Joint Replenishment Dealing with Various Inventory Policies

Victor Suhandi
Ph.D. Candidate, Department of Industrial and Systems Engineering
Chung Yuan Christian University
Chungli, Taoyuan, Taiwan

Lecturer, Department of Industrial Engineering,
Universitas Kristen Maranatha,
Bandung, Indonesia
g10702408@cycu.edu.tw

Ping-Shun Chen
Professor, Department of Industrial and Systems Engineering
Chung Yuan Christian University
Chungli, Taoyuan, Taiwan
pingshun@cycu.edu.tw

Abstract

The joint replenishment models usually use one type of inventory policy with several different parameters to handle several items. However, different demand patterns for each product can make use of all previous methods leading to unsatisfying performance. This study brings a joint replenishment procedure that accommodates more than one single-item inventory policy to give more flexibility. This study fills this gap by developing a methodology that consists of proposed general joint replenishment problem (JRP) models, potential order estimation procedures that bridge various single-item inventory policies to the JRP model, implementation procedure, and simulation models for comparisons. Comparisons to the adaptive JRP considering non-stationary demand patterns prove the advantages of the proposed procedure. Finally, a sensitivity analysis gives some managerial insights.

Keywords
Joint replenishment problem, multiple inventory policies, multi-item inventory, XYZ analysis, demand pattern

1. Introduction
Many industries, such as retail stores and pharmacy stores, involve hundreds or even thousands of items in their inventory systems. The relationship between a company and its suppliers usually involves more than one stock-keeping unit (SKU) item. For example, a supplier of dairy products can supply milk, cheese, yogurt, and the like. Moreover, each of these products can have various flavors, aromas, or sizes. This situation triggers multi-product problems in determining inventory policy (Goyal 1974).

The approach with ABC–XYZ classification is often used for the inventory system handler to become effective and efficient. The ABC analysis looks at the value of an item whereas the XYZ analysis looks at the demand pattern of each item. The XYZ analysis often produces items from the same supplier that have different demand patterns. Items with various demand patterns may require several inventory policies. The X class can use inventory policies with constant demand rates, and the Y class can use forecast-based inventory policies (Bialas et al. 2020). This situation makes multi-item inventory problems need to accommodate different inventory policies.

The situation of one supplier supplying more than one product is familiar in reality, and the existence of different demand patterns for these products poses a challenge for determining inventory policy. Therefore, a joint replenishment inventory policy is needed to accommodate various demand patterns or integrate several single-item inventory policies. This study shows the importance of multi-item inventory policy with multiple independent demand patterns.

The two objectives of this study are introduced as follows: First, this study established a JRP procedure with an optimization model in order to accommodate several single-item inventory policies. Second, this study compared the proposed procedure to relevant studies.

The simulation constructed in this study used the secondary data of Yang and Kim's (2020) research. However, the data need some adjustment to bring fair comparison. Although the previous study showed the flexibility to deal with non-stationary demand, it used a type of the inventory policy for all items. This point was different from this study.

2. Literature Review

The development of JRP has been going on for decades. Goyal and Satir (1989) reviewed several papers on JRP published before 1985 and divided them into deterministic JRP and stochastic JRP. Deterministic models mostly used the periodic review approach by looking for review or replenishment intervals for each item. Meanwhile, the stochastic model used the can-order policy. Furthermore, Khouja and Goyal (2008) reviewed JRP from 1989 to 2005 and added dynamic JRP as a grouping factor in addition to the constant demand rate and stochastic JRP. Dynamic JRP adopts a deterministic and dynamic lot-sizing technique on multi-item inventory policies. In this study, the development of the JRP model is classified into four groups. The first group consists of JRP models that use a can-order policy that follows a continuous review pattern by adding a can-order parameter for items that may be replenished in advance together with trigger items. The second group consists of JRP models based on a cyclical policy that follows a periodic review pattern by finding a common replenishment time that minimizes inventory costs. The third group consists of JRP models based on dynamic lot-sizing that combines several demands at multiple periods for each item. Finally, a forecast-based JRP model uses forecast data to update the parameters of the inventory policy.

In the first group, Balintfy (1964) introduced the concept of the JRP can-order parameter and integrated it into the continuous review model. Orders for items that have not reached the reorder level can be made by considering saving on ordering costs or setup costs by combining items that have reached the reorder point or trigger items. Several other studies belonging to this group include Ohno and Ishigaki (2001) improved the time to find the solution; Kalpakam and Arivarignan (1993) included the substitute items to deal with stock out; and Kiesmuller (2010) implemented can-order in container load problem.

In the second group, Goyal (1974) is one of the pioneers in the periodic review of JRP with a deterministic demand for cyclical policies. In this approach, a joint replenishment cycle is introduced for multiple items. Some other studies are as follows: Atkins and Iyogun (1988) used a lower bound to find the ordering interval; Viswanathan (2002) developed the cyclical type to accommodate bigger JRP; Tsao and Sheen (2012) used JRP with joint discount and term of the payment period; Amaya et al. (2013) applied linear programming to solve cyclic and continuous problems; X. Y. Ai et al. (2017) added the deterioration factor into JRP; X. Ai et al. (2021) used the discount concept in the cyclic policies; Braglia, Castellano, and Gallo (2016) optimized the lead time by using a stochastic model in the cyclic policies; Braglia, Castellano, and Frosolini (2016) and Braglia et al. (2017) proposed a model to optimize the joint ordering costs; and Braglia et al. (2021) optimized the individual ordering costs. In the periodic review JRP model with cyclical policy, each item is treated as a periodic order quantity (POQ), with several items being ordered simultaneously, thereby saving ordering costs. Ease of implementation is the main advantage of the method. Using an inventory policy with simple parameters, such as reorder level and maximum inventory, is often easier to implement than dynamic lot-sizing due to the integration of forecasting models. However, it is not friendly for items with trends and seasonal factors.

