Optimization of Multimodal Dial-A-Ride Systems with Fixed-Line Flexible Time Schedules

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Abstract

This paper investigates a multimodal system that integrates the general dial-a-ride problem with a set of fixed-line vehicles having a flexible schedule. The passenger trips are made in a three-stage journey using a dial-a-ride vehicle and a fixed-line bus. Passengers are picked up from an origin point and transferred to a collecting point via a dial-a-ride vehicle. Then, fixed-route buses transfer passengers to a second collecting point where another dial-a-ride vehicle will pick up and transfer them to their requested destination. The main objective of this integration is to reduce the total trip cost by carrying part of it through cheaper fixed-route service while maximizing the utilization of seats in the fixed-route vehicle. The model provides each fixed line with a suggested start time. A mixed-integer linear programming model was developed for the problem, and computational testing on small-size instances has been carried out to demonstrate the viability of the proposed system.

Keywords
Multimodal transportation, Pickup and delivery, Transfer points, Scheduled lines, Integrated dial-a-ride, and Mixed-integer linear programming (MILP).

1. Introduction
Modern communities need transport networks that provide convenience and excellent productivity with minimal noise, emissions, costs, and delays. Dial-a-Ride Problem (DARP) is a problem of managing a transport system that provides individuals with multi-occupancy, door-to-door transport services. It could be considered an optimization problem that satisfies a group of requests; each request deals with a set of passengers transferred from different origins (pickup points) to required destinations (delivery points).

1.1 Objectives
Balancing cost and time in transportation transit systems is always the primary challenge research should solve, as cost-efficient trips with lower time are conflicting objectives. While DARP is not the optimum economic solution, there was a need for a multimodal transportation system that combines more than one transportation system to reduce costs. Transit systems allow passengers to use more than one transportation type. Fixed-line buses are one of these types that are relatively cheap in comparison with others. So, each trip in multimodal transportation systems costs less than when carried out by DARP from origin to destination. Scheduled bus or train services can carry many passengers (thus are cost-efficient) but travel on fixed routes at scheduled times to which passengers must adjust their travel plans. Ho et al. (2018) have compared three transportation types of multimodal transportation systems that combine more than one transportation system (see Table 1). These types will be used in the present model formulation to exploit each type's benefits best and avoid its disadvantages as much as possible. Two transportation systems are used to complete each request: a dial-a-ride system (i.e., taxi service) and a fixed-line system (i.e., scheduled bus service).

Low demand for fixed lines makes it difficult to cover their operating cost, which makes their service unavailable or infrequent. Thus, a valid research interest is finding a flexible fixed-line schedule updated according to varying demand and the number of requests.

Table 1. A comparison of three transportation types (Ho et al. 2018).

	Bus	**Multimodal systems**	**Taxi**
Route	Fixed	Flexible	Customized
Schedule	Fixed	By request	By request
Speed	fixed	Medium	Fast
Cost	Low	Medium	High
Mode	Shared	Shared	Non-shared
Capacity	High	Medium	Low
Reservation	Not needed	Often needed	Not needed

Two transportation systems are used to complete each request: a dial-a-ride system (i.e., taxi service) and a fixed-line system (i.e., scheduled bus service). Low demand for fixed lines makes it difficult to cover their operating cost, which makes their service unavailable or infrequent. Thus, a valid research interest is finding a flexible fixed-line schedule updated according to varying demand and the number of requests.

Each client would complete his journey from an origin to a destination by a dial-a-ride vehicle that would pick up each client from his origin and transfer him to the first transit point. Transferring that client to the second transit point is considered the second stage, and a fixed-line bus does it. The third stage will transfer each client from the second transit point to the delivery point by another dial-a-ride vehicle. These three stages are illustrated in Figure 1.

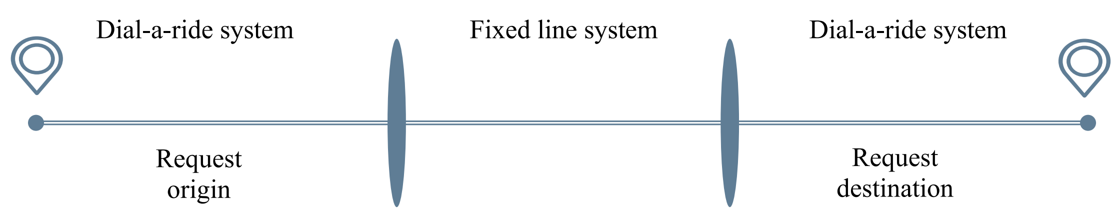

Figure 1. A three-stage transporting system.

