Home Health Care Routing and Scheduling Problem for Newborns and Mothers: A Case Study in Ho Chi Minh City

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Abstract

The Home Health Care Routing and Scheduling Problem (HHCRSP) has become one of the major consideration for many Home Health Care (HHC) service providers who deliver home and personal care services directly to customers' homes. Satisfying customers' needs and caregivers' well-being requires and saving a comprehensive approach that balances the quality of care provided to patients with the well-being and job satisfaction of caregivers.
This study has developed a mixed-integer programming model with the aim of minimizing travel distances while simultaneously achieving a balanced workload, meeting clients' requirements, and accommodating caregivers' lunch breaks. For small-scale problems, this study employed CPLEX software to solve to find the optimal solutions. For large-scale problems, these problems were divided into smaller problems by using (Agglomerative Hierarchical Clustering) AHC and K-meaning Algorithms. The obtained results indicate that the combination of the AHC algorithm and CPLEX yields superior solutions compared to the utilization of the K-means algorithm and CPLEX via testing randomly generated datasets. The proposed approach is put to the test in a real-world case study conducted in Ho Chi Minh City.

Keywords
Home Health Care Services, Routing and Scheduling Problem, Clustering, MILP, Newborns and Mothers.

1. Introduction
One of the fastest-growing sectors of the health care industry today, which is Home Health Care (HHC) service is studied. First of all, HHC is the delivery of social, medical and paramedical services to clients directly to their own home following their request. Every day, caregivers from a HHC center start their route at the center, receiving the list of the clients they need to service, visit them to provide the services and then return to the center. There are many

diverse types of home health care services available which depend on the client's needs, including: medical services, services of helping daily living, other kinds of housekeeping, etc. To receive home health care services, patients typically need to meet certain requirements such as the patient's medical condition, their available time and their need for skilled medical care.

The demand for home health care services has been increasing in recent years due to several factors: the rapid rise in the ageing population, life expectancy and the percentage of the working class. People nowadays prefer convenience more than in the past, so they want to receive care at their own home for time saving and comforting rather than going to residential, aged-care center. Furthermore, the COVID-19 pandemic has highlighted the importance of home health care services. Many individuals are hesitant to receive medical care in a hospital or clinic setting due to the risk of exposure to the virus. Home health care services can provide a safe and effective alternative for patients who need medical care but want to avoid exposure to COVID-19.

Despite benefits and potential developments in HHC, there are still many challenges in this service that need to be solved. The problems involve the allocation of resources, such as caregivers and medical equipment, to ensure that patients receive the care they need in a timely and efficient manner. However, this is a complex problem due to the diverse needs of patients, the varying levels of care required, the limited availability of resources, and the travel time and distance. Failure to address this problem can result in delays in care, increased costs, and decreased patient satisfaction. In order to provide high-quality care to patients, home health care providers must develop effective routing and scheduling strategies that balance the needs of patients with the resources available to the organization. Addressing this problem is crucial to improving the quality of care for patients, reducing healthcare costs, and increasing the efficiency of healthcare delivery. For that reason, this paper will explore the Home Health Care Routing and Scheduling Problem (HHCRSP) in more detail for further developments.

Ho Chi Minh City – one of the most developing cites in Viet Nam, become a potential place for the development of the health care industry, especially for the HHC services. Until now, there are many health care centers that have already delivered home care services for clients due to their busy life, saving time and the need of convenience. Those centers not only provide nursing care, personal care but also provide home care such as housekeeping, house cleaning, cooking, babysitters, etc.

Among those kind of services, the HHC services which deliver the care for newborn and the mother during and after pregnancy has become one of the most popular ones. It mainly contains the job of home cleaning for pregnant woman, baby bathing, cooking, massage both the mother and the infant, etc. Because of that, in this thesis, the problem of a HHC services provider for infant and woman during and after pregnant in Ho Chi Minh City called Baby & Mother Home Care Spa is studied. This company divided their services into to group: the first group called homecare (including: home cleaning and cooking when the mother cannot do the job while pregnant or right after gave birth) and personal care (including: bathing for baby, body massage for baby, body massage for pregnant woman, recovery care right after gave birth, clogged milk duct treatment).