In the third group, Boctor et al. (2004) summarized the models for the dynamic joint replenishment problem (DJRP). Boctor et al. (2004) developed two models, the DJRP2 and the DJRP3, as modifications of the classical model (DJRP1). The DJRP1 used a mixed-integer programming (MIP) model that minimized total inventory costs. Several studies belonging to this group are as follows: Narayanan and Robinson (2010) combined the output of the heuristics as the initial solution for the meta-heuristics algorithm; Cheung et al. (2016) introduced a sub-modular approach as DJRP, and Noh et al. (2020) implemented can-order policies using DJRP. The dynamic JRP, the basis for this study, provides the ability to deal with changes in demand.

The last group is the forecast-based JRP policy as an essential development responding to the changes in demand to maintain good JRP results. In this situation, the inventory policy parameters are updated and can reduce inventory decision errors. Thus, the forecast-based JRP can handle inventory systems with non-stationary demand. Some of the studies in this group are as follows: Lee and Chew (2005) proposed autoregressive to estimate future demand; Yang and Kim (2020) considered the error of last forecasting to estimate future demand.

The current study presents a JRP model that accommodates several different single-item inventory policies. This model must be able to implement JRP with the condition that management enforces different inventory policies for several items with the same supplier. This study shows a procedure for implementing the proposed JRP and compares it to the adaptive JRP of Yang and Kim (2020) with the adjustment of several conditions.

3. Methods

This section contains the notations, problem definition and assumptions, proposed JRP model, JRP input preparation procedures, implementation procedure, and simulation models.

3.1 Notations

The proposed model uses the following notations:

S_t common ordering cost at period t,
s_{it} individual ordering cost for item type i at period t,
h_i unit inventory holding cost for item type i,
d_{it} demand for item type i for period t,
I_{it} inventory level of item type i at the end of period t (I_{i0} represents the initial inventory level),
M sufficiently large number,
x_{it} replenishment quantity of item type i at the beginning of period t,
y_{it} binary variables = 1 if and only if item type i is replenished at the beginning of period t, i.e. y_{it}=1 if x_{it}>0, and
z_t binary variables taking the value 1 if an order is placed for period t.
Q_i estimated order quantity of item i,
δ_{it} time discrepancy due to early order for item i to order at period t,
τ_i estimated single-item ordering time of item i, and
υ_t potential multi-item ordering time of period t, $\upsilon_t = \tau_i$ if $t = i$.
HC_{it} total holding cost of item i at period t,
SC_{it} total shortage cost of item i at period t,
b_i shortage cost of item i,
DX_{it} variable for McCormick envelope approach of item i at period t,
LX_{it} lower bound of order quantity of item i at period t,
UX_{it} upper bound of order quantity of item i at period t,
LD_{it} lower bound of the discrepancy between ordering time and estimated ordering time of item i at period t, and
UD_{it} upper bound of the discrepancy between ordering time and estimated ordering time of item i at period t.
d_t a demand at period t,
w_1 a constant demand rate,
w_2 a linear trend,
c_t a seasonal index for period t,
ε_t an independent random variable with mean zero and constant variance.

3.2 Problem Definition and Assumptions

The objective of the current study is to propose a procedure with a general JRP optimization model that uses continuous-time and dynamic inventory policies. And build the simulation model to compare the proposed model with the current benchmark and with the joint replenishment incidentally (without planning). The contribution of this study is obtaining a JRP procedure with sufficient flexibility and eliminating the limitation to use one type of inventory policy when dealing with several different demand patterns items. A general input should be prepared to bridge various single-item inventory policies and the JRP optimization model. The JRP optimization model consists of the decision variables, the objective function, and the constraints.

The decision variables used are the quantity of each item ordered at some point in time. Ordering times can occur periodically or at points in continuous time, depending on the single-item inventory policy used. Joint replenishment may occur at these points in time. Some items may become a trigger to order, and some others that maybe join together to make it more efficient. The objective function is the total cost related to the inventory decision. It consists of common ordering costs, individual ordering costs, holding costs, and shortage costs. In this study, there are two models with and without shortage. A constraint is used to express the domain of ordering time that can make additional inventory costs due to early orders. There is also an ordering time limit related to additional shortage costs due to delays. Besides that, some constraints state the relationship between variables. The individual ordering quantity variables are related to their status variables and the common ordering status variable.

The length of storing time may vary according to the continuous-time interacting with the amount stored is a continuous variable forming a non-linear programming model. The solution is done by seeking convex non-linear programming and using CPLEX software. The simulation models are used to test the application of the models in inventory systems and are useful to compare with other models. Thus the advantages of using the proposed JRP model are known. The simulation model uses the Arena software.

The assumptions used in this study are as follows:

- All shortages can be backlogged for models with shortages.
- Each item has only one supplier.

3.3 Proposed JRP Model

This study develops a JRP model that can accommodate various inventory policies. Demand patterns that are not always constant require dynamic handling. Dynamic JRP has been widely developed, and Boctor et al. (2004) summarized the classic dynamic JRP. Dynamic JRP accommodated the demand dynamic like the lot-sizing technique. It modeled the demand for each item in each period becomes a direct input for the model, whereas the proposed model has two stages. The first stage converts the demand into an estimated order release according to the inventory policies (IP) of each item. Then in the second stage, the estimated order release becomes the input for the proposed model. This approach has the flexibility to accommodate various IPs in the first stage while input adjustments are made for the JRP model to be universally applicable to various IPs.

In the dynamic JRP, the number of periods in a planning horizon is adopted as one of the dimensions in the model, along with the number of items. In the proposed model, the number of periods will be equal to the number of items. Each item has one estimated single-item order time (τ_i), which leads to the potential multi-item order time (υ_t); order pooling can occur at these points in time. Hence, the proposed models can accommodate continuous time points in the continuous review inventory policy as well as periodic review inventory policies. This study proposed JRP models with fully backlogged shortages.

