Optimal Decisions for a Group Buying Problem with Two Retailers and Two Capacitated Suppliers

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Abstract

Retailers (companies) can leverage their collective buying power to negotiate lower prices through group buying. This study examines a group buying case study involving two retailers that are facing price sensitivity in the market in conjunction with quantity discount pricing from two suppliers who have limited capacities. To solve the problem, a non-linear optimization method based on Lagrangian multipliers is developed. Numerical examples are provided in order to test the validity of the proposed solution approach and gain insight into how the input parameters influence the solution.

Keywords
Group Buying, Optimal Order, Capacitated Suppliers, Non-linear optimization, Lagrange Multipliers.

1. Introduction
A company's procurement costs usually represent a significant portion of its expenses, and reducing these costs contributes directly to increased profits, especially in industries with low profit margins, such as retail(Handfield and Nichols 2002). Group Buying (GB) is one of the strategies for saving on procurement costs, in which different buyers aggregate their demands in order to enhance their negotiation power to get the most favorable price for their goods (Hsieh 2009). Many companies use GB, including Novation and Premier in healthcare(Weinstein 2006), Mfrall in manufacturing, Covinist in automotive, Polysort in plastics, and Transplace in logistics (Keskinocak and Savaşaneril 2008). This study will examine the case of GB for a single product with two suppliers with limited capacity who offer quantity discount pricing, and two competing buyers (retailers), who would like to replenish that product. By applying a mathematical approach, a method is developed for determining the optimal order quantity to each supplier and determining the retail prices.

2. Literature Review
This research is related to GB because the retailers cooperate to maximize their benefit from quantity discount pricing; it is also relevant to the newsvendor problem because each retailer aims to maximize their profit by optimizing its order quantity and retail price to the consumers. The study by Chen and Roma (2011) compares group buying with individual buying in a two-level distribution channel with competing retailers. The researchers found that in the case of symmetric retailers, GB benefits them always, while in the case of asymmetric retailers, GB benefits the weaker player. Based on Chen and Roma (2011), (Zhou and Xie 2014) developed their model where the supplier acts as Stackelberg leader and is an active member. It was found that in this case, GB may be detrimental to all members of the supply chain, except in cases where economies of scale are present. In a newsvendor framework, (Luo and Wang 2015) investigated the optimal discount, order quantity, and minimum order quantity with a fixed retail price for a single retailer and single customer. As a result of numerical results, they found that GB is more beneficial when demand changes in response to price changes. Karabağ and Tan (2015) examined a two-period GB supply chain, involving suppliers, Group Buying Organizations (GBOs), and retailers. In their model, the numerical results indicated that all suppliers and retailers would benefit from GB. One of the closest papers to this research is the research by Mohammdi and Ozelkan (2022) where they looked at a GB problem with one supplier and two competing retailers and proposed an analytical solution to the problem.

As the closest model to GB to incorporate stochastic demand, the Newsvendor model can be extended to GB by including additional retailers (Karabağ and Tan 2015, Wang and Chen 2015). A seminal study in the field of newsvendor research was conducted by Petruzzi and Dada (1999), who studied the newsvendor problem under joint replenishment and pricing decisions with additive and multiplicative demand scenarios. They demonstrated that the optimal price in the additive case is always lower than the riskless profit, while the reverse is true in the multiplicative case. Consumers can be strategic by waiting until the end of the selling season to purchase items at a lower price since the newsvendor always sells the remaining items at a reduced price. Ye and Sun (2016) examined an example in which consumers expect to be able to purchase the product at a bargain price at the end of the sales period, and retailers anticipate the reserved price for the customers. In some cases, such strategic behavior can even benefit retailers through increased profits.

3. Problem Description

Consider a supply chain with two competing retailers and two suppliers with a single product. The suppliers offer a quantity-based discount. The goal is to find out the optimal order quantity and retailer price for each retailer and to find out how much order should be allocated to each supplier knowing their individual pricing function and capacity such that the purchasing cost is minimized. The retailers also face a stochastic, price-sensitive demand from the market which affects their decision.

To simplify the problem, one can split the GP problem to two sub-problems, the retailers' problem and the suppliers' problem. Where in the retailers' sub-problem the goal is to find the optimal retail price and order quantity, while in the suppliers' sub-problem the order quantity is allocated to each supplier in the problem. The retailers' sub-problem can be described as below.

