A MILP Model for Single Depot, Single Truck and Multiple Drones for Last-Mile Delivery Problem

Saloni Jangid
Department of Mathematics
Central University of Rajasthan
Ajmer 305817, India
salonijangid2002@gmail.com

Rupkatha Ghosh and M. Mathirajan
Department of Management Studies
Indian Institute of Science
Bangalore 560012, India
rupkathag@iisc.ac.in, msdmathi@iisc.ac.in

Abstract

The high volume of daily e-commerce shipments in major urban areas require today's technology of drones for last-mile delivery (LMD) of parcels. The traditional method of delivering parcels using trucks is time-consuming due to traffic congestion, impeding their timely delivery. On the other hand, drones can avoid traffic congestion by flying over road networks. However, drones have a limited flight endurance due to their battery capacity constraints. At the same time, trucks possess long-haul capabilities. Hence, for achieving an efficient LMD, both trucks and drones should be integrated to offset each other's disadvantages. This paper addresses one such truck-drone based last-mile delivery (TD-LMD) problem. There are plenty of studies which have investigated TD-LMD problem with different configurations. For our study, we examine the TD-LMD problems involving single depot, single truck and multiple drones (SD-ST-MD) configuration. In the literature, various researchers have used mathematical models and heuristics to address this problem. In our study, we consider one Mixed-Integer Linear Programming (MILP) model given in the existing literature for the TD-LMD problem with SD-ST-MD configuration and relax one of their assumptions to give it a new dimension. In the process of extending the MILP model, we also introduce the CO_2 emission cost along with the economic cost for optimizing the TD-LMD problem. Accordingly, the workability of the MILP model is tested using a tiny numerical example and solved through developing a code in the Python-MILP solver module. Further, the computational complexity is analyzed by increasing the number of nodes in the SD-ST-MD based TD-LMD problem.

Keywords

Last-Mile Delivery, Truck, Drones, Emission Cost, Mixed-Integer Linear Programming Model

1. Introduction

In a last-mile delivery (LMD) system, the determination of an effective delivery path plays a pivotal role (Chang and Lee 2018). The usage of drones in an LMD can increase the overall efficiency of the system. This is because, drones can lead to a significant reduction in labour costs. Additionally, drones are faster than traditional vehicles like trucks, hence are used for delivering parcels. Moreso, they can fly over road networks to avoid traffic problems without any human intervention (Tinic et al. 2023). Despite these advantages, the utilization of drones in an LMD comes with a few drawbacks. Due to the physical characteristics of drones, they are subject to certain limitations. Firstly, their flight endurance is constrained by their battery capacity since they solely rely on battery power. Additionally, most delivery drones have a limited payload capacity, in terms of weight and size, often allowing for the transportation of only a single parcel at a time. In contrast, traditional vehicles like trucks possess the ability to transport multiple parcels simultaneously without being concerned about the individual parcel weight and size. Additionally, trucks possess long range travel capabilities, enabling them to visit multiple customers before returning to the depot for efficient deliveries. This brings the need to integrate both the trucks and drones for faster deliveries (Wohlsen 2014).

For achieving an efficient LMD structure, both trucks and drones should be synchronized so that each one's disadvantage will be nullified by other's advantage (Vasquez et al. 2021). By employing a combination of delivery trucks and drones, the endurance limitation of drones can be mitigated (Agatz et al. 2018). This approach also enables drones to be launched from nearby locations, near the customers, thereby reducing the overall distance and extending the effective range of the drones. Thus, combining drones with trucks results in efficient, faster, and in-time delivery which leads to more customer satisfaction. With this premise, this study addresses the truck-drone based LMD problem (TD – LMD).

The remainder of this paper is organized as follows. In Section 2, we discuss a review of the existing literature on the TD-LMD problem. Section 3 discusses the problem description of the TD-LMD, considered in this study, briefly. In Section 4, we present a MILP model and the workability of the same is demonstrated in Section 5. Section 6 discusses the computational complexity of the proposed MILP model. Finally, we conclude the paper in the last section.