The proposed model has an objective function and some constraints. The objective function is the Total Cost (*TC*) which consists of the common ordering cost in each period *t*, individual ordering cost for each item in each period, weighted time (δ_{it}) while holding the cost for each item in each period, and shortage cost (see Equation (1)). Delta (δ_{it}) reflects the length of time that an item uses an early or late order, which should have been ordered at τ_i but accelerated or delayed to υ_t (see Equation (2)). Equations (3) and (4) represent the total holding cost and total shortage cost, respectively.

Constraint (5) indicates that the orders have to fulfill the required quantity while Constraint (6) shows that an individual ordering cost occurs when an order is placed for an item. Constraint (7) requires common order cost in a period if there is at least one item ordered in that period. The order quantity x_{it} must be positive, while y_{it} and z_t are Boolean (see Constraints (8) to (10)).

$$TC = \sum_{t=1}^{T} \left[S_t z_t + \sum_{i=1}^{n} \left(s_{it} y_{it} + HC_{it} + SC_{it} \right) \right] \tag{1}$$

$$\delta_{it} = \tau_i - \upsilon_t ; (i = 1,...,n; t = 1,...,T) \tag{2}$$

$$HC_{it} = \begin{cases} h_i \delta_{it} x_{it} ; \text{if } (\delta_{it} \geq 0) \\ 0; \text{otherwise} \end{cases} ; (i = 1,...,n; t = 1,...,T) \tag{3}$$

$$SC_{it} = \begin{cases} -b_i \delta_{it} x_{it}; \text{if } (\delta_{it} < 0) \\ 0; \text{otherwise} \end{cases}; (i = 1,...,n; t = 1,...,T) \tag{4}$$

$$\sum_{t=1}^{T} x_{it} = Q_i; (i = 1,...,n) \tag{5}$$

$$x_{it} \le My_{it}; (i = 1,...,n; t = 1,...,T) \tag{6}$$

$$\sum_{i=1}^{n} y_{it} \le nz_t; (t = 1,...,T) \tag{7}$$

$$x_{it} \ge 0; (i = 1,...,n; t = 1,...,T) \tag{8}$$

$$y_{it} = \{0,1\}; (i = 1,...,n; t = 1,...,T) \tag{9}$$

$$z_i = \{0,1\}; (i = 1,...,n) \tag{100}$$

$$DX_{it} = \delta_{it} x_{it}; (i = 1,...,n; t = 1,...,T) \tag{11}$$

$$LX_{it} \le x_{it} \le UX_{it}; (i = 1,...,n; t = 1,...,T) \tag{12}$$

$$LD_{it} \le \delta_{it} \le UD_{it}; (i = 1,...,n; t = 1,...,T) \tag{13}$$

$$DX_{it} \ge LX_{it}\delta_{it} + LD_{it}x_{it} - LX_{it}LD_{it}; (i = 1,...,n; t = 1,...,T) \tag{14}$$

$$DX_{it} \ge UX_{it}\delta_{it} + UD_{it}x_{it} - UX_{it}UD_{it}; (i = 1,...,n; t = 1,...,T) \tag{15}$$

$$DX_{it} \ge UX_{it}\delta_{it} + LD_{it}x_{it} - UX_{it}LD_{it}; (i = 1,...,n; t = 1,...,T) \tag{16}$$

$$DX_{it} \ge LX_{it}\delta_{it} + UD_{it}x_{it} - LX_{it}UD_{it}; (i = 1,...,n; t = 1,...,T) \tag{17}$$

The proposed JRP model produces a non-convex quadratic programming formula. The McCormick envelopes approach is needed to obtain a convex solution. The formula in Equation (11) forms quadratic programming, so a limited domain is needed for its elements (Equations (12) and (13)). Equations (14) through (17) are the implementation of the McCormick envelopes on the model (Dombrowski, 2018).

3.4 JRP Input Preparation Procedures

Inputs for the proposed JRP need to be prepared based on the inventory policies used. These inputs are the estimated order time (τ_i) and order quantity (Q_i), which are calculated based on demand forecasting (d_{it}) and on-hand inventory (I_{it}) following each inventory policy.

This study uses several popular single-item inventory policies. The continuous review (Q, r) and the order-up-to-level (E, r) policies represent the constant demand rate and continuous-time inventory policies. Both of these models refer to Silver et al. (2017), and the economic order quantity (Q_i^*) formula is adjusted to Equation (18) in this study. For the safety factor (k_i) formula in Equation (19), a shortage cost (b_i) is used that considers the number of shortage events. This study assumes that the management follows the formula to find the safety factor according to Equation (19). Thus it simplifies the steps following Silver et al. (2017). The reorder level (r_i) calculation refers to Equation (20), where μ_{Li} and σ_{Li} are the demand in an order waiting period and its standard deviation, respectively. Equation (21) is used in the (E, r) inventory policy to obtain the maximum inventory level (E_i). This formula considers single-item inventory policies so that it treats each item type independently.

$$Q_i^* = \sqrt{\frac{2(S + s_i)d_i}{h_i}}; (i=1,...,n) \tag{18}$$

$$k_i = \sqrt{2 \ln \frac{d_i b_i}{\sqrt{2\pi} Q_i^* h_i \sigma_{Li}}}; (i=1,...,n) \tag{19}$$

$$r_i = \mu_{Li} + k_i \sigma_{Li}; (i=1,...,n) \tag{20}$$

$$E_i = r_i + Q_i^*; (i=1,...,n) \tag{21}$$

The potential order estimation (POE) procedures to estimate Q_i and τ_i depending on each single-item inventory policy are as follows:

a. Capture the inventory policy parameters and inventory level:
   i. Continuous review (*Q, r*) policy: economic order quantity (*Q*), reorder level (*r*), inventory level (*I*), and demand rate (*d*).
   ii. Order-up-to-level (*E*, *r*) policy: maximum inventory (*E*), reorder level (*r*), inventory level (*I*), and demand rate (*d*).
b. Calculate Q_i and τ_i,
   i. (*Q, r*) policy:

$$Q_i = Q; (i=1,...,n) \tag{22}$$

$$\tau_i = \max\left\{\left(I - r\right)/ d, 0\right\}; (i=1,...,n) \tag{23}$$

   ii. (*E*, *r*) policy:

$$Q_i = E - r; (i=1,...,n) \tag{24}$$

$$\tau_i = \max\left\{\left(I - r\right)/ d, 0\right\}; (i=1,...,n) \tag{25}$$

For forecast-based inventory policies, estimate order time and order quantity using Equations (18) through (25) with updated parameters. These parameters are calculated based on the assumption that demand (*d*) is the same as the forecast value (*f*). Forecast-based inventory policies include policies that handle trends and seasonal demand rates.