During the services operation of this HHC center, the question of how to develop an efficient and effective system is quite challenging for them. Since the company seeks to respect the clients' time window but the travel time or/and service times always happen stochastic. In addition, on some occasions, a large number of clients have been approved to receive different types of care packages, but the number of caregivers is limited, causing the delay in delivering services and client dissatisfaction. Recently, there is no model or approach available to handle this specific type of home care. Most of the researchers are just considering the general cases of HHC services or mostly nursing services at home. For newborn and mother, just theoretical research such as the factors that influence this type of care, the survey for the need of care, but not considering the technique and method of routing and planning caregivers to customers. As a result, the service provider has to do it manually, which is very time consuming and ineffective when dealing with a large number of clients. Furthermore, due to human error, most of the time the company cannot find the optimal route. Sometimes, they assigned clients live very far from each other together may make caregivers cannot deliver the required services in the right time windows. Furthermore, in practice, caregivers have to work 8-9 hours per day, so it is important for them to have a short break. This break time sometimes cannot happen because the caregivers have a lot of predefined clients to serve per day, they did not have enough time to afford the break.

Therefore, there is a need to find a new approach for solving the problem in HHC center. A mathematical model is developed to address the problem of how to define an optimal route for large number of client with different preferences, reducing the traveling cost for all caregivers as much as possible to ensure clients are served in their time windows but caregivers can also have time for lunch break. In addition, the model also minimizes the number of used caregivers so that the company can save the cost paid for them in each route. The solution determines which caregivers serve which client and at what time as well as what time during the day should caregivers take a short lunch break.

For further understanding, Fig. 1 below presents an example of HHCRSP with three caregivers (two with personal care services and one with homecare services) and nine clients. There are different services' requirements of clients for caregivers in Fig. 1. It is clear that caregiver number 1 only visits clients who offer home care services, and she is not willing to take on any task related to personal care. On the other hand, caregiver number 2 and 3 are responsible for taking care of the ones who offer personal care services following the predefined plan shown in the figure, resulting in the minimum travel distance.

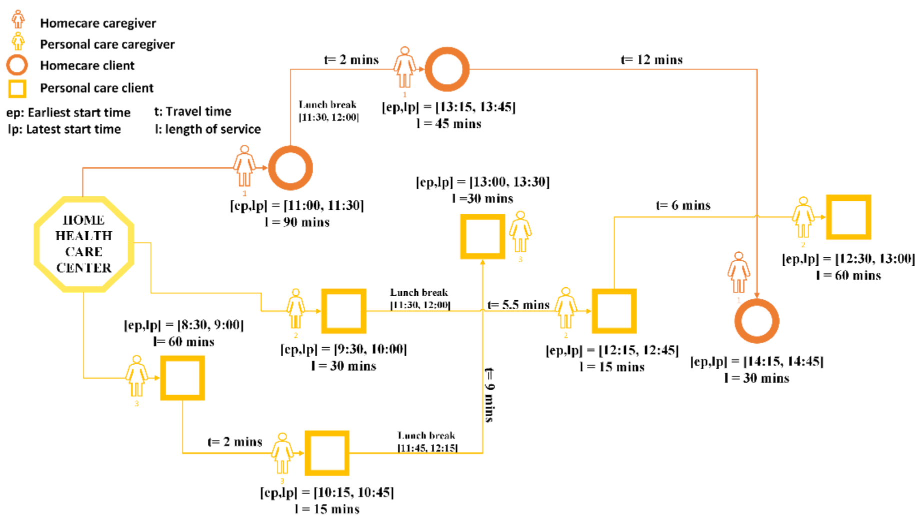

Figure 1. An example of VRPHHCS for newborns and mothers

This paper's goal is to develop a mathematical model that can optimize the routing and scheduling of a HHC services provider in Ho Chi Minh City with respect to the objective of minimizing the total traveling cost for all caregivers as well as the number of required caregivers to visit all clients. With these optimal solution, the service provider can know which clients will be served by which caregivers at what time while matching the skills with requirements and at what period time caregivers can take a break. In addition, the company can predefine the route for each caregivers which results in all nodes will be visited in a timely manner, preventing unexpected traveling costs, avoiding violating the customers' time window and caregivers' working hours.