2. Literature Review

The problem of synchronizing drones with traditional delivery trucks was first introduced by Murray and Chu (2015). The authors studied two types of problems namely, the flying sidekick travelling salesman problem (FS-TSP) and the parallel drone scheduling TSP (PDS-TSP) that aimed to minimize the total travel time of the truck and the drone. For each problem, the authors proposed a MILP model and heuristics respectively. Along the same lines, Agatz et al. (2018) modified the original FS-TSP, as introduced by Murray and Chu (2015) and renamed it as "Traveling Salesman Problem with Drone" (TSP-D). This problem allowed the truck to wait at the launch location for the return of drones. The authors proposed an integer programming model with an objective of minimizing the completion time. They also proposed efficient heuristics based on local search and dynamic programming. Unlike the previous studies, Ha et al. (2018) formulated the TSP-D problem with the objective of minimizing the total operational cost of truck along with the waiting cost of drone. They provided a MILP model and two different heuristics. Vasquez et al. (2021) proposed a MILP formulation and an exact two-stage decomposition method for the TSP-D problem. The computational results on benchmark instances indicated that the optimal solution of instances could be found only up to 25 nodes. They further observed that the drone speed strongly affected the computational performance of the proposed method.

At the same time since 2018, various researchers started exploring the usage of multiple drones, integrating the TSP with multiple drones (MD). Kitjacharoenchai et al. (2020) addressed the TSP-MD problem with multiple trucks where every truck had a limited capacity and could carry multiple drones that could be launched to serve one or multiple customers. The authors proposed a MILP formulation to solve small instances and two heuristics to solve large-scale instances for their problem. Poikonen and Golden (2020) considered the Multi-visit Drone Routing Problem, where a truck and multiple drones were assigned to work in tandem. They proposed a constructive heuristic approach for their problem. Murray and Raj (2020) considered a last-mile delivery (LMD) system in which a delivery truck operated in coordination with a fleet of drones which aims to leverage the delivery truck and the fleet of drones to complete the delivery process and return to the depot in the minimum amount of time. Due to NP-hard nature of the MILP model, the authors proposed a heuristic solution approach that consisted of solving a sequence of three subproblems.

Moshref-Javadi et al. (2020) considered the setting where truck and drone operations are synchronized, with the objective of minimizing the waiting time of customers and proposed a MILP formulation which solved instances with up to 11 nodes to optimality. Additionally, to be able to solve real-world problem size instances, the authors proposed an efficient Truck and Drone Routing Algorithm (TDRA). Similarly, Cavani et al. (2021) provided a compact MILP formulation for the TSP-MD problem with the goal of minimizing completion time. They also proposed a decomposition based exact algorithm and claimed that their method could solve instances involving up to 24 customers to proven optimality. Salama and Srinivas (2022) introduced a new variant of the TSP-MD problem that allowed the truck to stop at non-customer locations for drone launch and recovery operations. The proposed variant accounted for three decisions namely, a) assignment of each customer location to a vehicle, (b) routing of truck and drones, and c) scheduling drone and truck operations at each stop. The authors solved a small numerical problem with only 8 customers using a MILP formulation with an objective to minimize the completion time.

Like Ha et al. (2018), Tinic et al. (2023) addressed the TSP-MD problem with the objective of minimizing the total operational cost including the vehicles' operating and waiting costs. The authors proposed flow based and two cut based MILP formulations strengthened with valid inequalities. Further, their analysis revealed that solutions obtained from the models with an objective of minimizing cost have much lower cost as compared to models addressing the minimization of completion time. For the first time in the literature, Meng et al. (2023) proposed an innovative dualobjective MILP model to explore the environmental and economic impacts of drone-assisted truck delivery under the carbon market price. The objective of their study was to minimize carbon emissions and total cost, including energy consumption, carbon emissions, and driver's wage. Their results indicated that drone-assisted delivery as compared to traditional truck delivery, reduced carbon emissions by 24.90%, total cost by 22.13%, and shortened delivery time by 20.65%.

The above briefly reviewed literature on the TD-LMD problem is summarized in Table 1. From Table 1, it is observed most of the studies address the TD-LMD problem with an objective of minimizing the time. There are very limited studies which have minimized the cost component. Moreover, there is only one study with an objective of minimizing both the economic and environmental cost simultaneously. This research gap is addressed in this study.