4. Solution Procedures

4.1 Implementation Procedure

The procedure developed in this study considers the ordering process for several different items together. These items have inventory policies that may differ from one another. When an item becomes a trigger for an order, it is time to consider whether or not the other items can join together. The joint ordering can create savings related to common ordering costs. Calculating savings use the estimation of the order time and the order quantity on its inventory levels and inventory policies.

The joint replenishment for multiple inventory policies (JRMIP) procedure for implementing the proposed JRP model is as follows:

1) Capture the trigger items.
2) Estimate the order time and order quantity for related items using the POE procedure.
3) Perform the JRP optimization model using Equations (1) through (17).
4) Execute the closest order according to the output of the JRP optimization.

4.2 Simulation Models

The simulation model is used to compare the performance of the proposed JRP model to other systems including the adaptive JRP method. Figure 1 shows some of the processes required to simulate the JRP model. The first process is to read the model parameters and inventory policies used. This process is only done once at the beginning of the simulation. Then, the second process is the potential order estimation using single-item inventory policies, followed by the JRP optimization process as the third process. The demand fulfillment process and updating inventory system statistics as the fourth process. The last one is writing the output to a file for every period. The second through fifth processes are repeated until the simulation time is complete.

Arena simulation software is used in conjunction with CPLEX to model the inventory system with the proposed procedure (JRMIP). The Arena software will activate the JRP optimization model on the CPLEX software via a VBA block when running the proposed model. VBA blocks are also used to prepare the input for CPLEX and capture the output from CPLEX. The simulation time is three years or 156 weeks following the previous study Yang and Kim (2020). During the simulation time, the processes in the inventory model occur more than hundreds of times so that the resulting output is relatively stable.

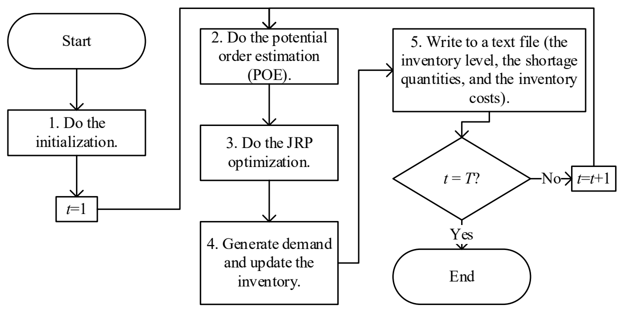

Figure 1. Flowchart of the simulation process.

5. Result and Discussion

Adequate forecasting is needed to support an inventory system. This study uses the basic forecasting models. Demand data are generated according to the forecasting model used and the normal distributed random fluctuations. It is different from Yang and Kim (2020), who used random variables for several parameters in forecasting models. This study focuses on the JRP optimization technique that the discrepancy of forecasting techniques only occurs because of the irregular random fluctuation (ε). In this case, the hypothetical demand data generated follow Equation (26) for constant rate, trend, and seasonal forecasting models (Silver et al., 2017).

$$d_t = (w_1 + w_2 t)c_t + \varepsilon_t ; (t=1,...,T) \tag{26}$$

Table 1. Parameters for comparison between JRMIP and MIVL procedures.

items	w_1	w_2	c_1	c_2	c_3	c_4	ε	k	s ($)	h ($/#/week)	b ($)	S ($)	R (weeks)*
item 1	30	1.5	0.5	2.8	0.5	0.2	0.05	1.96	400	1	50	500	1
item 2	30	1.5	1.5	0.35	1.5	0.5	0.05	1.96	400	1	50	500	1
item 3	30	1.5	0.5	0.2	0.5	2.8	0.05	1.96	400	1	50	500	1
item 4	50	1	0.5	2.8	0.5	0.2	0.05	1.96	400	1	50	500	1
item 5	50	1	1.5	0.35	1.5	0.5	0.05	1.96	400	1	50	500	1
item 6	50	1	0.5	0.2	0.5	2.8	0.05	1.96	400	1	50	500	1

*R replenishment interval

As shown in previous research, the multi-item variable level (MIVL) procedure (Yang and Kim 2020) is the best procedure when handling non-stationary demand. Thus, it can be chosen as a benchmark in this study. Various demand patterns are used to represent seasonal influences and trends that can occur. This study used six demand patterns to represent the demand pattern used by Yang and Kim (2020). Table 1 presents data for this comparison, where item 1 has a peak in the summer season, item 2 has a peak in the spring and fall seasons, and item 3 has a peak in the winter season. Items 1, 2, and 3 have smaller intercepts and steeper slopes compared to items 4, 5, and 6. Figure 2 shows the demand fluctuation along with the forecast data for item 1. The simulation model generates the same demand for the proposed procedure (JRMIP) and MIVL. Thus the differences that occur will be more focused on the performance of the joint replenishment model developed. Likewise, the error level used is 5, 10, and 15 percent so that the accuracy of forecasting can be controlled to see the performance of the joint replenishment model.

This study uses the data from Yang and Kim's (2020) research with some adjustments. This study eliminates the data with a negative trend. Parameter w_1 only uses 30 and 50. Parameter w_2 chooses the median (1.5 and 1) of the uniform distribution available. The error level (ε) uses 0.05 in the beginning and then changes it by 0.1 and 0.15 to represent the error with uniform distribution in the secondary data. Seasonal factors (c_1, c_2, c_3, and c_4) for items 1 to 3 are the same as the data and are repeated for items 4 to 6. Parameters k, S, and R are the same as the data. Parameters s and b are chosen inside the range of uniform distribution specified by the data. Simplifying some random data becomes a constant parameter to reduce the uncertainty and can focus on comparing between procedures.