The demand function for each retailer i and competitive retailer j can be shown with the following function (1):

$$D_i(p_i, p_j, \epsilon) = y_i(p_i, p_j) + \epsilon = a_i - (b_i + \gamma_i) * p_i + \gamma_i * p_j + \epsilon \quad (1)$$

Where $y_i(p_i, p_j)$ is the price-sensitive demand function, decreasing with retailer i's price and increasing with retailer j's price, $a_i > 0$ is the base demand of the retailer, $b_i > 0$ is the price elasticity of the customers to retailer i and γ_i is the price elasticity of customers to retailer i's competitors. The last part of the demand functions, ϵ captures the random factor with probability density function $f(.)$ and cumulative distribution function $F(.)$ on the range $[A, B]$. To make sure that $D(p, \epsilon)$ is positive for some range of p, we assume $A > -a$ as in this case (Petruzzi and Dada 1999). This type of linearly decreasing demand function is common in the economic literature (Yao, Leung et al. 2008, Wang and Chen 2015). If the retail price and order quantity for retailer i is represented as p_i and q_i; and for competitor retailer j as p_j and q_j; the profit function for each retailer i can be written as:

$$\pi_i(z_i, p_i, p_j) = \begin{cases} p_i D_i(p_i, p_j, \epsilon) - cq_i + v_i[q_i - D_i(p_i, p_j, \epsilon)], & D_i(p_i, p_j, \epsilon) \le q_i, j \ne i \\ p_i q_i - cq_i - s_i[D_i(p_i, p_j, \epsilon) - q_i], & D_i(p_i, p_j, \epsilon) > q_i, j \ne i \end{cases} \quad (2)$$

Where p_i and q_i are the decision parameters indicating retail price and order quantity, c is the purchasing price from supplier, v_i and s_i are salvage price and shortage cost. The first case happens if there is excess inventory, and the retailer faces a salvage cost $(c - v_i)$ per excess item, on the other hand if shortage happens the retailer faces a shortage cost s_i for each missed sale.

In the suppliers' sub-problem, there are two suppliers offering a single product to the market using quantity-based pricing and they have a limited supply capacity. The pricing function for each supplier k is as below:

$$w_k(q_k) = m_k + \frac{d_k}{q_k^{e_k}} \quad (3)$$

Where $m_k \ge 0$ is the base price, d_k is the discount scale, e_k $(-1.00 \le e \le 1)$ is the steepness and q_k is the order quantity assigned to supplier k. This model is proposed by (Schotanus, Telgen et al. 2009) and is claimed to have flexibility to represent 66 practical discount schedules. To ensure that the purchasing price is decreasing with q_k, we assume $d_k e_k > 0$. Knowing the QDF and capacity C_k for each supplier; and the total order quantity from retailers Q; the order assignment problem can be modeled as the below optimization problem:

$$Min \sum_k w_k(q_k).q_k \tag{4}$$

s.t: (5)

$$\sum_k q_k = Q \tag{6}$$

$$q_k \le C_k \quad \forall k \tag{7}$$

Where the objective function is minimizing the total purchasing cost. The first constraint tries to satisfy all the demand and the second constraint is the capacity requirement for each supplier.

Proceeding formally, a solution method is provided below to first solve the retailers' and suppliers' problems separately, and then combine them to solve the GP problem. Figure 1 displays a schematic of the problem at a high level.