3. Problem Description

There are plenty of studies in the literature addressing the TD-LMD problem with various configurations. For our study, we have considered only single depot, single truck and multiple drones (SD-ST-MD) configuration. Moreover, most of the studies have minimized the total completion time. However, with the rising environmental awareness, most governments have mandated to practice sustainability measures in their supply chain (Meng et al. 2023). Hence, minimizing the cost from an economic and environmental perspective as compared to the total completion time of delivery, would be a better objective. Accordingly, let N be the total number of nodes available in the network of the TD-LMD problem. Out of these 'N' nodes, node 0 represents a single depot (SD). The remaining 'N-1' nodes represent the customer delivery points. We consider a single truck (ST) from which multiple drones (MD) can be launched for delivery of the parcels. The values of the additional parameters are: time required for ST to travel from node 'i' to node 'j' (t_{ij}), time required for a drone to travel from node 'i' to node 'j' (t_{ij}'), distance from node 'i' to node 'j' (d_{ij}), operating cost of truck (CoT), operating cost of drone (CoD), waiting cost of truck (CwT) and waiting cost of drone (CwD), speed of truck (vt) and speed of drone (v). With these given data, the economic cost for the truck (EC_t) and drones (EC_d) can be computed, as follows:

$$EC_t = (CoT \times \frac{d_{ij}}{vt} \times X_{ij}) + (CwT \times S_k) \text{ and,}$$
$$EC_d = \left(CoD \times (A_i^k t'_{ik} + B_i^k t'_{ki})\right) + (CwD \times U^k)$$

where, X_{ij} = 1, if ST travels from node 'i' to node 'j'
and S_k = time that ST waits at node 'k'
A_i^k = 1, if drone visits node 'k' from node 'i'
B_i^k = 1, if drone returns from node 'k' to node 'i'
U^k = time that the drone returning from node 'k' waits for the ST at meeting point

Similarly, for calculating the environmental cost of the truck (ENC_t) and environmental cost of the drone (ENC_d), the carbon price (Cp) is known. The CO_2 emission for the truck is computed using the formula defined by Ubeda et al. (2014) as follows:

$$ENC_t = Cp \times F(z) \times d_{ij} \times X_{ij} \text{ - where F(z) is emission factor.}$$

For the TD-LMD problem considered in our study, we assume the truck to be fully loaded with diesel used as fuel. Accordingly, F(z) = 1.018 as per the classification scheme of trucks defined in Ubeda et al. (2014). On the other hand, a drone is an electric vehicle with prima facie zero emissions. However, the energy consumed by the drone is produced by power generation facilities. Thus, we consider the CO_2 emitted by power generation facilities. Goodchild and Toy (2018) estimated that 0.3773 kg of CO_2 is emitted for each kWh produced. They also considered β as the Wh consumed by the drone per km. The value of β depends on the characteristics of the drone and is considered to be in the range (10,100). Substituting these values in the formula defined by Pugliese et al. (2020), the CO_2 emission for the drone is estimated as follows:

$$ENC_d = Cp \times \beta \times 0.3773(10^{-4}) \times \left(d_{ik} + d_{kj}\right)Y_{ij}^k$$

where, Y_{ij}^k = 1, if drone visits node 'k' while truck travels from 'i' to 'j'

Table 1. A summary on the existing literatures on the TD – LMD problem

Sl. No.	**Authors**	**Year**	**Problem Configuration**				**Objective is to Minimize**			**Methodology**	**Type of Model**	**Source of Data**
			SD	**ST**	**ND**		**Time**	**Cost**				
					SD	**MD**		**Eco.**	**Env.**			
1	Murray and Chu	2015	✓	✓	✓		✓			MM & HA	MILP; NHA, SaHA	ED
2	Agatz et al.	2018	✓	✓	✓		✓			MM & HA	ILP; R1C2HA	ED
3	Ha et al.	2018	✓	✓	✓			✓		MM & HA	MILP; GRASP	ED
4	Kitjacharoenc -hai et al.	2020	✓	✓		✓	✓			MM & HA	MILP; DTRC, LNS	ED
5	Poikonen and Golden	2020	✓	✓		✓	✓			HA	RTS	ED
6	Murray and Raj	2020	✓	✓		✓	✓			MM & HA	MILP; 3PHA	ED
7	Moshref-Javadi et al.	2020	✓	✓		✓	✓			MM & HA	MILP; GHA, SA	ED
8	Cavani et al.	2021	✓	✓		✓	✓			MM & HA	MILP; DA, BaC	ED
9	Vasquez et al.	2021	✓	✓	✓					MM & HA	MILP; DA	ED
10	Salama and Srinivas	2022	✓	✓		✓	✓			MM & HA	MILP; SA, VNS	ED
11	Tinic et al.	2023	✓	✓		✓		✓		MM & HA	MILP; BaC	ED
12	Meng et al.	2023	✓	✓	✓			✓	✓	MM	MILP	ED