This study is organized as follows. Section 2 presents the literature review for The Routing and Scheduling Problem in Home Health Care. The problem and mathematical model are described and developed in Section 3. The solution approach for this problem is discussed in Section 4. In section 5, a case study of a HHC center in Ho Chi Minh is studied and illustrated. In order to make further development and recommendations, sensitivity analysis is made in Section 6. Finally, conclusions are drawn in Section 7.

2. Literature Review

The Routing and Scheduling Problem in Home Health Care (HHC) services until now has gained much interest from researchers. There are various aspects of HHC that have been studied. Moussavi et al.(2019) considered the problem for serving all of the customers when they offer the services from one day to another day – multiple days. They can require to be serviced for many time period in a day with different services (Moussavi et al. 2019). In order to deliver the services to all clients, there is a need for designing an optimal plan with respect to the objective of minimizing the total daily traveling distance of the staff, the travel distance traveled by one employes as well as the total distance for the whole the time horizon. Cinar et al. (2021) provided a solution method to schedule a large number of patients while the number of nurses is limited. In this case, they decide to make the client's priority according to their condition and need Cinar et al. (2021). If the priority is high, the patient will be considered to be served first Cinar et al. (2021). However, it seems to be hard to afford all patients such that they will come up with the solution of avoiding delay as much as possible. Since the priorities are updated dynamically the unvisited ones will be checked again for the next planning horizon Cinar et al. (2021). On the contrary, Bazirha et al. (2022) decided to skip the patients if their time window is violated since they consider the hard time window. However, similar to other cases, they still want to minimize the total traveling cost of their staff and the average number of unvisited patients.

Another aspect which is also considered in HHC problem of Afifi et al. (2016) and Decerle et al. (2019) are the synchronized visits. It means that a client in a day can require to be visited two or more times on the same location, depending on the number of vehicles needed to deliver the service. Decerle et al. (2019) suggested the case when two visit points are associated to the same client, they are required to be visited by two distinct vehicles. Decerle et al. (2019) want to solve various objective of minimization including: the total travel time, total assigned negative preferences as well as the dissimilarities in the workload characteristics. In order to so in the most effective way and time saving, they developed a new approach based on Simulated Annealing Local Search Decerle et al. (2019). On the contrary, Afifi et al. (2016) seek to minimize the staff total working time and clients' time window while maximizing working time difference among 2 types of nurses. By applying memetic algorithm, they want to develop a solution which highlights the relationship between working time, quality of service and route balancing Afifi et al. (2016).

As in Khodabandeh et al. (2021), they introduce an interesting constraint following the HHC problem – the satisfaction of the nurse or they named it downgrading cost. Different from other researchers where they only consider much more about the customers, Khodabandeh et al. (2021) thought that a more skillful nurse should be paid more but they also need more training than others which lead to the increase in cost. If the nurse is arranged to serve the patient without using all their skills, this leads to the nurse's dissatisfaction. Such that, a bi-objective model was proposed with respect to the nurse downgrading cost to design the best solution which skillful nurse can be used in an appropriate way (Khodabandeh et al.2021). Besides, Nasir & Dang (2018) had a tendency of solving the daily scheduling and routing problem considering existing patients, new patients, waiting list patients with different requirements and skills for 2 types of nurses: new nurses and existing nurse.

Different from the above papers with the case of only one home center as the depot, Fathollahi-Fard et al. (2020) and Bahadori-Chinibelagh et al.(2022) conducted research for HHC problem based on the perspective of multi – depots, there are existing various kinds of pharmacies and laboratories. That means nurses will start their route at the pharmacy, traveling to visit patients. After that, they collect the pharmacy requirements and biological tests, travel and bring them to the laboratory. There is only one pharmacy, and one laboratory should be considered to one patient. However, despite the similarities of giving services from pharmacy and laboratory at home, Fathollahi-Fard et al., (2018), Shi et al.(2017) and Liu et al. (2013) just considered only one pharmacy and one laboratory for the whole problem – becoming the single depot HHC problem. One interesting thing that is Fathollahi-Fard et al. (2018) paid more attention to the environment and air pollution from transportation when delivering HHC services. This demerits from HHC services give them the motivation to discuss the impact of GHG emissions for the home health care system, find an optimal route in which can reduce the pollution. In regardless of green home health care, Shi et al. (2017) simulated a real scenario in which vehicles have their own capacity, the drugs cannot be loaded unlimitedly. So, if there is not enough drugs provided for patients at their home, the nurses must come back to the pharmacy for more pickup, causing the failure in routes. Shi et al. (2017) want to come up which an optimal solution to minimize the distance if there is additional routes result from failure route.