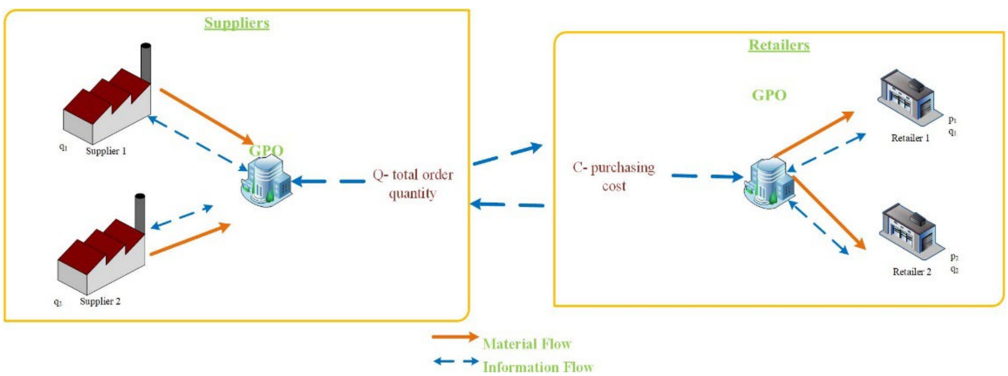

Figure 1. Two Retailer- Two Supplier Group Purchasing

4. Methodology

The approach developed to solve this problem is based on a method developed by Petruzzi and Dada (1999) for the newsvendor problem. The profit function displayed in the previous section can be simplified to:

$$E\big[\pi_i(z_i, p_i, p_j)\big] = \Psi_i(p_i, p_j) - L_i(z_i, p_i) \tag{8}$$

Where the first part is $\boldsymbol{\Psi_i(p_i,p_j) = (p_i - c)\big[y_i(p_i,p_j) + \mu\big]}$, which can be called risk-less profit function because it calculates the profit function in case the random variable hits its mean value; and the second part is $\boldsymbol{L_i(z_i,p_i) = (c - v_i)\Lambda(z_i) + (p_i + s_i - c)\Theta(z_i)}$. This term calculates the loss due to shortage and surplus; where $\boldsymbol{\Theta(z_i) = \int_{z_i}^{B}(u - z_i)f(u)du}$ and $\boldsymbol{\Lambda(z_i) = \int_{A}^{z_i}(z_i - u)f(u)du}$.

To maximize the expected profit, we need to first find the first and second order derivative with respect to z_i and p_i. Next, we need to find p_i^0 which is the optimal price for the riskless profit function Ψ_i, it can be found by solving below system of equations:

$$\begin{cases} \dfrac{\partial \Psi(p_i,p_j)}{\partial p_i} = 0 \\ \dfrac{\partial \Psi(p_j,p_i)}{\partial p_j} = 0 \end{cases} = \begin{cases} y_i(p_i,p_j) + \mu - (b_i + \gamma_i)(p_i - c) = 0 \\ y_j(p_j,p_i) + \mu - (b_j + \gamma_j)(p_j - c) = 0 \end{cases} \tag{9}$$

By solving the above system and substituting $t_i = a_i + (b_i + \gamma_i)c + \mu$, we will have the value of riskless profit for retailer 1 and 2 as:

$$p_1^0 = \frac{\gamma_1 t_2 + 2(b_2 + \gamma_2)t_1}{4(b_1 + \gamma_1)(b_2 + \gamma_2) - \gamma_1\gamma_2} \tag{10}$$

$$p_2^0 = \frac{\gamma_2 t_1 + 2(b_1 + \gamma_1)t_2}{4(b_1 + \gamma_1)(b_2 + \gamma_2) - \gamma_1\gamma_2} \tag{11}$$

By replacing these values in partial derivative of the expected profit function with respect to each p_i and solving it for p_i we can convert it to a function of z_i which provides the relationship between p_i and z_i at the optimal point. Then, we can calculate the partial derivative with respect to z_i for each retailer as:

$$\frac{\partial E[\pi_1(z_1, \mathrm{p}_1(\mathrm{z}_1), \mathrm{p}_2(\mathrm{z}_2))]}{\partial z_1} = -(c - v_1) + \left(1 - F(z_1)\right)\left(p_1^0 + s_1 - c - \frac{\Theta(z_1)}{2(b_1 + \gamma_1)}\right) \tag{12}$$

$$\frac{\partial E[\pi_2(z_2, \mathrm{p}_2(\mathrm{z}_2), \mathrm{p}_1(\mathrm{z}_1))]}{\partial z_2} = -(c - v_2) + \left(1 - F(z_2)\right)\left(p_2^0 + s_2 - c - \frac{\Theta(z_2)}{2(b_2 + \gamma_2)}\right) \tag{13}$$

Which is a function of only z_i; by finding the roots of this function for both retailers we can find the optimal profit function for each retailer. We can proof that z_i^* is the largest z in the range $[A, B]$ that satisfies $dE\left[\pi_\mathrm{i}\left(\mathrm{z_i}, \mathrm{p_i}(z_i), \mathrm{p_j}(\mathrm{z_j})\right)\right] \big/ d\mathrm{z_i} = 0$ which can be used to identify z^* for each retailer and thus p^* and q^*.