Meaning of Abbreviations used: SD – Single Depot, ST – Single Truck, ND – Number of Drones, SD – Single Drone, MD – Multiple Drones, MM – Mathematical Modelling, HA – Heuristic Algorithm, NHA – Neighbour HA, SaHA – Savings HA, R1C2HA – Root 1 Cluster 2 HA, ILP – Integer Linear Programming, MILP – Mixed ILP, GRASP – Greedy Randomized Adaptive Search Procedure, DTRC – Drone Truck Root Construction, NS – Neighbourhood Search, LNS – Large NS, RTS – Root, Transform, Shortest Path, 3PHA – 3 Phase HA, GHA – Greedy HA, SA – Simulated Annealing, DA – Decomposition Algorithm, BaC – Branch and Cut, VNS – Variable NS ED – Experimental Design

Considering the above data, applicable for every node, the objective of the TD-LMD problem considered in this study is to deliver parcels to 'N-1' customer locations using an integrated ST-MD configuration by minimizing the total economic and the environmental costs for the truck and drones respectively.

4. Proposed MILP Model

In this study, the (0-1) mixed integer linear programming (0-1 MILP) model proposed by Tinic et al, (2023) is considered to model the problem described in section 3. Effectively, the (0-1) MILP model proposed in Tinic et al (2023) is extended by (a) relaxing one of the assumptions in Tinic et al. (2023): ***there is no limit on number of drones to be utilized*** and (b) introducing environmental cost along with economic cost while optimizing the cost for modelling the problem described in this study. Accordingly, the proposed MILP model for the problem described in section 3 is presented as follows:

Notations

Let $L_N = L \setminus N$ for any set $N \subset L$. We use L_i and $L_{i,j}$ for $N=\{i\}$ and $N=\{i,j\}$ respectively.

Parameters Used:

n Number of node/customer location except location 0 (depot)

t_{ij} Time required for truck to travel from $i \in L$ to $j \in L$

t_{ij}' Time required by drone to travel from $i \in L$ to $j \in L$

d_{ij} Distance from customer location $i \in L$ to customer location $j \in L$

CoT Operating cost of truck

CoD Operating cost of drone

CwT	Waiting cost of truck
CwD	Waiting cost of drone
Cp	Carbon price
F(z)	Emission factor in kg $C0_2$/km
E	Endurance value of drone
m	Maximum number of drones used in the whole delivery process
Ca	Amount of carbon emitted (in grams) for each kWh power

Decision Variables:

$X_{ij} \in \{0,1\}$	$X_{ij} = 1$ if the truck travels from $i \in L$ to $j \in L_i$.
$Y_{ij}^k \in \{0,1\}$	$Y_{ij}^k = 1$ if a drone visits $k \in L_0$ when the truck travels from $i \in L_k$ to $j \in L_{i,k}$
$A_i^k \in \{0, 1\}$	$A_i^k = 1$ if a drone is launched from $i \in L_k$ to visit $k \in L_0$.
$B_j^k \in \{0,1\}$	$B_j^k = 1$ if a drone returning from $k \in L_0$ is retrieved at $j \in L_k$.
$U^k \geq 0$	Time that the drone returning from $k \in L_0$ waits for the truck at their meeting point, if the drone arrives earlier.
$V^k \geq 0$	Time that the truck waits for the drone returning from $k \in L_0$ at their meeting point, if the truck arrives earlier.
$S_k \geq 0$	Time that the truck waits at $i \in L$.
$W_i^k \geq 0$	Time that the truck waits at $i \in L_k$ while the drone visiting $k \in L_0$ is airborne unless the drone is retrieved at i.
$T_{ij} \geq 0$	Total travel time by the truck up through $j \in L$ if the truck travels from $i \in L$ to $j \in L_i$.