This paper is organized as follows: section 2 reviews the literature on some contributions of multimodal transportation systems of which dial-a-ride problems are part. Section 3 introduces methods and problem formulation. Section 4 proposes data collection. Section 5 reports the results obtained and gives a solution visualization. The conclusion and future research directions are finally presented in section 6.

2. Literature Review

The dial-a-ride problem (DARP) is one of the vehicle routing problems (VRP) variants. The VRPs consist of designing vehicle routes to perform transportation requests by a given fleet at minimum cost and under a set of constraints. Mor and Speranza (2020) discussed the classification of vehicle routing problems based on their objectives. In addition to the classical objectives of assigning customers to vehicles and sequencing each trip, they surveyed the literature on periodic routing problems, inventory routing problems, vehicle routing problems with release dates, and multi-trip vehicle routing problems. Their survey did not include the study of problem formulations and their solution methods., Bai et al. (2022) in food delivery, addressed a vehicle routing problem that manages cold-chain logistics, they aimed to find a route with minimum delivery cost and minimum carbon emissions that occur during the distribution of cold-chain logistics products. They guided cold chain logistics companies to select a distribution strategy according to their needs and for the governments to develop a carbon tax. A Nondominated Sorting Genetic Algorithm (NSGA) was developed and constraints that satisfy customers' demands, and all types of costs (vehicle operating cost, product freshness cost, quality loss cost, transportation cost, carbon emission cost, and penalty cost) were added. Thus, the model can precisely describe the distribution costs and carbon emissions produced and provide the distribution strategies and the necessary prerequisites for achieving a low-carbon state.

Dursun and Özger (2022) studied another modeling approach for vehicle routing problems, they formulated a model for multi-depot heterogeneous routing problems that deal with airline and roadway vehicles. Firstly, they developed a mixed integer linear programming (MILP) model. Secondly, they solved the model using a hybrid genetic algorithm. They recommended new different transportation types and their related constraints to be added in future work. Guillermo et al. (2017) presented a model for collecting food donations, their study used the capacitated vehicle routing problem with time windows (CVRPTW) problem to collect donations from a food bank. The primary goal is to reduce the expenditures and operational times that are impacted by the fact that donor visits are subject to dynamic changes in scheduling. This problem was tackled using a mixed integer linear programming model that determines the optimum route while computing the distance matrix using data retrieved from the Google Maps application programming interface (API). The model determines the order in which the vehicles must visit the major, donor, and secondary nodes connecting points between the routes. They considered their study a pilot test that could be applied to national food bank research.

Cavagnini and Morandi (2021) in transporting goods, studied a static share-a-ride problem (SARP) that serves people and transfers parcels in an integrated way, and the benefits of using horizontal cooperation between a set of service providers were analyzed. After comparing different four scenarios of cooperative models, their results proved that all agents in the system benefited from horizontal cooperation. They proposed a new mathematical programming formulation for cooperative SARP (coop-SARP), and results showed that cooperation leads to reduced travel times and improved vehicle occupancy rates, service levels, and profits, which make such a cooperative system more appealing for service providers. They proved that the benefits arising from the coalition are bigger at the aggregate level than those which could be achieved in a non-cooperative system because stakeholders increase their efficiency by sharing resources, producing less pollution, and lowering transporting prices. Dragomir and Doerner (2020) decomposed a routing problem that transforms parcels between two cities into a three-part network structure and solved a pickup and delivery problem with long hauls without direct shipments between regions. The fixed-line part was a scheduled line, and the model met all capacity and timing constraints.

Since the (DARP) is the basis for integrated (multimodal) dial-a-ride problems (IDARP), contributions to solving the DARP can be valuable when studying IDARP. Dial-a-ride problems aim to design vehicle routes and their time schedules and keep operational costs minimal, with an acceptable increase in each trip’s total time.