The problem of suppliers' order assignment is an optimization problem with k variables and k+1 parameters. This problem can be solved using a non-linear optimization method. Here we will approach the problem using Lagrange multipliers.

Edwards and Penney (2013) define the condition for Lagrangian method such that the gradients of the constraints should be:

(1) nonzero
(2) non-parallel
(3) equality

The constraints in this problem meet the first and second conditions, to meet the third condition, we introduce a slack variable r to the problem:

$$Min\ f(q_k) = \sum_k w_k(q_k).\, q_k \tag{14}$$

s.t:

$$h(q_k) = \sum_k q_k = Q \tag{15}$$

$$g_k(q_k, r_k) = q_k + r_k^2 = C_k \quad \forall k \tag{16}$$

The slack variable is added in squared form to make sure it is non-negative. The Lagrangian for this problem can be written as:

$$\mathcal{L}(q_k, r_k, \lambda, \mu_k) = f(q_k) + \lambda[h(q_k) - Q] + \sum_k \mu_k[g_k(q_k, r_k) - C_k] \tag{17}$$

Calculating the gradient of $\boldsymbol{\mathcal{L}}$ and setting it equal to zero will include all the equations we need to solve our problem. The gradient operates on four sets of variables $(\boldsymbol{q_k}, \boldsymbol{r_k}, \boldsymbol{\mu_k}, \boldsymbol{\lambda})$. Finding the partial derivatives will result in a system of equations, which can be solved to obtain the solution to the original optimization problem. In the next sections a problem consisting of two retailer sand to suppliers will be solved using a combination of the methods developed for the retailers' problem and suppliers' problem.

5. Results

In this section we are going to test the developed algorithm to solve a test case problem. The provided case is a two-retailer and two-supplier problem where the input to the retailers' and suppliers' problem are presented in Table 1 and Table 2:

Table 1. Input Parameters for Retailers

Retailer	s_i	v_i	a_i	b_i	γ_i
1	60	30	500	0.5	0.3
2	60	30	500	0.5	0.3

Table 2. Input Parameters for Suppliers

Supplier	m_k	d_k	e_k	c_k
1	65	-0.02	-1	1,000
2	75	-0.025	-1	1,000

To initiate the algorithm the starting price c_0 is set to 50 and $\epsilon = 10^{-3}$. At the convergence point the optimal results are as the following for retailers and suppliers in Table 3 and Table 4:

Table 3. Optimal Results for Retailers

Decisions	Retailer 1	Retailer 2
p_i	490.41	490.41
q_i	541.61	541.61
Profit	153,950	153,950

Table 4. Optimal Results for Suppliers

Decisions	Supplier 1	Supplier 2
c_k	45	72.92
q_k	1,000	83.22
Revenue	45,000	6,069

Based on the results, since the retailers are symmetric, they have similar decision parameters. But comparing the suppliers' price and order quantity we observe that Supplier 1 receives most of the orders, thus even though its unit price is much less, its profit level is higher compared to Supplier 2.

6. Summary
In this research a GB problem with two competing retailers and two suppliers was studied where the suppliers have a limited capacity and the retailer practice GB to save on purchasing cost. The problem was split into two sub-problem; retailers' problem and suppliers' problem. For each sub-problem an analytical approach was proposed to solve the problem and then the two solutions were used sequentially to solve the GB problem. Finally a test case was solved using the proposed solution to show the applicability of the problem. One can expand this research to bigger and more generic GB problems or do more experimental analysis to study the effect of GB and usefulness of the developed solution on different problems.

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Biographies
Abdollah Mohammadi is a Ph.D. candidate in Infrastructure and Environmental Systems (INES) program, his research is about Optimal Decision Making in Group Purchasing. He holds a M.Sc. degree in Industrial Engineering from University of Tehran where he worked on developing a multi-objective optimization model for a cross-dock scheduling problem. He is currently working as an Operations Research Analyst in Datos Technologies inc. where he is working on developing optimization models for applications in different industries such as transportation, manufacturing, and warehousing. Abdollah is a veteran member of Charlotte Dragonboat Association where he paddles in competitive races throughout U.S.

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