As we have said before, challenges in HHC are only satisfy clients but also satisfy the caregivers. In order to do so, besides reducing the workers' downgrading, we can boost their desire by giving them a short break to relax and have lunch during their working hours. Both Liu et al. (2017) and Xiao et al. (2018) considered this aspect in their research. Their paper seeks to find solutions which minimize the penalty cost for unvisited customers as well as the company operation cost such as traveling cost or car rental costs.

If other papers let services providers have quite flexible time for scheduling the visit, Demirbilek et al. (2021) addressed a problem in which services must be made immediately as soon as the customers' request is placed. A client is served on the same day and time, and by the same staff the whole time as long as his/her order has been accepted.

To do so, there are serval questions that need to be answered to come up with the solution: should the patient be accepted? If yes, which nurse will deliver the services? Which days and times should the serviced happen? In Demirbilek et al. (2021) it would be many cases that services are accepted but others may be rejected due to conflict. Or one more special cases, Yang et al. (2021) besides taking into consideration of realistic constraint such as services consistency, workload balance and the objective of cost optimization, they pay equal attention to customer satisfaction and caregiver satisfaction. Yang et al. (2021) want to minimize not only the total operation cost but also minimize caregivers' inconsistency and workload imbalance to maximize the caregivers' consistency.

After prevising all the previous paper related to HHC problem, besides the balanced workload allocation, I found that one of the most important characteristics need further extension is the matching skill and requirements with respect to not violating the time window. There are some studies attempting to capture this aspects such as Pahlevani et al., (2022), Hosseinpour-Sarkarizi et al. (2020) and Yuan et al. (2015). Between them, Pahlevani et al. (2022) as the key preference for this thesis. However, there is one gap in Pahlevani et al.(2022) that is they did not propose the break time for their caregivers despite the 8-9 hours working per day. Therefore, to fill this gap, in this paper, HHCRSP is studied to find the optimal solution that can serve all of the clients without violating the time window as well as the skill and requirement constraints but still consider caregivers' satisfaction by letting them have their short lunch break.

3. Problem description and Modeling

The problem is modeled through a graph G (N, A), where N is the set of nodes and A is the sets of arcs connecting these nodes. For (i, j) arc ∈ A, t_{ij} denotes the travel time from node i to node j, and d_{ij} is the travel distance. Each task which requires a given service time must be started in between a time window $[ep_i, lp_i]$. In addition, two preferences vector pp_i for clients' required service and pc_k for caregivers provided services are introduced. These vectors must align with each other to create a feasible schedule that avoids conflicts between service requirements and caregiver skills. These skills and requirements of clients are known as hard constraints; therefore, no visits are planned if a caregiver cannot meet the expectation. In addition, each caregiver is subject to maximum working durations hour of 9 hours. They have designated time availability within the interval $[ec_k, lc_k]$. Furthermore, addressing the matter of caregivers' lunch breaks during their work shifts, a lunch break duration and time window are established. These parameters, shared across all staff members, are denoted as B for lunch break duration and [bb, ab] for the designated time window.

Furthermore, addressing the matter of caregivers' lunch breaks during their work shifts, a standardized lunch break duration and corresponding time window are established. These parameters are denoted as B for lunch break duration and [bb, ab] for the designated time window. HHCRSP for newborns and mothers considers a flexible time window concept for clients which denotes the period when clients are available to receive service. Within a 30- minute time window, the client must be visited and served. The assumptions of this problem are defined as follows:

- The problem considers only a single home care center.
- A caregiver must start her route from the home center (node 0) and return to node 0 for end of the route.
- Each client must be served by exactly one caregivers;
- All clients must be visited.
- Each client will request one type of care and be served with a different time window.
- The daily wage for each caregiver on a route remains constant.
- All the services are limited to a single day.
- No cancellation and lateness are allowed.
- The distance between clients i and j is calculated by the Spherical Law of Cosines.