Molenbruch et al. (2017) presented The IDARP variants and their classification, they introduced the characteristics and classification of different models of dial-a-ride problems. They also extracted a general definition of the problem followed by solution methods from an algorithmic point of view. In solution methods, they got an exact solution for small instances formulated by branch-and-cut algorithm, column generation, or branch-and-price algorithms. For larger instances, metaheuristics based on local search gave reasonable results. Moreover, hybridization was presented as a recent way to improve the performance of solving the problem by combining both exact and approximate methods. Johnsen and Meisel (2022) studied rural areas’ transportation and showed that due to the few offers of public transit in rural areas, the rural population is still reliant on motorized individual mobility. Autonomous vehicles can narrow this mobility gap since it moves the responsibility of driving from humans to machines and opens new opportunities for vehicle sharing by removing economic and service time constraints (e.g., labor costs and working hour regulations). In their study, they provided a model of interrelated trip problems in the rural dial-a-ride with autonomous vehicles. They defined the problem as a mixed-integer model and propose an adaptive variable neighborhood search heuristic (AVNS) to solve it, the computational results showed that the heuristic model performs very well and is superior to typical MIP solvers. They also explored the effect of trip interrelationships (vertical and horizontal interrelationships) on the problem's solvability, service quality, and total transportation costs of the solutions. Chen et al. (2019) solved a model for taxi pooling, their model showed how more than one passenger could share each taxi in a dial-a-ride system on (almost) the same route. The model solved a taxi pooling problem with stochastic vehicle travel times. They simplified the stochastic taxi pooling model and built a deterministic taxi pooling model by replacing all stochastic traveling arcs with deterministic traveling arcs and removing all unanticipated penalty costs in the stochastic model. Hang et al. (2015) developed a memetic algorithm to solve a multi-trip dial-a-ride (MTDARP) problem and presented a mathematical description of the problem in their study. The model deals with patient transportation processes through a set of ambulances. Patients need to be transported from one location to another to receive medical services, and multiple trips for each ambulance were needed. A set-up time between two consecutive trips for each vehicle included ambulances disinfection at the depot (hospital) to prevent the spread of disease.

Integrated dial-a-ride problems (IDARP) is an extended model for DARP. Using integrated dial-a-ride service is considered a cost-effective way as it combines two or more transportation modes and optimizes the total cost of each trip.

Häll et al. (2009) implemented some early work in formulating an integrated dial-a-ride problem (IDARP). The authors proposed a fixed route service as a part of each journey, the passenger can either be carried by the dial-a-ride service from the pick-up point to the drop-off point directly or from the pick-up point to a transfer point where the passenger can change to a fixed route service. They solved the model with an exact solution and presented a small-sized instance with a graphical representation of its solution. Shiri et al. (2020) examined the impact of transfers on pickup and delivery systems, and many parameters were considered in their experiments. These parameters included modeling parameters (distance metric and objective function), system design parameters (number and location of the transfers points in the network), operational parameters (number, capacity, time window and cost of vehicles, and length, time window, and volume to capacity ratio of requests). They ended with that: firstly, there is a 5.7% reduction in the trip cost when transfers are used under normal conditions, secondly: the cost reduction of trips could decrease more when having the heterogeneity of vehicles, shorter time windows, and an increase in the length of the request. They also expected that the profitability of transfers would increase by problem size. Posada et al. (2017a) presented another aspect of the problem of integrating demand-responsive systems with a known schedule fixed routes. They presented an extended model of DARP with scheduled fixed-line to serve older adults by forcing a fleet of dial-a-ride system vehicles to reach each transfer point at a specific time. Posada et al. (2017b) studied door-to-door transport systems for older and/or disabled people who need some requirements more than others. Direct transport is available and practical but also quite expensive compared to traditional public transport. These modes of transport were integrated. The main models were transfer points between direct and traditional public transport vehicles. They added fixed route timetables to IDARP and then forced vehicles to carefully plan their arrival at the transfer points according to those added timetables. Brand et al. (2017) presented a study that showed a new technique for assessing integration in bus networks, the combination of bikes and buses in a transportation trip, and the impact of integration on the bus system. The framework is divided into two parts: first, to evaluate the characteristics of various bus services and their access and egress modes, and second, to evaluate the impact of integrating these services with access and egress modes. Several bus lines have been evaluated using the proposed framework. According to their study, systems with a higher frequency and speed can attract twice as many bikes on the entry and egress side. They assessed and studied different bus routes depending on their work, performance, and capabilities to integrate with access and egress modes. Integration is a result of the framework presented, and it could be then easily assessed by analyzing many bus lines belonging to different types of bus services (e.g., conventional lines and high-quality lines).

Charisis et al. (2018) solved unsatisfied demand in transit transportation as they constructed a cost-effective demand-responsive feeder network for first-mile problems while having the ability to tackle unsatisfied demand by introducing additional (repair) routes. Since there were multiple origins and destinations, the problem was formulated as a capacitated vehicle routing problem (CVRP) with a many-to-many demand pattern, unlike most transit problems with many-to-one demand patterns. A mathematical programming model was introduced, and a genetic algorithm was used to solve it. A parallel representation technique was used, which allowed for the optimization of many chromosomes simultaneously, giving the model flexibility and the capacity to integrate changes in demand and network design easily.

