Minimizing Job Tardiness in One-Dimensional Cutting Stock Problem Using Mixed Integer Linear Programming (MILP)

Sokunthea Sar
School of Manufacturing Systems and Mechanical Engineering
Sirindhorn International Institute of Technology
Thammasat University
Pathum Thani, 12120, Thailand
m6522040028@g.siit.tu.ac.th

Pham Duc Tai
School of Manufacturing Systems and Mechanical Engineering
Sirindhorn International Institute of Technology
Thammasat University
Pathum Thani, 12120, Thailand
tai.pham@siit.tu.ac.th

Jirachai Buddhakulsomsiri
School of Manufacturing Systems and Mechanical Engineering
Sirindhorn International Institute of Technology
Thammasat University
Pathum Thani, 12120, Thailand
jirachai@siit.tu.ac.th

Abstract

The one-dimensional cutting stock problem is a fundamental problem in manufacturing that involves the preparation of raw materials for production. An example of a cutting stock problem in the industry is steel pipe manufacturing, where the raw material preparation operation requires cutting steel sheets from stock rolls based on job specifications. Steel manufacturers usually perform job planning and scheduling on the cutting process to satisfy the production requirement, i.e. job due date, from subsequent operation. The key performance measure is job tardiness. In this paper, we propose a mixed integer linear programming model to solve the one-dimensional cutting stock problem so as to minimize total job tardiness. A numerical example is presented to demonstrate the effectiveness of the model in performing job scheduling to meet the due dates.

Keywords
One-dimensional cutting stock problem, Mixed integer linear programming, Due dates, and Job tardiness.

1. Introduction
The steel pipe manufacturing industry is a major industry in Thailand. Pipe products are in different shapes, sizes, lengths, and thicknesses. They are produced by a two-stage manufacturing process. In the first stage known as coil-cutting, a steel coil with a thickness is cut into coils with smaller widths. After that, these small coils are fed to a machine to produce steel pipes in the second stage. This stage is referred to as pipe-forming. To cope with the increasing demand for steel pipes in terms of the number of orders and variety of pipes in each order, manufacturers are trying to maximize the efficiency of their production process by synchronizing the coil-cutting and pipe-forming stages. The synchronization is achieved by generating a cutting plan such that coils with the right widths and

thicknesses are produced on time for further processing by the pipe-forming stage. However, developing such a plan is a challenging problem as many constraints, such as the required specification in terms of shapes, sizes, thickness, and lengths of pipes in order, the due dates of the order, the production schedule of the pipe-forming stage, as well as the capability of cutting machinery, must be considered. As a result, the main objective of this paper is to formulate a mathematical model that solves the steel coil-cutting problem in the steel pipe manufacturing industry. Specifically, it is formulated as a mixed integer programming model based on the one-dimensional cutting problem. Instead of minimizing the leftover stock, the model in our paper aims at minimizing the total tardiness of the small steel coils. The tardiness of a small steel coil is defined as the difference between when it is needed and when it is ready for further processing in the pipe-forming stage. To accommodate the minimization of tardiness in our model, additional constraints are included in the mathematical model to determine the completion time and the lateness of the small steel coils. The performance of our formulated model is investigated by building it in a spreadsheet, which can be solved to optimal by using the Open Solver add-in and testing it on a series of benchmark problem instances. The results indicate that our model can generate a cutting plan that meets the production schedule of the pipe-forming stage, which, in turn, satisfies the due date of customer orders.