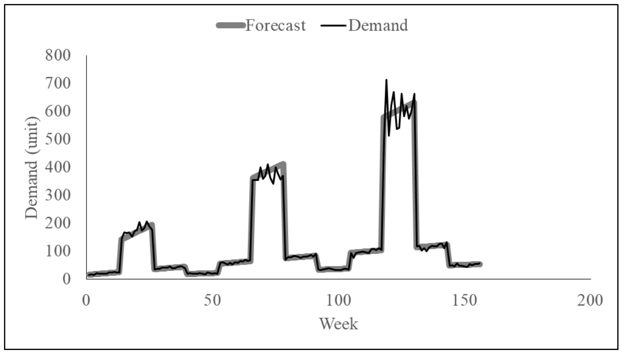

Figure 2. Forecast and demand item 1.

In addition, the JRMIP can accommodate various inventory policies, so this study chooses one that can be best for comparison. MIVL uses periodic reviews similar to orders-up-to-level on single-item inventory policy. MIVL uses the actual demand for the past week to predict demand in one future period. However, for a fairer comparison in this study, the MIVL procedure also uses a forecasting model with a controlled error level. Thus, it has a better accuracy of forecasting. The MIVL procedure calculates the target inventory based on forecast demand for each item, followed by calculating the cost of whether an item is ordered or not. These costs are input for the JRP model used to decide whether an order should be released or not at that time. The MIVL procedure calculates the order quantity from the target inventory reduced by the current inventory level. In this study, JRMIP will use an order-up-to-level inventory policy for each item to be similar to MIVL and easier to compare. The calculation of the target inventory level uses Equation (21) and is then followed by calculating the estimated order time and estimated order quantity using Equations (24) and (25). JRMIP uses a single-item inventory policy as a first step to estimate the order time of each item and continues using the optimization model (Equations (1) through (17)) to determine which items should be ordered at that time.

The simulation outputs are recorded as text files that can be converted to spreadsheet files and used to verify the models. Appendix A shows the results of the JRMIP procedure. The statistics collected for three years from week 1 to week 156. The detail for all items (1 – 6) is recorded weekly. The forecast data follow the forecasting model for each item. The demand data are generated following the forecast model with random fluctuation based on the error set (5, 10, and 15%). The potential order quantities are used as the input for the JRP optimization model. The inventory level shows the ending inventory after the decision to replenish and is calculated based on the previous inventory level, demand, and replenishment quantity for each item. The replenishment quantity for each item is the output of the JRP optimization model using the CPLEX software. Finally, the costs are accumulated from the first week until week 156. The total costs for comparison are in the last row. Appendix A shows the output statistics for the MIVL procedure. The forecast and demand data are similar to the JRMIP procedure. Several variables follow the MIVL procedure by Yang and Kim's (2020) research. Finally, the total costs accumulated from the first week until week 156. The last row's total costs are compared to the JRMIP outputs.

The comparison between the proposed procedure (JRMIP) and MIVL using the total costs as the ultimate performance. Figure 3 shows that JRMIP produces a better total cost. Savings are 6 percent for the 5 and 10 percent forecast error levels, and 3 percent for the 15 percent forecast error level compared to MIVL. Greater accuracy in forecasting will provide more advantages to JRMIP.

Figure 4 shows the inventory level for item 1 using JRMIP and MIVL procedures. The JRMIP procedure produces a higher inventory level than MIVL. However, the JRMIP can be superior in the total cost. By having a higher inventory, the JRMIP can reduce the replenishment frequency.

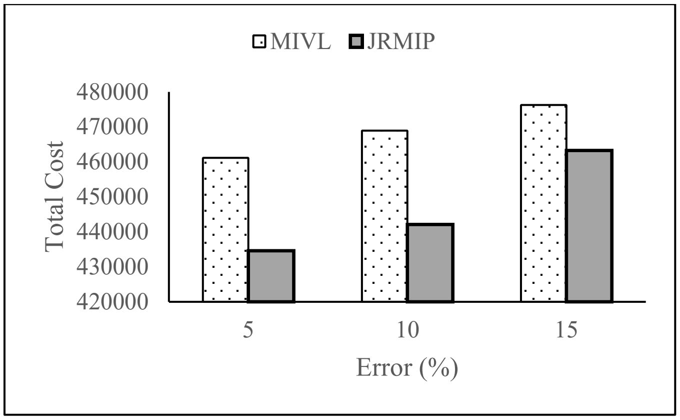

Figure 3. Comparison of total costs between the MIVL and JRMIP procedures.

Appendix A shows the MIVL has more often replenishment than the JRMIP procedures. It can be seen from the ordering decision related to all items for three years of simulation time. This ability is owned by the single-item inventory policy maintained in the JRMIP procedure. The proposed procedure converts the advantages in total ordering costs become more efficient total costs.

The shortages for both JRMIP and MIVL are rare. Figure 5 shows that JRMIP experiences no shortages, whereas MIVL has three times. These shortages are relatively small compared to a three-year observation period. Appendix A also shows that the shortage cost is close to zero.