Since transportation is an essential mechanism for economic growth but also has a negative environmental impact, recent research called 'green' logistics aims to minimize the harmful impact of transportation on the environment. Ibhaze et al. (2019) in electric vehicles, expanded constraints to include battery management, detours to charging stations, recharge times, and selecting destination depots, besides classic DARP features. They added a problem variant for the DARP, which considers using autonomous electric vehicles (e-ADARP), and they solved small-to-medium-sized problems. A solution algorithm and test results were developed using real data. Abdoli et al. (2019) discussed a modern aspect of the green environment and using clean energy sources integrated with DARP research. They addressed the green vehicle routing problem with the alternative-fuel vehicles (AFVs) fleet, where refueling decisions are also considered during route planning to allow all vehicles to use clean fuel at the right time.

Thus, adding a transit transportation system was studied, and a solution was presented by other research, while all presented models had a fixed schedule for fixed-line transportation. The present work models an integrated vehicle routing problem with a flexible schedule for a fixed-line transportation system. It presents a combination of a dial-a-ride problem with a flexible scheduled fixed line, schedules adapted according to request numbers and requests' time windows. This problem was solved while keeping an optimum cost.

3. Methods

Multiple users request transportation from their origins to a specific destination (pickup and drop-off/delivery points). It is assumed that all requests are made in advance. All requests are received, then **routes for dial-a-ride vehicles** and **schedules for fixed-line buses** are arranged and optimized with minimum cost. Traveling time and service time are assumed to be well-known. In the same vehicle, there could be more than one user simultaneously, but with different requests (different pickup, delivery points, and time windows), so vehicles are shared, which affects cost too. Each pickup and delivery request is defined by time window, origin point, and destination point for each, and each vehicle has its capacity, while fixed-line vehicles are defined by their maximum capacity. Fixed-line vehicles do not have a schedule for their trips (as presented in the literature), but their schedule is updated according to the number of clients and the time they arrive at each station, giving **a flexible schedule**.

Each request may be served by two types of transportation arranged in a three-step journey as follows:

a. Private PD vehicle (that will pick up the client from his/her pickup point and deliver him/her to the next stage, which is a fixed route that starts at a station called “transfer point 1”)
b. Fixed route (that will start and end with “transfer station” and have a higher capacity, lower cost, and faster service)
c. Private PD vehicle (that will pick up the client from the second transfer station, “transfer point 2”, to his/her final delivery point or destination)

Figure 2 shows a schematic representation of the multimodal network, where each request is made by a passenger or a group of passengers (up to the maximum capacity of each PD vehicle), and passengers are collected at a transfer point to continue their trip via fixed-line transportation, as shown. The destination of the fixed-line bus is the pickup point (a second pickup for any passenger) for a second pickup and delivery cycle that end with the destination for each passenger(s).