Indices and sets

Notation	Meaning
N	Set of all nodes including home healthcare office {0, 1, …, N}
P ⊂ N	Set of all locations of clients {1, …, P}
C	Set of all caregivers {1, …, C}

Decision Variable

Notation	Meaning
Z_{ik}	1 if caregiver k is assigned to client i 0 otherwise
X_{ijk}	1 if caregiver k traversed from node i to node j 0 otherwise
S_{ik}	Arriving time of caregiver k to node i

A	Set of arcs A $\subseteq \{(i, j) \mid i \neq j \in$ N$\}$	y_{ik}	1 if caregiver k takes a break at client i before service. 0 otherwise
W_k^0	Minimum working hours for caregiver k	y'_{ik}	1 if caregiver k takes a break at client i after service. 0 otherwise
W_k	Maximum working hours for caregiver k	tb_k	the start time of the lunch break of caregiver k
M	A big number		
f_k	Daily wage of caregiver k		
tc	Traveling cost per kilometer		
ep_i	Earliest time that the service for client i can be started		
lp_i	Latest time that the service for client i can be started		
ec_k	Earliest time that caregiver k can start the service		
lc_k	Latest time that caregiver k can finish the service.		
t_{ij}	Travel time from node i to node j		
d_{ij}	Distance from node i to node j.		
l_i	Service time of client i		
pp_i	Preference vector for client i. It takes binary value 0 or 1.		
pc_k	Preference vector for caregiver k. It takes binary value 0 or 1.		
B	The lunch duration for lunch break		
bb	Earliest time that caregiver k can start the lunch break		
ab	Latest time that caregiver k can start the lunch break		

Using above notations, The HHCRSP for newborns and mothers is formulated as:

$$Minimize \sum_{i \in N} \sum_{j \in P} \sum_{k \in C} d_{ij} X_{ijk}\, tc + \sum_{j \in P} \sum_{k \in C} f_k X_{0jk} \quad (1)$$

Subject to

$$\sum_{k \in C} \sum_{i \in N} X_{ijk} = 1 \qquad \forall j \in P \qquad (2)$$

$$\sum_{i \in P} X_{ijk} = Z_{jk} \qquad \forall k \in C, \forall j \in P \qquad (3)$$

$$\sum_{j \in P} X_{0jk} = \sum_{j \in P} X_{j0k} \qquad \forall k \in C \qquad (4)$$

$$\sum_{i \in N} X_{ijk} - \sum_{i \in N} X_{jik} = 0 \qquad \forall k \in C, \forall j \in P \qquad (5)$$

$$\sum_{j \in P} \sum_{k \in C} X_{0jk} \leq C \qquad \forall j \in P, \forall k \in C \qquad (6)$$

$$S_{ik} + M(1 - Z_{ik}) \geq ec_k \qquad \forall k \in C, \forall i \in P \qquad (7)$$

$$S_{jk} + l_j - M(1 - Z_{ik}) \leq lc_k \qquad \forall k \in C, \forall i \in P, \forall j \in P \qquad (8)$$

$$S_{ik} + t_{ij} + l_j - S_{jk} \leq M(1 - X_{ijk}) \quad \forall k \in C, \forall i \in P, \forall (j \neq i) \in P \qquad (9)$$