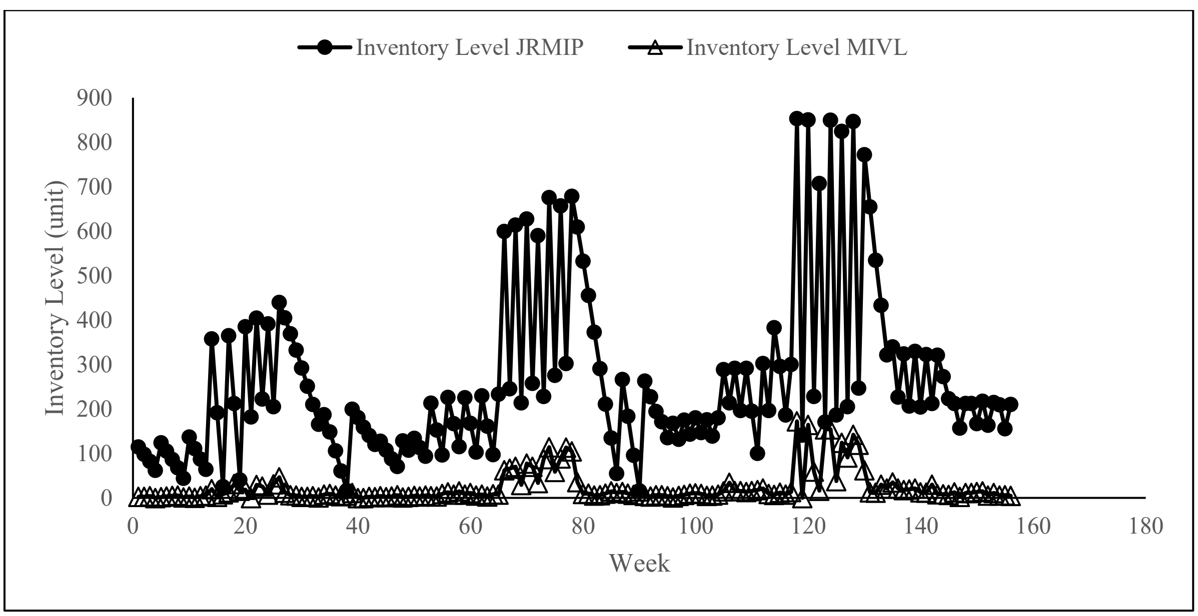

Figure 4. Comparison of inventory level between the MIVL and JRMIP procedures for item 1.

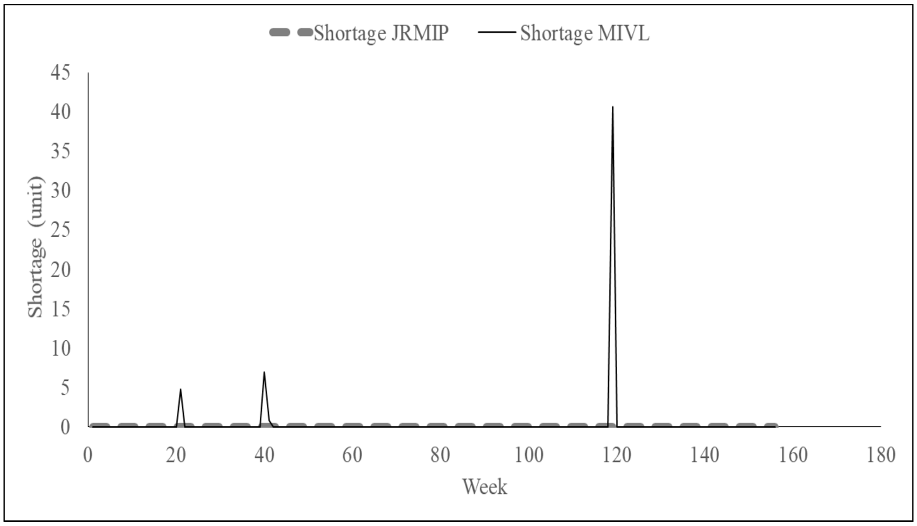

Figure 5. The number of shortage of item 1.

6. Conclusion

The implementation of joint replenishment by accommodating various single-item inventory policies can facilitate the implementation of the XYZ analysis. The results of the comparison with the latest adaptive JRP show better results. Thus, the application of JRMIP can become a new standard for benchmarks in future studies. Moreover, JRMIP can work on forecast-based inventory policies, non-forecast-based inventory policies, or a combination of both. The JRMIP procedure can be an innovation in completing a real problem that often uses several inventory policies.

Future research from this study can include the development of constraints related to an industry that can apply them specifically to provide better results that are more in line with the industry. The application of JRP in a supply chain is also a challenging topic to develop.

Acknowledgments

This research is supported by the Ministry of Science and Technology, Taiwan under contract no. MOST 110-2221-E-033-032- and MOST 111-2221-E-033-026-.

References

Ai, X., Yue, Y., Xu, H., and Deng, X., Optimizing multi-supplier multi-item joint replenishment problem for non-instantaneous deteriorating items with quantity discounts, *PLOS ONE*, vol. 16, no. 2, pp. e0246035, 2021.

Ai, X. Y., Zhang, J. L., and Wang, L., Optimal joint replenishment policy for multiple non-instantaneous deteriorating items, *International Journal of Production Research*, vol. 55, no. 16, pp. 4625-4642, 2017.

Amaya, C. A., Carvajal, J., and Castano, F., A heuristic framework based on linear programming to solve the constrained joint replenishment problem (C-JRP), *International Journal of Production Economics*, vol. 144, no. 1, pp. 243-247, 2013.

Atkins, D. R., and Iyogun, P. O., Periodic versus 'can-order' policies for coordinated multi-item inventory systems, *Management Science*, vol. 34, no. 6, pp. 791-796, 1988.

Balintfy, J. L., On a basic class of multi-item inventory problems, *Management Science*, vol. 10, no. 2, pp. 287-297, 1964.

Bialas, C., Revanoglou, A., and Manthou, V., Improving hospital pharmacy inventory management using data segmentation, *American Journal of Health-System Pharmacy*, vol. 77, no. 5, pp. 371-377, 2020.

Boctor, F. F., Laporte, G., and Renaud, J., Models and algorithms for the dynamic-demand joint replenishment problem, *International Journal of Production Research*, vol. 42, no. 13, pp. 2667-2678, 2004.

Braglia, M., Castellano, D., and Frosolini, M., Joint-replenishment problem under stochastic demands with backorders-lost sales mixtures, controllable lead times, and investment to reduce the major ordering cost, *Journal of the Operational Research Society*, vol. 67, no. 8, pp. 1108-1120, 2016.