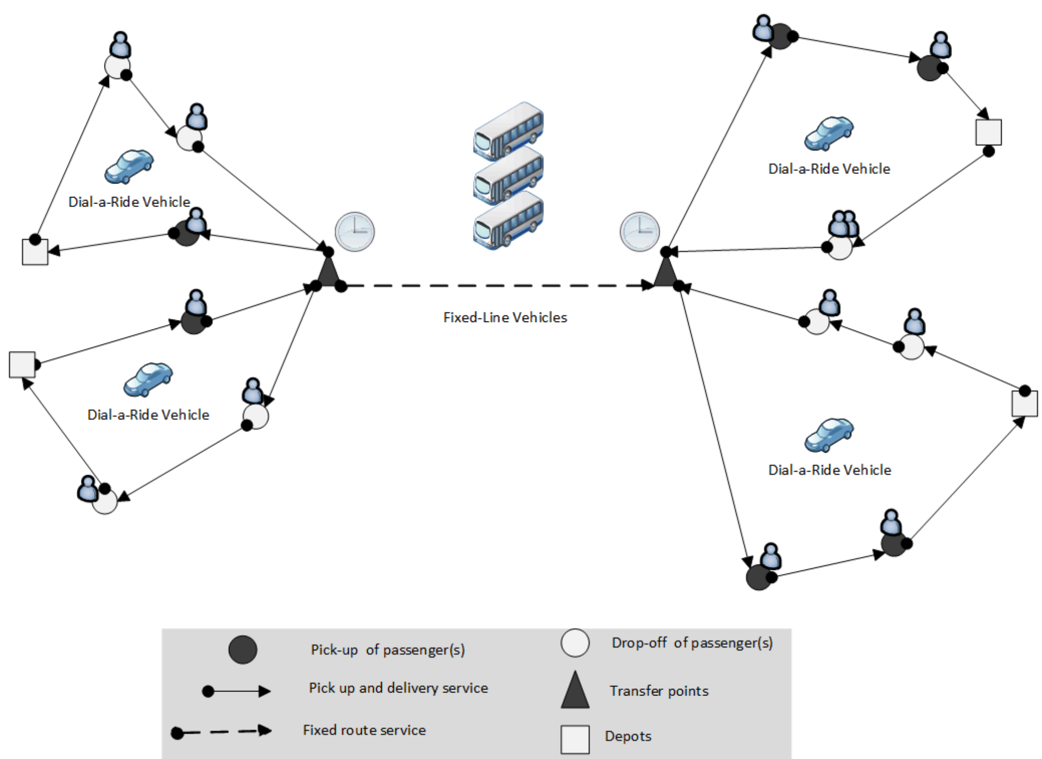

Figure 2. An illustration of the multimodal transportation network

The typical assumptions of the problem include the following:

Visit: Each request starts from an origin to a destination.

Time window: Each user can specify the earliest and latest times of pickup and/or drop-off (i.e., the departure time from the origin and the arrival time at the destination).
Depot(s): The starting and ending location(s) of a trip (or route) of a vehicle.
Trip: A PD vehicle finishes a trip once it returns to the depot.
Vehicle capacity: The maximum number of users in a vehicle simultaneously. The number of users in the vehicle is known as a load.
Ride time: The time a user spends in a vehicle (i.e., the difference between the scheduled pickup and delivery times).
Route duration: The time a vehicle travels for a trip (i.e., the difference between the times of leaving from and returning to a depot).

Any business's objective is minimizing costs while keeping an acceptable service level for users. Also, transportation has a matter of cost and luxury trade-off. This work presents a model that integrates both modes of transportation: the fixed-line problem with an optimum cost, the dial-a-ride problem, and the optimum comfort capability. Cost reduction could be made through both dial-a-ride and fixed-line systems as follows:
Cost reduction in dial-a-ride trip: Each trip could be shared with more than one passenger so each could pay a lower price. So, a model was presented to solve a vehicle routing problem concerned with picking up all passengers with a minimum number of vehicles and transferring them to their transit point.
Cost reduction in fixed-line trips: Each line should have a fixed schedule that controls bus kick-off time, whether occupied with one passenger or fully occupied. In rural places, it raised trip costs and made it difficult to have frequent buses, making fixed-line systems an irrelevant service. Cost reduction was reached by a flexible schedule that will be efficiently utilized by updating the number of clients and their arrival time at each transit point and providing a kick-off time for each fixed-line vehicle.
This proposed system forms an integrated service that will optimize fixed-route vehicles and dial-a-ride vehicles by utilizing all their possible capacity, so minimizing the overall costs.

3.1 Mathematical Formulation

Sets, parameters, and decision variables are the constituents that define the model. The problem is defined by digraph G = (N, A), where N represents the set of points, and A represents the set of arcs.

The sets used in the model can be described as follows:
Sets

O	Set of depots
R	Set of requests
P	Set of pickup nodes
D	Set of delivery nodes
T	Set of transfer nodes
V	Set of pickup and delivery vehicles
N	Set of all nodes
N_1	P U D
N_2	P U D U T
B	Set of fixed-line buses

The first step in formulating a mixed-integer programming model is defining the decision variables under the programmer's control and parameters imposed by the external environment and out of the programmer's control. Decision variables were presented as input to this model.

Parameters

T_{ij}	Traveling time from point i to j
S_i	Service time at point i
C_B	The capacity of fixed-line buses that move between transfer stations (from node i to j)
C_v	The capacity of pickup and delivery vehicle
$[e_i, l_i]$	Time window at point i

ϕ_v	The cost of pickup and delivery of one passenger with a vehicle (v)
ϕ_B	The operating cost of each trip of a fixed-line bus (b) between two points
M	A large positive number

$$fir = \begin{cases} 1 \text{ if point i is the start point of request r} \\ 0 \text{ if point i is an intermediate point} \\ -1 \text{ if point i is a destination of request r} \end{cases}$$

Decision variables

A performance measure can be associated with the resulting decision variable values, so decision variables reflect the modeling purpose. This model includes three types of variables: binary, integer, and continuous.