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$$\sum_{i \in P} Z_{ik} = 0 \quad \forall k \in C \quad where\ pp_i \neq pc_k \tag{10}$$

$$W_k^0 \leq \sum_{i \in P} Z_{ik} l_i \leq W_k \quad \forall k \in C \tag{11}$$

$$ep_i \leq S_{ik} \leq lp_i \quad \forall k \in C, \forall i \in P \tag{12}$$

$$\sum_{i \in P} y_{ik} + \sum_{i \in P} y'_{ik} = 1 \quad \forall k \in C \tag{13}$$

$$y_{ik} + y'_{ik} \leq \sum_{j \in N} X_{ijk} \quad \forall k \in C, \forall (i \neq j) \in P \tag{14}$$

$$tb_k + By_{jk} \leq S_{jk} + (1 - y_{jk})ab \quad \forall k \in C, \forall j \in P \tag{15}$$

$$S_{ik} + (t_{ij} + l_i)(X_{ijk} + y_{jk} - 1) \leq tb_k + (2 - X_{ijk} - y_{jk})lp_i \quad \forall k \in C, \forall j \in P, \forall (i \neq j) \in P \tag{16}$$

$$tb_k + (t_{ij} + B)(X_{ijk} + y'_{ik} - 1) \leq S_{jk} + (2 - X_{ijk} - y'_{ik})ab \quad \forall k \in C, \forall j \in P, \forall (i \neq j) \in P \tag{17}$$

$$S_{ik} + l_i\, y'_{ik} \leq tb_k + (1 - y'_{ik})lp_i \quad \forall k \in C, \forall i \in P \tag{18}$$

$$bb \leq tb_k \leq ab \quad \forall k \in C \tag{19}$$

$$X_{ijk}, Z_{ik}, y_{ik}, y'_{ik} \in \{0,1\} \quad \forall k \in C, \forall j \in P, \forall i \in P \tag{20}$$

Objective function (1) aims to minimize all caregivers' total travel cost as well as the number of required caregivers to serv all customers. Constraint (2) makes sure there must be only one visit per day for each client. Constraint (3) ensures only one caregiver is used to deliver services to a client. Constraint (4) indicates that each caregiver must start her route at node 0 (the home center) and also return to node 0 at the end of the route. Constraint (5) ensures when a caregiver visits client i, there must be an out arc going out this node (i.e., the flow conservation for a caregiver). Constraint (6) presents that the amount of service assignments must never exceed the total number of carers that are available. Constraints (7) and (8) have represented staff availability. Constraint (9) guarantees when caregiver k completely finishes serving client i and there is enough time to arrive at node j, she can continue to go from client i to client j for another visit. Constraint (10) ensures that caregiver k is assigned to client i if client i has the same preference as caregiver k. Constraint (11) guarantees the total duration of all assigned services to caregiver k is within the expected workload of caregiver k. Constraint (12) expresses the service time must start in between the client's time window. Constraint (13) determines whether caregiver k should start to have a break before or after visit node i. Constraint (14) denotes that caregivers k can take a break at client i if client i is visited by this caregiver. Constraints (15) – (18) ensure that the pauses for caregivers to rest at clients before and after services begin at the proper times. Constraint (19) ensures the lunch break must happen completely in the defined time window. Constraint (20) presents the binary decision variables.

4. Proposed solutions

This study focuses on developing methodology to enable service provider to routing and scheduling caregivers to their customers easily without violate time window and workload constraint as well as ensure that there must a match between caregivers' skill and clients' service requirement. To take advantage of multiple approaches, a Mixed Integer Linear Programming Model (MILP) is developed and using CLPEX Optimization Solver to achieve the optimal solutions. However, in the case of solving the problem with larger size, CLPEX cannot provide the optimal solution in an appropriate computational time due to the complexity of the proposed model. Therefore, the Clustering Algorithm was decided to use. Figure 2 shows the flowchart of the proposed approach.

4.1 K-means clustering

K-means clustering is a popular algorithm that has a long history, providing us the way to partition data points into k clusters based on their similarity. The algorithm starts by randomly selecting k centroids, and then assigns each data point to the nearest centroid. The centroids are then updated based on the meaning of the data points in each cluster, and the process is repeated until convergence. This method has its own advantages such as flexibility, can be applied to a variety of data including continuous, binary and categorical data; quickly identify cluster in large dataset; suitable for real – world dataset that may contain errors.