2. Literature Review

The cutting stock problem (CSP) poses significant scientific challenges and is commonly encountered in various practical industrial applications (Oliveira et al. 2021), including automobile (de Lara Andrade et al. 2021), paper (Pierini and Poldi 2023), glass (Parreño and Alvarez-Valdes 2021), and marble (Baykasoğlu and Özbel 2021) manufacturing. The problem refers to cutting an inventory of stock materials into smaller items based on a pattern to fulfill customer orders or for the next stage of processing. CSP, especially the one-dimensional problem, is studied by many researchers, including (de Lara Andrade et al. 2021; Muter and Sezer 2018; Pierini and Poldi 2021; Silva et al. 2023). Muter and Sezer (2018) propose an algorithm to solve a two-stage one-dimensional CSP that the total number of stock rolls to be cut. de Lara Andrade et al. (2021) aim to reduce inventory costs and material losses in the production of automotive springs by integrating a one-dimensional CSP with a lot-sizing model. Pierini and Poldi (2021) develop a model that simultaneously solves the capacitated lot sizing and one-dimensional CSP in the context of multiple paper production facilities such that the total cost, including setup, inventory, and production is minimized. Despite its popularity, studies of one-dimensional CSP usually focus on minimizing scrap and material costs when specifying cutting patterns. This may not be adequate when meeting scheduling and due date restrictions or delivering orders on time is of greater importance (Reinertsen and Vossen 2010). As a result, incorporating job tardiness as an objective function in the one-dimensional CSP is a research gap that remains to be filled. To address this gap in the research literature, this paper formulates a mixed-integer linear programming (MILP) model for the one-dimensional CSP such that total job tardiness is minimized. The performance of this model is illustrated by a numerical experiment based on data from a steel pipe manufacturer.

3. Problem Statement and Mathematical Model Formulation

The one-dimensional cutting stock problem with job tardiness minimization involves determining the optimal cutting patterns for a given set of raw materials, i.e. stock rolls of steel sheet while satisfying the demand requirements. The cutting patterns can be derived from the optimal assignment of jobs to stock rolls. The objective is to minimize the total job tardiness. The problem can be formulated as a mixed integer linear programming (MILP) model, which uses the following notation.

Indices

- j Index of job, where N denotes the set of jobs, $j \in N = \{1, 2, \dots, n\}$,
- i Index of sub-job, where M denotes the set of sub-jobs, $i \in M = \{1, 2, \dots, m\}$, and $m \geq n$. That is, each job may be separated into multiple sub-jobs, which are a subset of M. The pairs of jobs and sub-jobs are given using a set A, where $(j, i) \in A$, and
- k, l Indices of stock rolls, where R denotes the set of stock rolls, $k, l \in R = \{1,2, \dots, r\}$, and R_0 denotes the set of stock rolls, including a dummy roll 0, $R_0 = \{0\} \cup R$.

Parameters

- P_l Processing (cutting) time of stock roll l (minute),
- S Required setup time of the machine (minute) to adjust the cutter positions and load the next stock roll,
- WS_l Width of stock roll l (mm),
- TS_l Thickness of stock roll l (mm),

WJ_j Width of job j (mm),
TJ_j Thickness of job j (mm),
D_j Due date of job j (minute),
D_i Due date of sub-job i (minute), and
V A large value.

Decision Variables

$X_{i,l}$ A binary variable that takes on a value of 1, if stock roll l is used in the cutting plan for sub-job i, or 0 otherwise.
$Y_{k,l}$ A binary variable that takes on a value of 1, if stock roll l succeeds stock roll k, or 0 otherwise.
L_l Width of the leftover of stock roll l (mm)
C_l Completion time of stock roll l (min)
C_k Completion time of previous stock roll k (min)
B_j Completion time of job j (min)
B_i Completion time of sub-job i (min)
T_j Tardiness of job j (min)
E_j Earliness of job j (min)

Objective Function

$$\text{Minimize} \quad \sum_{j \in N} T_j \tag{1}$$

Subject to:

$$\sum_{l \in R} X_{i,l} TS_l = TJ_j \qquad \forall (j,i) \in A \tag{2}$$

$$\sum_{l \in R} X_{i,l} = 1 \qquad \forall i \in M \tag{3}$$

$$\sum_{l \in R} Y_{0,l} = 1 \qquad \forall i \in M \tag{4}$$

$$\sum_{k \in R_0, k \neq l} Y_{k,l} = 1 \qquad \forall l \in R \tag{5}$$

$$\sum_{l \in R_0, l \neq k} Y_{k,l} = 1 \qquad \forall k \in R \tag{6}$$

$$\sum_{h \in R_0, h \neq k} Y_{h,k} - \sum_{l \in R_0, l \neq k} Y_{k,l} = 0 \qquad \forall k \in R \tag{7}$$