Proposed 0-1 MILP model:

$$Min \sum_{i \epsilon L} \sum_{j \epsilon L_i} X_{ij} t_{ij} CoT + \sum_{k \epsilon L_0} S_k\, CwT + \sum_{i \epsilon L} \sum_{k \epsilon L_{0,i}} (A_i^k\, t_{ik}{}' + B_i^k t_{ki}{}') CoD + \sum_{k \epsilon L_0} U^k CwD$$
$$+ \left(Cp \times \sum_{i \epsilon L} \sum_{j \epsilon L} F(z) d_{ij}\, X_{ij} \right) + \left(Cp \times \beta \times Ca \times \sum_{i \epsilon V} \sum_{j \epsilon L} \sum_{k \epsilon L_0} (d_{ik} + d_{kj}) Y_{ij}^k \right)$$

subject to

$\sum_{i \epsilon L_j} (X_{ij} + A_i^j) = 1$ ……(1)

$\sum_{i \epsilon L_j} X_{ij} = \sum_{i \epsilon L_j} X_{ji}$ ……(2)

$\sum_{i \epsilon L_K} A_i^k = \sum_{i \epsilon L_K} B_i^k$ ……(3)

$\sum_{j \epsilon L_{i,k}} Y_{ij}^k - \sum_{j \epsilon L_{i,k}} Y_{ji}^k = A - B_i^k$ ……(4)

$\sum_{k \epsilon L_{0,i,j}} Y_{ij}^k \leq (n-2) X_{ij}$ ……(5)

$A_i^k \leq \sum_{j \epsilon L_i} X_{ij}$ ……(6)

$Y_{i0}^k \leq B_0^k$ ……(7)

$A_0^k + B_0^k \leq 1$ ……(8)

$\sum_{i\epsilon L_k} A_i^k t_{ik}' + \sum_{j\epsilon L_k} B_i^k t_{kj}' + U^k \leq E$ .... (9)

$\sum_{i\epsilon L_k} A_i^k t_{ik}' + \sum_{j\epsilon L_k} B_i^k t_{kj}' - \sum_{i\epsilon L} \sum_{j\epsilon L_k} Y_{ij}^k t_{ij} - \sum_{i\epsilon L_{0,k}} W_i^k \leq V^k$ ....(10)

$\sum_{i\epsilon L} \sum_{j\epsilon L_k} Y_{ij}^k t_{ij} + \sum_{i\epsilon L_{0,k}} W_i^k - \sum_{i\epsilon L_k} A_i^k t_{ik}' - \sum_{j\epsilon L_k} B_i^k t_{kj}' \leq U^k$ ....(11)

$V^k - E\left(1 - B_i^k\right) \leq S_i$ .... (12)

$S_i \leq E \sum_{k\epsilon L_{0,i}} B_i^k$ .... (13)

$S_i \leq T\left(1 - \sum_{j\epsilon L_k} Y_{ij}^k\right) \leq W_i^k$ ....(14)

$W_i^k \leq S_i$ .... (15)

$W_i^k \leq E\left(\sum_{j\epsilon L_k} Y_{ij}^k\right)$ ....(16)

$\sum_{i\epsilon L} \sum_{j\epsilon L_i} X_{ij} \geq 3$ .... (17)

$\sum_{j\epsilon L_i} T_{ij} - \sum_{j\epsilon L_i} T_{ji} - \sum_{j\epsilon L_i} t_{ij} X_{ij} = 0$ ....(18)

$T_{0i} = t_{0i} X_{0i}$ .... (19)

$T_{ij} \leq \left(M - t_{j0}\right) X_{ij}$ .... (20)

$T_{i0} \leq M X_{i0}$ .... (21)

$T_{ij} \leq \left(t_{0i} - t_{ij}\right) X_{ij}$ .... (22)

$\sum_{k\varepsilon L_{0,i}} \left(\sum_{j\epsilon L_{i,k}} \left(Y_{ij}^k + Y_{ji}^k \right) + A_i^k + B_i^k \right) \leq m$ .... (23)

$X_{ij}, A_i^j, B_i^j \in \{0,1\}$

$Y_{ij}^k \in \{0,1\}$

$V^i, S_i \geq 0$

$W_i^k \geq 0$

The objective function is to minimize the total economic cost consisting of both the operating and waiting costs of truck and drones as well as CO_2 emission costs of truck and drones respectively.