Binary variables

$$X_{ij}^{v} = \begin{cases} 1 \; if\ vechicle\ v\ travelled\ from\ (i)\ to\ (j) \\ 0\ otherwise \end{cases}$$

$$y_{ij}^{r} = \begin{cases} 1 \; if\ arc\ (i,j)\ is\ traversed\ by\ request\ (r) \\ 0\ otherwise \end{cases}$$

$$q_{ij}^{b} = \begin{cases} 1 \; if\ fixed\ line\ moves\ from\ i\ to\ j\ \ is\ traversed\ by\ fixed\ line\ bus\ (b) \\ 0\ otherwise\ , (where\ i\ and\ i\ are\ transfer\ points) \end{cases}$$

Integer variable

w_{iv}	The used capacity of the vehicle (v) upon leaving node (i)

continuous variables

β_i	A variable that indicates the departure time of a vehicle from a node (i)
γ_{ir}	A variable that indicates the departure time of request r from a node (i)
α_{tv}	A variable that indicates the time that each vehicle (v) reaches the transfer point
SC_t	A variable that indicated buses schedule at each transfer point

Objective Function

The objective function of this model aims to minimize the total travel cost of pickup and delivery vehicles and the cost of fixed lines as follows:

$$Minimize \sum_{i,j\,\in\,A} \sum_{v\in V} \phi_v T_{ij} X_{ij}^{v} + \sum_{b\in B} \sum_{i,j\,\in T} \phi_b\, q_{ij}^{b} \tag{1}$$

Constraints

Constraints represent relationships of decision variables and parameters obtained from the characteristics of our current study problem. In our case, constraints deal with three issues: a route for each pickup and delivery vehicle, capacity for each pickup and capacity vehicle and fixed-line buses, and finally, timing and scheduling for fixed-line buses. These characteristics are presented in a precise and quantitative form.

In the first group of constraints, constraint (2) ensure that all pickup and delivery points are visited by a vehicle once. Constraint (3) ensures that each pickup or delivery vehicle is used at most once, and constraint (4) ensures that each transfer point is visited by a vehicle once. To ensure flow balance, constraints (5) and (6) make sure that each request flow is balanced, and constraint (7) makes sure that each transfer point of the fixed line is a point for pickup/delivery that is done by a vehicle.

$$\sum_{i\in N} \sum_{v\in V} X_{ij}^{v} = 1 \qquad \forall j \in N \tag{2}$$

$$\sum_{i\in N_1} X_{o,i}^{v} \le 1 \qquad \forall v \in V,\ o \in O \text{ (set of depots)} \tag{3}$$

$$\sum_{i\in N}\sum_{v\in V} X_{it}^{v} \le 1 \qquad \forall t \in T \qquad (4)$$

$$\sum_{j\in N} X_{ij}^{v} - \sum_{j\in N} X_{ji}^{v} = 0 \qquad \forall I \in N, v \in V \qquad (5)$$

$$\sum_{i\in N_2} y_{ij}^{r} - \sum_{i\in N_2} y_{ji}^{r} = f_i^{r} \qquad \forall r \in R, I \in N_2 \qquad (6)$$

$$\sum_{i\in N}\sum_{v\in V} X_{it}^{v} \le \sum_{r\in R}\sum_{i\in N_2} y_{it}^{r} \qquad \forall t \in T \qquad (7)$$

Thus, all constraints related to flow control and routing for pickup and delivery, or fixed line and their capacities, are covered.In the second group of constraints, constraint (8) makes sure that the capacity of each vehicle for pickup and delivery will not be exceeded, while constraint (9) makes sure that the capacity for each fixed line is not exceeded too, and finally constraint (10) defines and control vehicle's used capacity.

$$\sum_{r\in R} d_i y_{ij}^{r} \le \sum_{v\in V} C_v X_{ij}^{v} \qquad \forall\, i \in N, j \in N \qquad (8)$$

$$\sum_{r\in R} d_r X_{ij}^{v} \le \sum_{b\in B} C_B q_{ij}^{b} \qquad \forall\, i \in N, j \in N \qquad (9)$$

$$X_{ij}^{v} = 1 \rightarrow wjv \ge (wiv + dj) \qquad \forall\, v \in V, \forall\, i \in N, j \in N \qquad (10)$$

After linearizing using the big-M technique:

$$w_{jv} \ge (w_{iv} + d_j) - M\,(1 - X_{ij}^{v})$$

In the last group of constraints, constraint (11) controls the timing for each request, while in constraint (12), the same timing is considered for each vehicle. In constraint (13), the precedence constraint is covered, and the time window (TW) for each point is assured by constraints (14). Constraint (15) is conserved by giving each vehicle time to reach the transfer point. The target of getting each bus schedule is given in constraint (16), considering fixed-line bus capacity and each vehicle capacity. Thus, all constraints related to timing and scheduling are covered.

$$y_{ij}^{r} = 1 \rightarrow \quad \gamma_j^{r} \ge \gamma_i^{r} + T_{ij} \qquad \forall\, r \in R, i, j \in N_2 \qquad (11)$$