$$\sum_{i \in M} X_{i,l} WJ_i \leq WS_l \qquad \forall l \in R \tag{8}$$

$$C_l - C_k + V\left(1 - Y_{k,l}\right) \geq S + P_l \qquad \forall k \in R_0, \forall l \in R: k \neq l \tag{9}$$

$$V\left(1 - X_{i,l}\right) \geq C_l - B_i \qquad \forall i \in M, \forall l \in R \tag{10}$$

$$B_j - D_j = T_j - E_j \qquad \forall j \in N \tag{11}$$

$$B_i \leq B_j \qquad \forall i \in M, \forall j \in N \tag{12}$$

$$C_0 = 0 \tag{13}$$

$$X_{i,l} \in \{0,1\} \qquad \forall l \in R \tag{14}$$

$$Y_{k,l} \in \{0,1\} \qquad \forall k \in R \tag{15}$$

$$T_j, E_j, B_j, B_i, C_l \geq 0 \qquad \forall j \in N, \forall i \in M, \forall l \in R \tag{16}$$

In the model, the objective function (1) is to minimize the total job tardiness, which is the total amount of time by which the completion times of all jobs exceed their respective due dates. Constraints (2) enforce that sub-job i, which is tied to job j, and stock roll l have the same thickness. Constraints (3) force that each sub-job i can be assigned only once on stock roll l. Constraints (4) indicate that one of the stock rolls is the first stock roll to be cut, and it must succeed the dummy stock roll $k = 0$. Constraints (5) and (6) specify that each stock roll has only one preceding stock roll and exactly one succeeding stock roll. Constraints (7) represent the flow balance of each stock roll on the slitting line machine. Constraints (8) limit the total widths of all sub-jobs assigned to a stock roll to be within the stock roll width.–Constraints (9) to (12) compute the completion time of stock rolls and sub-jobs, job tardiness, and job completion time, respectively. Constraint (13) specifies the completion of the dummy roll. Finally, Constraints (14) to (16) define the types of decision variables.

4. Numerical Data

In this numerical data, there are four stock rolls that are assigned to process 10 jobs. These jobs are further divided into 15 sub-jobs. This is due to the lengths of jobs 1, 2, 3, 7, and 8, which exceed the length of a stock roll. As a result, those jobs need to be cut more than once, either on two separate stock rolls or in parallel on the same stock roll. To accommodate each of those jobs, the original job is split into two sub-jobs that can be combined to make up the original job. The sub-jobs have the same thickness, width, and due date as their original jobs, whereas the length of the sub-jobs must add up to the length of their original job. The due date, processing time of each stock roll, job and sub-jobs are presented in the tables below:

Table 1. Jobs and sub-jobs specification

Jobs	Sub-jobs	Length (m)	Width (mm)	Thickness (mm)	Due Date (min)
1	1	900	152	2	26
1	2	100	152	2	26
2	3	900	152	2	51
2	4	300	152	2	51
3	5	600	200	3	42
3	6	50	200	3	42
4	7	540	200	3	20
5	8	900	150	2	9
6	9	500	190	3	37
7	10	900	150	2	30
7	11	350	150	2	30
8	12	600	190	3	8
8	13	160	190	3	8
9	14	420	200	3	58
10	15	370	150	2	35

The splitting of job 1 into sub-jobs is illustrated in the figure 1.