Constraint (1) limits the customer deliveries to be done either by a truck or by drone. Constraint (2) ensures that if a truck visits a customer location, it also departs from the same location. Similar to constraint (2), Constraint (3) ensures that if a drone visits a customer location, it will depart from that location itself. Constraint (4) ensures that the truck starts its journey from where a drone is launched and ends its journey at the spot where that launched drone needs to be collected. Constraint (5) ensures that if a drone is launched to another node while the truck travels along its designated path, then the path should be selected on the truck's tour. Constraint (6) permits a drone to be launched at a customer's location only if that specific location is part of the truck's tour. Constraint (7) ensures that the drones are required to return to the depot after their customer visits so as to meet the truck. Constraint (8) describes the assumption that the depot cannot serve as both launch and retrieval location of a drone mission. Constraint (9) shows the limited endurance of the drone. Constraints (10) and (11) are the waiting time constraints for both truck and drones.

Constraint (12) determines the time that the truck needs to wait at a location for the last arriving drone. Constraint (13) makes sure that the truck waits at a customer's location only for retrieving the drones. Constraints (14) and (15) together balance the truck's waiting time. Constraint (16) exclude the retrieval location of the drone from the total waiting time of the truck associated with the particular drone delivery. Constraint (17) defines the requirement of minimum three nodes for a truck route. Constraint (18) represent elimination of subtours while constraints (19), (20), (21), (22) are time bounding constraints as stated in Tinic et al. (2023), where M denotes a sufficiently large number. Finally, Constraint (23) endogenously imposes the restriction on the usage of maximum number of drones in the whole delivery process.
The rest of the constraints are non-negativity restrictions and binary restrictions on the variables.

5. Demonstrating Workability of the Proposed 0-1 MILP Model

Numerical Example: The numerical example in our study considers 11 nodes, with the first node as depot. This leads to 10 customer locations. For generating the location of the nodes, we use uniform instances in the range (0,100). Considering the first node as depot, we generate its coordinates uniformly in the range (0,1). The remaining coordinates of customer locations are chosen to be integers and generated in the range (0,100). Using these location coordinates, we find the Euclidean distance and round them off to the nearest integer. This distance remains the same for truck and drones. Further, using the obtained Euclidean distance and known speed of truck and drones, we determine the time required by both to move from one customer location to the other. Further, the value of drone's limited endurance E is restricted only to 50th percentiles i.e., median of all possible drone sorties.
Accordingly, the following parameters as defined earlier in Section 3 have been considered from various studies which are listed in Table 2.

Table 2. Values of the Parameters for the TD-LMD problem considered in our study.

Parameter	Values	Reference
N	11	*Assumed*
vt	0.6	Tinic et al. (2023)
v	1	
CoT	2	
CoD	1	
CwT	1.5	
CwD	1	
Cp	1	*Assumed*
F(z)	1.018	Ubeda et al. 2014
Ca	$3.773(10^{-4})$	Goodchild and Toy (2018)

The optimal solution yielded by the proposed MILP model is presented in Figure 1. The objective function value is 1328.9. From Figure 1, it is indicated that the model utilizes 2 drones to deliver parcels at customer locations 8 and 10. The first drone is launched from customer location 3 to visit customer location 8 and finally the drone is retrieved at customer location 5. Similarly, the second drone visits customer location 10 while the truck travels from customer location 5 to customer location 1 and the drone is retrieved at customer location 2 to meet the truck. The remaining 8 customer locations (excluding starting depot 0) are served by the truck. Further, the model ensures that the truck returns from the last customer location i.e., location 4 back to starting depot 0 after finishing all the parcel deliveries.

