$ ) in the time window, then up to 0,17% of the distribution value can be stored. For instance B, we've got determined that generally for every 10% increase in  $\alpha$ , the distribution cost rises by around 0.012% although then arrived at  $\alpha=0.3$  charges extended through nearly 0,17%.

The overall computational results are summarized that Additional expenditures are incurred as a result of the time constraint. In this instance, the decision maker has two options: decrease the level of service or pay additional charge to reach the customer at a specific time.

## Conclusion

This paper offered an open vehicle routing problem as well as a literature review on the subject and its variations. This work makes a modest addition to the current topic of the open vehicle routing problem with fuzzier time windows. concentrated not only on open vehicle routing problem but also on the fuzzy logic, In a vehicle routing problem, carrier degree concerns connected with time window violations are addressed the use of fuzzy membership features, and the idea of Fuzzy time windows is suggested. And Based at the idea of fuzzy time windows, the Vehicle routing problem with fuzzy time windows is proposed.

A service level issue connected with time window violation in a vehicle routing problem is presented utilizing fuzzy membership functions and the idea of fuzzy time windows. a mathematical model for a open vehicle time windows with fuzzy time windows(OVRPFTW) are presented. In this work we modify the fuzzy time windows constraint by a decomposition of this original problem (OVRPFTW) into a set of crisp models for each value of alpha through the use of  $\alpha$  cut method. distinctive numerical tests are conducted to compare the by and large execution of models of OVRPTW and OVRPFTW when benefit level issues are included.

The results obtained by using CLPEX solver indicate that this approach solution is suitable for the small instance, but it not suitable for the large instance. therefore, some heuristic has required to solve this problem and a few other questions stay to be addressed.

Clearly, research on the topic would be of interest based on the encouraging findings reported in this paper, and work on the remaining challenges is ongoing and will be addressed in future studies.

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