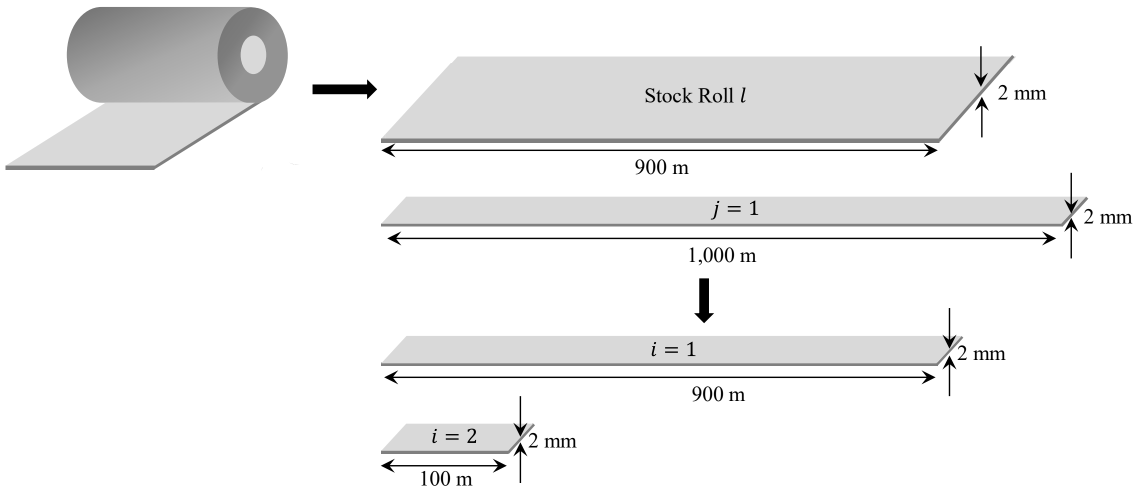

Figure 1. The splitting of job 1 into sub-jobs

There is one slitting line machine in the cutting process. Four stock rolls are available, two of them with a thickness of 2 millimeters and a length of 900 meters, and the other two with the thickness of 3 millimeters and a length of 600 meters. The stock rolls have the same width of 1,000 millimeters. Moreover, all stock rolls require the same setup time of 15 minutes for loading stock roll onto the slitting line machine and adjusting blade positions to the width of each sub-jobs that are assigned to the stock roll

Table 2. Stock roll specification

Specification	Stock roll 1	Stock roll 2	Stock roll 3	Stock roll 4
Thickness (mm)	2	2	3	3
Length (m)	900	900	600	600
Width (mm)	1,000	1,000	1,000	1,000
Processing time (min)	9	9	9	9
Setup time (min)	15	15	15	15

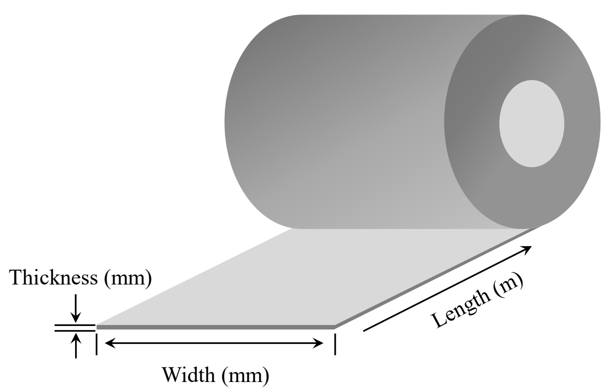

Figure 2. Stock roll's identifications

. The processing time of each stock roll depends on their length and the speed of the machine on the different stock roll thickness. The operating speed of machine is 100 meters per minute for the stock rolls with 2 mm thickness and is 70 meters per minute for the stock rolls with 3 mm thickness. Base on the length and thickness of the four stock rolls above, they contain the same processing time that is 9 minutes per roll.

5. Result and Discussions

The numerical example is used to demonstrate the effectiveness of our proposed MILP model in meeting the job due date requirements. The optimal solution that specifies the sub-jobs that are assigned to each stock roll is shown in Table 3 below:

Table 3. Stock roll sequence, sub-jobs assignment, and job tardiness

Stock rolls	Sub-jobs	Completion time (min)	Jobs	Completion time (min)	Due Date (min)	Tardiness (min)
1	1	24	1	24	26	0
	2	24				
	8	24	5	24	9	15
	10	24	7	24	30	0
	11	24				
	15	24	10	24	35	0
3	5	48	3	48	42	6
	6	48				
	7	48	4	48	20	28
	9	48	6	48	37	11
	14	48	9	48	58	0
2	3	72	2	72	51	21
	4	72				
4	12	96	8	96	8	88
	13	96				