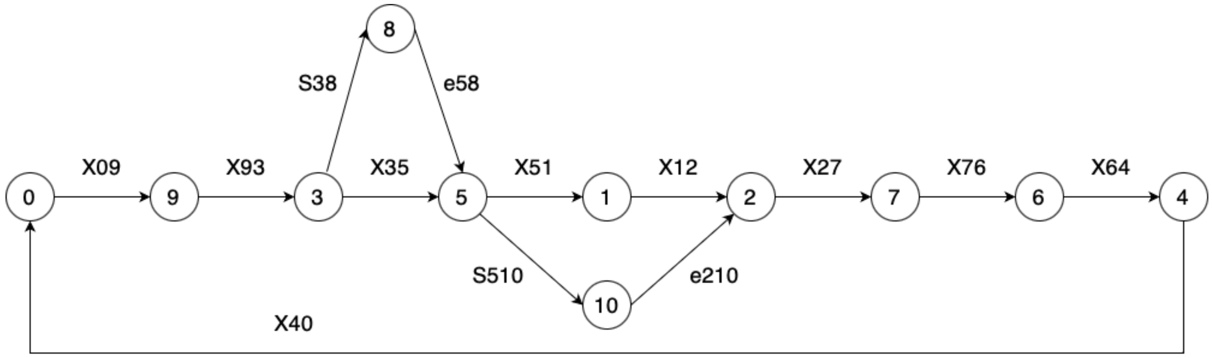

Figure 1. Optimal solution for 11 nodes in the TD-LMD network

6. Computational Complexity of the Proposed 0-1 MILP Model

The computational complexity of the proposed 0-1 MILP model is analyzed by increasing the number of nodes in the parcel-delivery network. Table 3 illustrates the CPU time (in seconds) taken on Mac OS system running at 3.5 GHz using 8 GB of RAM for obtaining the optimal solutions. For the last row having 14 customer locations in the parcel-delivery problem, the time exceeded 20 hours to get optimal solution after which we had to stop running the program. This shows the computational intractability of the 0-1 MILP model and further highlights the need to develop heuristic methods for tackling the TD-LMD problem involving more than 13 nodes.

Table 3. Computational Complexity of the extended MILP model

Number of Nodes	**Optimal Solution**	**Problem Configuration**			**Number of Drones Used**	**CPU Time (seconds)**
		Number of Constraints	**Number of Decision Variables**			
			Binary	**Continuous**		
4	1021.23	130	48	31	0	0.02
5	1176.86	212	100	49	0	0.9
6	1235.76	314	180	71	1	1.9
7	1291.86	436	294	97	2	2.4
8	1252.16	578	448	127	0	16.4
9	1365.49	740	648	161	4	33.9
10	1241.89	922	900	199	3	228.3
11	1328.91	1124	1210	241	2	334.9
12	1466.58	1346	1584	287	4	2205.3
13	1512.22	1588	2028	337	6	4651.5
14	**-**	**1850**	**2548**	**391**	**-**	**> 72904.8**

7. Conclusions

In this study, we considered a single depot, a single truck and multiple drones (SD-ST-MD) configuration for a truck-drone based last-mile delivery (TD-LMD) problem. Due to the increasing environmental concerns, we considered minimizing the cost from an economic and environmental perspective as compared to the total completion time of delivery of TD-LMD. With this, the existing Mixed-Integer Linear Programming (MILP) model in the literature is appropriately extended for the additional dimension considered in the problem configuration of TD-LMD problem. Then, the proposed MILP model was demonstrated by developing a suitable numerical example to reflect the problem configuration considered in this study. Further, the computational complexity of the proposed MILP model was analysed by increasing the number of delivery points. The computational complexity analysis clearly indicated that the simpler version of TD-LMD is computationally intractable to get an optimal solution. So, the TD-LMD problem requires an alternate method such as heuristic algorithm and this becomes the immediate future research issue for addressing TD-LMD problem considered in this study.

References

Agatz, N., Bouman, P., and Schmidt, M., Optimization Approaches for the Traveling Salesman Problem with Drone. *Transportation Science*, vol. 52, no. 4, pp. 965–981, 2018.

Cavani, S., Iori, M., and Roberti, R., Exact Methods for the Traveling Salesman Problem with Multiple Drones. *Transportation Research Part C: Emerging Technologies*, vol. 130, pp. 103280, 2021.

Chang, Y. S., and Lee, H. J., Optimal Delivery Routing with Wider Drone-Delivery Areas along a Shorter Truck-Route. Expert Systems with Applications, vol. 104, pp. 307–317, 2018.

Goodchild, A. and Toy, J., Delivery by drone: An evaluation of unmanned aerial vehicle technology in reducing CO_2 emissions in the delivery service industry. *Transportation Research Part D: Transport and Environment,* vol. 61, pp. 58–67, 2018.

Ha, Q. M., Deville, Y., Pham, Q. D., and Hà, M. H., On the Min-Cost Traveling Salesman Problem with Drone. *Transportation Research Part C: Emerging Technologies*, vol. 86, pp. 597–621, 2018.