Braglia, M., Castellano, D., and Gallo, M., An extension of the stochastic Joint-Replenishment Problem under the class of cyclic policies, *Operations Research Letters*, vol. 44, no. 2, pp. 278-284, 2016.
Braglia, M., Castellano, D., Santillo, L., and Song, D. P., Controlling lead times and minor ordering costs in the joint replenishment problem with stochastic demands under the class of cyclic policies, *International Transactions in Operational Research*, vol. 28, no. 1, pp. 376-400, 2021.
Braglia, M., Castellano, D., and Song, D. P., Distribution-free approach for stochastic Joint-Replenishment Problem with backorders-lost sales mixtures, and controllable major ordering cost and lead times, *Computers & Operations Research*, vol. 79, pp. 161-173, 2017.
Cheung, M., Elmachtoub, A. N., Levi, R., and Shmoys, D. B., The submodular joint replenishment problem, *Mathematical Programming*, vol. 158, no. 1-2, pp. 207-233, 2016.
Dombrowski, J. Global Optimization - McCormick Envelopes. 1-5. Available: https://arch.library.northwestern.edu/concern/generic_works/ft848q804?locale=en, 2018.
Goyal, S. K., Two methods for the analysis of a joint replenishment system with known order frequencies, *International Journal of Production Research*, vol. 12, no. 6, pp. 721-724, 1974.
Goyal, S. K., and Satir, A. T., Joint Replenishment Inventory Control - Deterministic and Stochastic-Models, *European Journal of Operational Research*, vol. 38, no. 1, pp. 2-13, 1989.
Kalpakam, S., and Arivarignan, G., A Coordinated Multicommodity (S, S) Inventory System, *Mathematical and Computer Modelling*, vol. 18, no. 11, pp. 69-73, 1993.
Khouja, M., and Goyal, S., A review of the joint replenishment problem literature: 1989-2005, *European Journal of Operational Research*, vol. 186, no. 1, pp. 1-16, 2008.
Kiesmuller, G. P., Multi-item inventory control with full truckloads: A comparison of aggregate and individual order triggering, *European Journal of Operational Research*, vol. 200, no. 1, pp. 54-62, 2010.
Lee, L. H., and Chew, E. P., A dynamic joint replenishment policy with auto-correlated demand, *European Journal of Operational Research*, vol. 165, no. 3, pp. 729-747, 2005.
Narayanan, A., and Robinson, P., Efficient and effective heuristics for the coordinated capacitated lot-size problem, *European Journal of Operational Research*, vol. 203, no. 3, pp. 583-592, 2010.
Noh, J., Kim, J. S., and Hwang, S.-J., A multi-item replenishment problem with carbon cap-and-trade under uncertainty, *Sustainability*, vol. 12, no. 12, 2020.
Ohno, K., and Ishigaki, T., A multi-item continuous review inventory system with compound Poisson demands, *Mathematical Methods of Operations Research*, vol. 53, pp. 147-165, 2001.
Silver, E. A., Pyke, D. F., and Thomas, D. J., *Inventory and production management in supply chains*, 4th Edition, CRC Press Taylor & Francis Group, Boca Raton, 2017.
Tsao, Y.-C., and Sheen, G.-J., A multi-item supply chain with credit periods and weight freight cost discounts, *International Journal of Production Economics*, vol. 135, no. 1, pp. 106-115, 2012.
Viswanathan, S., On optimal algorithms for the joint replenishment problem, *Journal of the Operational Research Society*, vol. 53, no. 11, pp. 1286-1290, 2002.
Yang, Y. H., and Kim, J. S., An adaptive joint replenishment policy for items with non-stationary demands, *Operational Research*, vol. 20, no. 3, pp. 1665-1684, 2020.

Biographies

Victor Suhandi is a Ph.D. candidate in the Department of Industrial and Systems Engineering, Chung Yuan Christian University, Taiwan. He obtained a Master's degree in the Department of Industrial Engineering and Management, Bandung Institute of Technology, Indonesia, and Bachelor's degree from the Department of Industrial Engineering, Maranatha Christian University, Indonesia. His research interests are production and inventory management, supply chain, and simulation modeling.

Ping-Shun Chen is a professor in the Department of Industrial and Systems Engineering, Chung Yuan Christian University, Taiwan. He obtained a Ph.D. degree in the Department of Industrial and Systems Engineering, Texas A&M University, USA. His research interests are supply chain management, system simulation, and healthcare simulation applications.

Appendix A

The simulation output with 15% error for JRMIP and MIVL procedures.