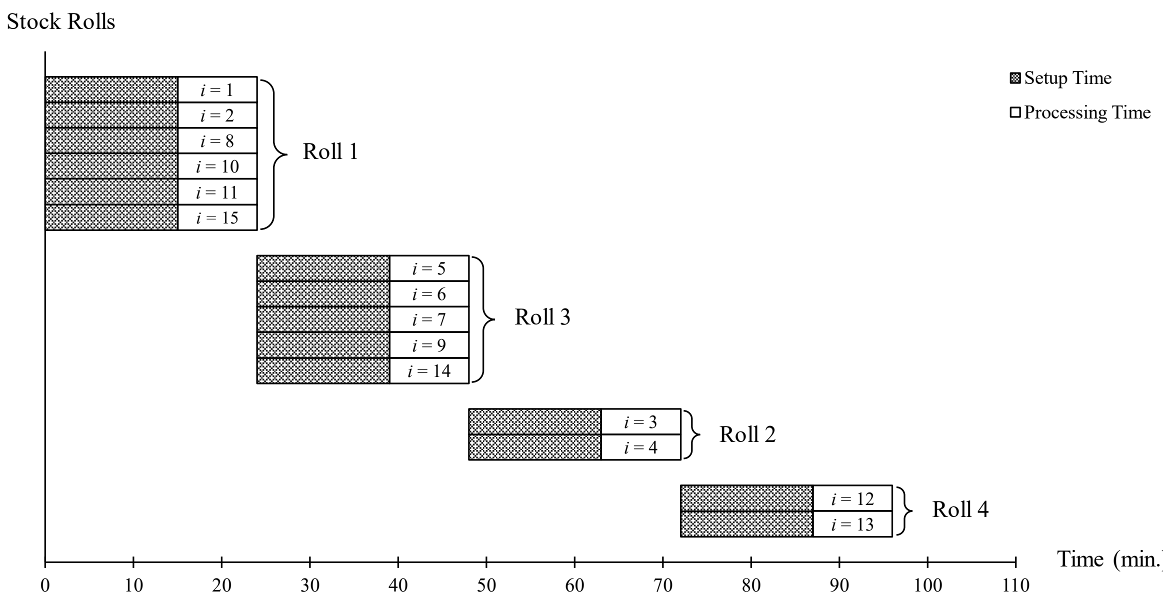

Figure 3. Stock rolls' sequence and the completion time with the assigned sub-jobs

In the cutting plan, the four stock rolls are cut in the sequence, 1 → 3 → 2 → 4, which yield the completion times that results in the minimum total tardiness for the original 10 jobs. Specifically, sub-jobs 1, 2, 8, 10, 11, and 15 are assigned to stock roll 1. After cutting, the original jobs 1, 5, 7, and 10 are completed. Sub-jobs 5, 6, 7, 9, and 14 are assigned to stock roll 3, which complete the original jobs 3, 4, 6, and 9. Sub-jobs 3 and 4, which make up the original job 2, are assigned to stock roll 2, and sub-jobs 12 and 13, which complete the original job 8, are assigned to stock roll 4, respectively. According to the solution, there are six jobs that are tardy, and the total tardiness of all jobs is 169 minutes. The specific jobs that are tardy and their corresponding tardiness values are shown in the figure.