Kitjacharoenchai, P., Min, B. C., and Lee, S., Two Echelon Vehicle Routing Problem with Drones in Last Mile Delivery. *International Journal of Production Economics*, vol. 225, pp. 107598, 2020.

Meng, Zhiyi, et al. Environmental and Economic Impacts of Drone-Assisted Truck Delivery under the Carbon Market Price. *Journal of Cleaner Production*, vol. 401, pp. 136758, 2023.

Moshref-Javadi, M., Hemmati, A., and Winkenbach, M., A Truck and Drones Model for Last-Mile Delivery: A Mathematical Model and Heuristic Approach. *Applied Mathematical Modelling*, vol. 80, pp. 290–318, 2020.

Murray, C. C., and Chu A. G., The Flying Sidekick Traveling Salesman Problem: Optimization of Drone-Assisted Parcel Delivery. *Transportation Research Part C: Emerging Technologies*, vol. 54, pp. 86–109, 2015.

Murray, C. C., and Raj, R., The Multiple Flying Sidekicks Traveling Salesman Problem: Parcel Delivery with Multiple Drones. *Transportation Research Part C: Emerging Technologies*, vol. 110, pp. 368–98, 2020.

Poikonen, S., and Golden, B., Multi-Visit Drone Routing Problem. *Computers & Operations Research*, vol. 113, pp. 104802, 2020.

Pugliese, L. D. P., Guerriero, F., and Macrina, G., Using drones for parcels delivery process, *Procedia Manufacturing*, vol. 42, pp. 488-497, 2020.

Salama, M. R., and Srinivas, S., Collaborative Truck Multi-Drone Routing and Scheduling Problem: Package Delivery with Flexible Launch and Recovery Sites. *Transportation Research Part E: Logistics and Transportation Review*, vol. 164, pp. 102788, 2022.

Tiniç, G. O., Karasan, O. E., Kara, B. Y., Campbell, J. F., and Ozel, A., Exact Solution Approaches for the Minimum Total Cost Traveling Salesman Problem with Multiple Drones. *Transportation Research Part B: Methodological*, vol. 168, pp. 81–123, 2023.

Ubeda, S., Faulin, J., Serrano, A., Arcelus, F.J.. Solving the green capacitated vehicle routing problem using a tabu search algorithm*. Lecture Notes in Management Science,* vol. 6, pp. 141–149, 2014.

Vásquez, S. A., Angulo, G., and Klapp, M. A., An Exact Solution Method for the TSP with Drone Based on Decomposition. *Computers & Operations Research*, vol. 127, pp. 105127, 2021.

Wohlsen, M., The next big thing you missed: Amazon's delivery drones could work—they just need trucks, 2014.

Acknowledgement

The first author thanks the Operational Research Society of India – Karnataka (ORSI-KA), who selected her for Prof B G Raghavendra's Internship Paid Award, for the opportunity of developing this research paper.

Biographies

Saloni Jangid is currently studying in an integrated MSc programme at the Department of Mathematics in Central University of Rajasthan, Ajmer, India. Her areas of research interest include mathematical optimization for supply chain management, and last-mile logistics.

Rupkatha Ghosh is currently a Research Scholar at the Department of Management Studies in the Indian Institute of Science (IISc), Bengaluru, India, pursuing her research in Urban Road Transportation Organizations. She has a bachelor's degree (B.Tech.) in Civil Engineering from the National Institute of Technology (NIT), Durgapur, India. She has 1.5 years of experience as a senior civil engineer at Larsen and Toubro (L&T) Construction. Her areas of research interest include operations management, heuristic optimization, and urban road transport.

Dr. M. Mathirajan obtained his Ph.D. in Operations Management and MS degree by research in Applied Operations Research (OR) from IISc, Bangalore. He also received his MSc in Mathematics from Madurai Kamaraj University and Postgraduate Diploma in OR from College of Engineering, Guindy, Anna University. He has been working as faculty of IISc Bangalore since 1986 and has been a Chief Research Scientist / Professor since 2013. He is a Fellow of the Operational Research Society of India (FORSI) and currently he is the national president of ORSI. His areas of research interest include mathematical/heuristic optimization and research methods for operations and supply chain management, sequencing and scheduling, personnel scheduling, routing and scheduling of logistics, urban road transport, and container terminal logistics problems.