After linearizing using the big-M technique:

$$\gamma_i^{b} + T_{ij} + S_i \le \gamma_j^{b} - M\,(1 - y_{ij}^{r})$$

$$\textstyle\sum_{v\in V} X_{ij}^{v} = 1 \rightarrow \beta j \ge \beta i + Tij + Si \qquad \forall\, i, j \in N_1 \qquad (12)$$

After linearizing using the big-M technique:

$$\beta_i + T_{ij} + S_i \le \beta_j - M\,(1 - \textstyle\sum_{v\in V} X_{ij}^{v})$$

$$\beta_{r+n} \ge \beta_r + T_{r,\,r+n} + S_{r+n} \qquad \forall\, r \in R \qquad (13)$$

$$e_i \le \beta_i - S_i \le l_i \qquad \forall\, i \in N_1 \qquad (14)$$

$$\textstyle\sum_{v\in V} X_{it}^{v} = 1 \rightarrow \alpha_{tv} = \beta_i + T_{jt} \qquad \forall\, i \in N_1, \forall\, v \in V, (i,j) \in T \qquad (15)$$

After linearizing using big-M technique:

$$\alpha_{tv} \ge \beta_i + T_{jt} - M\,(1 - \textstyle\sum_{v\in V} X_{it}^{v})$$

$$SC_t \ge \max\{\alpha_{tv}\} \qquad v \in V, \text{T is a transfer point} \qquad (16)$$

Decision variables domains

$$X_{ij}^{v} \in \{0,1\} \qquad \forall\, (i,j) \in N, v \in V \qquad (17)$$

$$y_{ij}^{r} \in \{0,1\} \qquad \forall\, (i,j) \in N, r \in R \qquad (18)$$

$q_{ij}^{b} \in \{0,1\}$ $\forall (i,j) \in T, b \in B$ (19)

$w_{iv} \in Z+$ $\forall j \in N_1, v \in V$ (20)

$\beta_i \in R+$ $\forall i \in N_1, v \in V$ (21)

$\gamma_i^r \in R+$ $\forall i \in N_1, r \in R$ (22)

$\alpha_{tv} \in R+$ $\forall t \in T, v \in V$ (23)

$SC_t \in R+$ $\forall t \in T$ (24)

4. Data Collection

Four sets of instances were generated, namely 4R, 8R, 16R, and 32R, which differ in the number of requests (i.e., 8R instances operate eight requests) and the geographical distribution of each request. All node locations were randomly generated. The demand of each request was randomly chosen from one or two, and the width of time window values was assumed. It was assumed that all vehicles used the same fuel type. While the cost for each passenger in the fixed-line buses is one (unit of currency), and the cost in the pickup and delivery vehicles is a proportion of distance, it is constant for all points and requests.

5. Results and Discussion

This part presents the results obtained from solving the previously proposed mixed-integer program (arc formulated) using GUROBI. The model is implemented in Python 9.0. All experiments are run on a CPU @ 2.70 GHz, with two cores. The model has been solved for a relatively small number of instances.

5.1 Numerical Results

Table 2 summarizes the result obtained after running the model and applying the different numbers of requests. Each instance name format shows the (instance name, number of vehicles, and number of requests) knowing that each request is served by two PD vehicles and a fixed-line vehicle.

Table 2. Results obtained with a different number of requests.

Instance	Objective function value	Number of Buses with schedules	GAP (%) *	CPU time (sec)
8R_1	341211.0	2 buses	0.0	3.80
8R_2	333878.0	2 buses	0.0	3.58
8R_3	327117.0	2 buses	0.0	3.64
8R_4	328993.0	2 buses	0.0	3.36
8R_5	341388.0	2 buses	0.0	5.32
8R_6	330563.0	2 buses	0.0	3.75
8R_7	334432.0	2 buses	0.0	2.98
8R_8	331700.0	2 buses	0.0	3.68
8R_9	340661.0	2 buses	0.0	3.96
8R_10	351210.0	2 buses	0.0	3.96
16R_1	14069300.00	4 buses	0.0	102.84
16R_2	13743700.00	4 buses	0.0	115.93
16R_3	14014438.53	4 buses	0.0	101.05
16R_4	13959500.00	4 buses	0.0	100.75
16R_5	14051000.00	4 buses	0.0	117.56
15R_6	14077100.00	4 buses	0.0	128.86
16R_7	14113700.00	4 buses	0.0	106.28
16R_8	140633e0.00	4 buses	0.0	103.05
16R_9	14056300.00	4 buses	0.0	104.98
16R_10	14033900.00	4 buses	0.0	146.84
32R_1	28594300.00	8 buses	0.4695	1500.25
32R_2	28520500.07	8 buses	0.0000	1477.73
32R_3	28421347.40	8 buses	0.8398	1501.09
32R_4	2842e000.00	8 buses	1.1017	1504.37
32R_5	28620600.00	8 buses	1.5627	1501.61
32R_6	28550000.00	8 buses	1.0102	1501.05