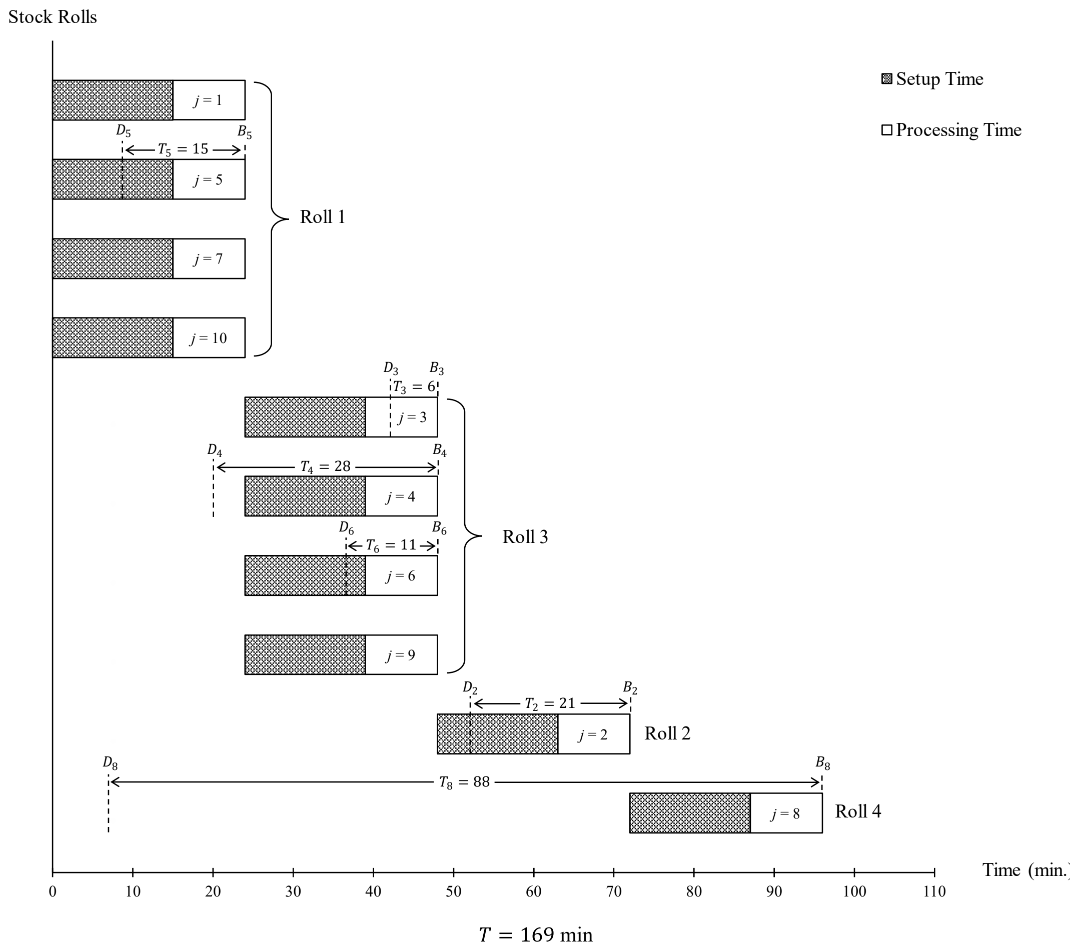

Figure 4. Job tardiness

6. Conclusion and Future Work

In this paper, a mixed integer linear programming (MILP) model is developed for the coil-cutting production stage. The model is a variant of one-dimensional CSP, which aims at minimizing total job tardiness. The effectiveness of our proposed model is demonstrated by a numerical experiment. The results indicate that our model can generate a cutting plan with minimal tardiness. This implies an opportunity to further extend this model. For instance, our model can be integrated with the scheduling model for pipe-forming production stage to be a unified planning model for steel pipe production industries. Another possible extension of our proposed model is to develop algorithms that generate cutting plans for large problem instances, involving dozens or hundreds of steel pipe orders.

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Biographies

Sokunthea Sar is a master’s degree student in Logistics and Supply Chain Systems Engineering in the international program at Sirindhorn International Institute of Technology, Thammasat University in Thailand. She earned an bachelor’s degree in Logistics and Cross-Border Trade Management from Burapha University, Thailand.

Pham Duc Tai is a lecturer at Sirindhorn International Institute of Technology (SIIT), Thammasat University, Pathum Thani, Thailand. Formerly, he was a lecturer at the International Academy of Aviation Industry, King Mongkut's Institute of Technology Ladkrabang, Bangkok, Thailand. He received a bachelor's degree in Industrial Systems Engineering from Ho Chi Minh City University of Technology, Vietnam, a master's degree in Logistics and Supply Chain Systems Engineering, and a doctoral degree in Engineering and Technology from SIIT, Thammasat University, Thailand. His research interests include those related to supply chain management, such as supply chain network design (SCND), supply chain optimization, retail management, inventory management, supply contract, marketing-operations interface, bullwhip effect, and information sharing.

Jirachai Buddhakulsomsiri is an Associate Professor in the School of Manufacturing Systems and Mechanical Engineering, Sirindhorn International Institute of Technology (SIIT), Thammasat University, Thailand. He received his Ph.D. in Industrial Engineering from the Department of Industrial and Manufacturing Engineering, Oregon State University, USA, in 2003. His research interests include (1) logistics and supply chain systems in the problem areas of supply chain network design, vehicle routing problems, inventory optimization, (2) data analysis using machine learning, computer visions, statistical analysis, and (3) system modeling using discrete-event system simulation, Monte Carlo simulation, system dynamics.