JRMIP

Week	Item	Forecast	Demand	PotOrdQty*	InvL*	Short*	TOrdCost*	Total Cost
1	1	15.75	15.32	130.36	115.05	0.00	2900.00	3015.05
1	2	47.25	44.66	248.76	204.10	0.00	2900.00	3219.15
1	3	15.75	13.86	130.36	116.50	0.00	2900.00	3335.65
1	4	25.50	23.31	172.15	148.84	0.00	2900.00	3484.49
1	5	76.50	69.54	335.36	265.82	0.00	2900.00	3750.31
1	6	25.50	28.58	172.15	143.58	0.00	2900.00	3893.89
2	1	16.50	16.65	18.82	98.40	0.00	2900.00	3992.29
2	2	49.50	53.48	51.82	150.62	0.00	2900.00	4142.91
2	3	16.50	17.77	17.36	98.74	0.00	2900.00	4241.65
2	4	26.00	23.77	25.28	125.07	0.00	2900.00	4366.72
2	5	78.00	83.47	73.68	182.34	0.00	2900.00	4549.06
2	6	26.00	29.38	30.54	114.20	0.00	2900.00	4663.26
3	1	17.25	15.66	38.91	82.74	0.00	2900.00	4746.00
3	2	51.75	51.43	112.36	99.19	0.00	2900.00	4845.19
3	3	17.25	17.37	38.57	81.37	0.00	2900.00	4926.56
3	4	26.50	25.42	51.01	99.65	0.00	2900.00	5026.21
3	5	79.50	74.12	161.27	108.22	0.00	2900.00	5134.43
3	6	26.50	24.65	61.88	89.55	0.00	2900.00	5223.98
				…				
154	1	52.20	54.79	49.41	209.59	0.00	226200.00	453975.64
154	2	130.50	137.79	115.26	335.39	0.00	226200.00	454311.03
154	3	730.80	698.69	810.20	906.35	0.00	226200.00	455217.38
154	4	40.80	45.91	41.39	181.67	0.00	226200.00	455399.06
154	5	102.00	99.30	115.26	303.66	0.00	226200.00	455702.71
154	6	571.20	475.38	548.33	857.49	0.00	226200.00	456560.20
155	1	52.50	54.04	55.72	155.55	0.00	226200.00	456715.75
155	2	131.25	134.05	139.58	201.35	0.00	226200.00	456917.10
155	3	735.00	738.24	705.71	168.11	0.00	226200.00	457085.21
155	4	41.00	41.50	46.58	140.17	0.00	226200.00	457225.38
155	5	102.50	104.91	100.58	198.74	0.00	226200.00	457424.12
155	6	574.00	516.72	480.25	340.77	0.00	226200.00	457764.90
156	1	52.80	55.66	110.69	210.59	0.00	229100.00	460875.48
156	2	132.00	149.21	275.42	327.56	0.00	229100.00	461203.04
156	3	739.20	750.31	1450.97	868.77	0.00	229100.00	462071.81
156	4	41.20	41.63	88.75	187.30	0.00	229100.00	462259.11
156	5	103.00	111.38	206.76	294.12	0.00	229100.00	462553.23
156	6	576.80	650.84	1001.84	691.77	0.00	229100.00	463245.00

* PotOrdQty: Potential order quantity based on POE procedure
InvL: Inventory level
Short: Shortage quantity
TOrdCost: Total ordering costs

MIVL

Week	Item	Forecast	Demand	Target Inv*	Oit*	Nit*	OrdDec*	InvL*	Short*	Total Cost
1	1	15.75	15.32	18.11	400.20	787.50	1	2.80	0.00	2902.80
1	2	47.25	44.66	54.34	400.59	2362.50	1	9.68	0.00	2912.48
1	3	15.75	13.86	18.11	400.20	787.50	1	4.25	0.00	2916.73
1	4	25.50	23.31	29.33	400.32	1275.00	1	6.01	0.00	2922.74
1	5	76.50	69.54	87.98	400.96	3825.00	1	18.43	0.00	2941.18
1	6	25.50	28.58	29.33	400.32	1275.00	1	0.75	0.00	2941.92
2	1	16.50	16.65	18.98	400.21	685.15	1	2.33	0.00	5844.25
2	2	49.50	53.48	56.93	400.62	1991.04	1	3.44	0.00	5847.69
2	3	16.50	17.77	18.98	400.21	612.28	1	1.21	0.00	5848.90
2	4	26.00	23.77	29.90	400.33	999.36	1	6.13	0.00	5855.03
2	5	78.00	83.47	89.70	400.98	2978.46	1	6.23	0.00	5861.26
2	6	26.00	29.38	29.90	400.33	1262.59	1	0.52	0.00	5861.78
3	1	17.25	15.66	19.84	400.22	746.21	1	4.18	0.00	8765.96
3	2	51.75	51.43	59.51	400.65	2415.34	1	8.08	0.00	8774.04
3	3	17.25	17.37	19.84	400.22	802.01	1	2.47	0.00	8776.51
3	4	26.50	25.42	30.48	400.33	1018.64	1	5.06	0.00	8781.57
3	5	79.50	74.12	91.43	400.99	3663.66	1	17.30	0.00	8798.87
3	6	26.50	24.65	30.48	400.33	1298.84	1	5.82	0.00	8804.69
					…					
154	1	52.20	54.79	60.03	400.65	2049.80	1	5.24	0.00	469719.55
154	2	130.50	137.79	150.08	401.63	4737.94	1	12.28	0.00	469731.84
154	3	730.80	698.69	840.42	409.14	34919.33	1	141.73	0.00	469873.57
154	4	40.80	45.91	46.92	400.51	1741.28	1	1.01	0.00	469874.58
154	5	102.00	99.30	117.30	401.28	4963.09	1	18.00	0.00	469892.58
154	6	571.20	475.38	656.88	407.14	23049.79	1	181.50	0.00	470074.08
155	1	52.50	54.04	60.38	400.66	2363.03	1	6.33	0.00	472980.41
155	2	131.25	134.05	150.94	401.64	5948.31	1	16.89	0.00	472997.30
155	3	735.00	738.24	845.25	409.19	29663.74	1	107.01	0.00	473104.31
155	4	41.00	41.50	47.15	400.51	1999.55	1	5.65	0.00	473109.96
155	5	102.50	104.91	117.88	401.28	4225.11	1	12.96	0.00	473122.92
155	6	574.00	516.72	660.10	407.18	19625.40	1	143.38	0.00	473266.31
156	1	52.80	55.66	60.72	400.66	2323.32	1	5.06	0.00	476171.37
156	2	132.00	149.21	151.80	401.65	5755.55	1	2.59	0.00	476173.96
156	3	739.20	750.31	850.08	409.24	31609.77	1	99.77	0.00	476273.73
156	4	41.20	41.63	47.38	400.52	1777.52	1	5.75	0.00	476279.49
156	5	103.00	111.38	118.45	401.29	4501.79	1	7.07	0.00	476286.56
156	6	576.80	650.84	663.32	407.21	21671.25	1	12.48	0.00	476299.03

* Target Inv: Target inventory level
Oit: Estimated costs if to order item i at period t
Nit: Estimated costs if not to order item i at period t
OrdDec: Order decision (1: to order; and 0: not to order)
InvL: Inventory level
Short: Shortage quantity