32R_7	28360400.00	8 buses	0.0000	1494.72
32R_8	28487900.00	8 buses	0.0000	1431.52
32R_9	28484669.66	8 buses	0.5275	1500.58
32R_10	285066e0.00	8 buses	0.8683	1503.88

Computational results in that table show that instances of up to 16 requests can be solved optimally (without GAP value) in a reasonable time. Optimality GAP value raised in 32-request models. Table 3 shows the resulting bus kick-off timetable at each transfer station.

Table 3. Buses schedule at transfer stations.

Transfer station A	Buses schedule	Transfer station B	Buses schedule
	Bus 1: 3702.40 (units of time)		Bus 1: 13607.43 (units of time)
	Bus 2: 28738.70 (units of time)		Bus 2: 30483.79 (units of time)
	Bus 3: 31691.99 (units of time)		Bus 3: 33164.79 (units of time)
	Bus 4: 41157.36 (units of time)		Bus 4: 55600.13 (units of time)

***Gap percentage and time limit,** Since DARP is an extension of the VRP, an NP-hard problem, time limits must be used to restrict the time allotted for solving the model. In most cases, time limits result in a gap between the current best solution and the lower bound. The computation time limit for 32-request instances was set to 30 minutes, and the optimality gap was consistently below 2%, as noticed in Table 2. Thus, the results indicated that the algorithm can efficiently solve instances of up to 32 requests in a reasonable time with an accepted time gap.

5.2 Graphical Results

Figure 3 presents the solution visualization by the resulting route for the fourth group (32R) that operates 32 requests.

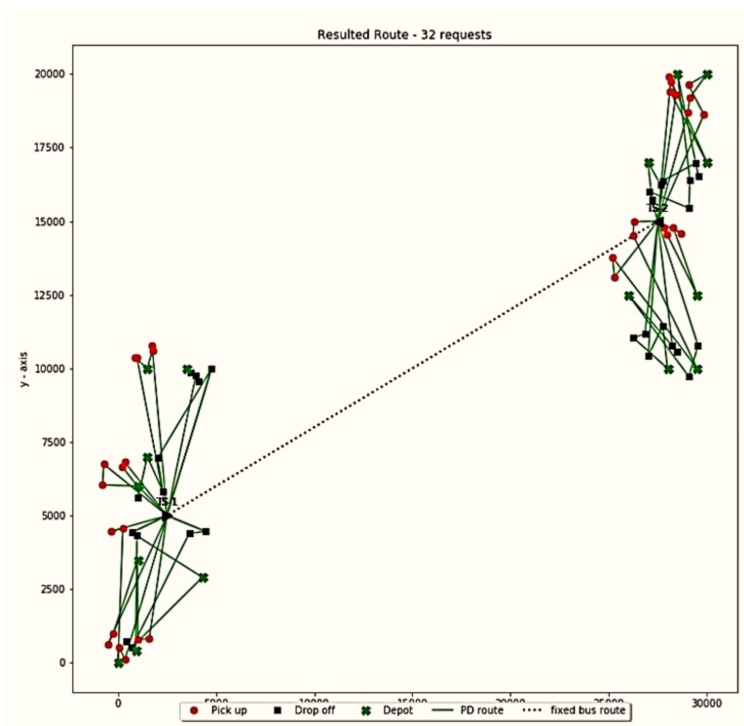

Figure 3. The resulting route for instance R32_1

6. Conclusion and Future Work

This paper has extended the model for a multimodal dial-a-ride system with the opportunity to have a flexible schedule that aims to minimize the cost of all trips while giving an optimum kick-off schedule for fixed-line vehicles, along with all constraints of timing and locations. A mixed-integer model for several small-seized problems was proposed and solved, and an optimum solution that minimizes the overall cost of the transportation system as well as the optimum schedule of used fixed routes that compromise its capacity and cost was found. The model has been tested on a different number of requests and different geographic distributions. The computational results provided the cost of each run, complete paths for each vehicle route, and fixed-line vehicle schedule.

The proposed model is essential for further research to be more efficient by using heuristic and meta-heuristic methods that can be developed later to solve medium to large-sized problem instances. The proposed model could be investigated on a broader scale by a greater number of requests and routes, which will lead to a more realistic model that can be applied to serve society. Moreover, the transportation environment could change (e.g., cancellation requests). In terms of realistic aspects, algorithms for solving a dynamic situation of the proposed model can be developed.

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