4.2 Agglomerative Hierarchical Clustering (AHC)

Agglomerative Hierarchical Clustering (AHC) is a bottom-up approach. The principle of how AHC works is quite simple: firstly, with n data point, n cluster will be created, one cluster for each data point. After that we calculate the distance between each data point to each other by some linkage methods and then merge closet clusters together. This process continues until all data points are in a single cluster. This method is useful for visualizing the relationships between data points, but it can be computationally expensive for large datasets. This method has its own advantages such as resulting dendrogram can provide insights into the underlying structure of the data and can be easily interpreted; does not require prior knowledge of the number of clusters required, unlike the k-means algorithm; can handle non-convex clusters, which the k-means algorithm cannot.

In order to calculate the distance between object A and object B, Euclidean distance measure and Ward's linkage are generally preferred. The Ward linkage method works by minimizing the sum of squared differences within all clusters. In other words, it seeks to minimize the variance within each cluster. The distance between two clusters is calculated as the increase in the sum of squared differences when the two clusters are merged. Linkage function to group objects into hierarchical cluster tree (dendrograms). We can use dendrogram as the reference to determine a partition of the data by deciding where to cut the hierarchical tree into clusters.

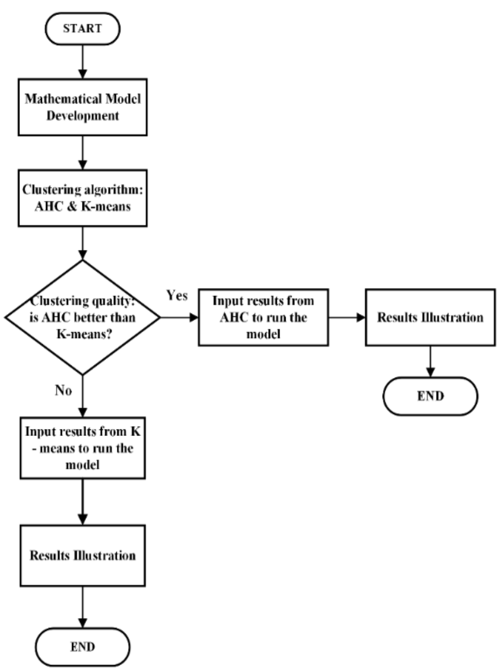

Figure 2. Flowchart of the proposed approach

K-means Algorithm

1. Initialize cluster centroids randomly
2. Repeat until stabilization {
2.1 For every patient calculate distance from every centroid
2.2 Attribute every patient to the closest cluster

$$C(i) = argmin_j d_{ij}$$

2.3 Update cluster (set of patients)
2.4 Calculate the new centroid for every cluster

The centroid $C_j = (x_j, y_j) = \left(\frac{\sum_{i=1}^{m} x_i}{m}, \frac{\sum_{i=1}^{m} y_i}{m} \right)$

}

Figure 3. K – means algorithm flow

5. Case study

The main purpose of this research is assisting a home health care provider in Ho Chi Minh City providing personal care and home care for newborns and mothers.

A real-world dataset comprises 156 clients who predominantly reside in proximity to the center's location, encompassing various districts including Tan Phu, Tan Binh, Binh Tan, Binh Thanh, Go Vap, Phu Nhuan, District 1, District 3, District 4, and District 5.

Based on clients' specific requirements, we used two different codes for their categorization. For instance, clients seeking homecare services like house cleaning or cooking were assigned the code "1" in the client's reference vector. On the contrary, those in need of personal care services such as baby bathing, baby body massage, and clogged milk duct treatment were designated as code "0" in the client's reference vector. The availability of services for different clients varies in terms of the earliest and latest times a service can commence, falling within the time range of 8:00 AM to 5:00 PM. The duration of each service varies based on the nature of the service and the requirements of the customers. However, these durations are standardized to convenient intervals such as 15 minutes, 30 minutes, 45 minutes, 60 minutes, or 90 minutes per service.

Similarly, for caregivers, the same technique is employed, using a binary vector <0,1> to indicate their skill sets in personal care and homecare, respectively. In order to sufficiently handle 156 clients, there is a set of 40 caregivers (20 caregivers have personal care skills and 20 caregivers have homecare skills). According to the information available from the HHC center, each caregiver adheres to a 9-hour workday, from 8:00 am to 5:00 pm. Furthermore, a mandatory 30-minute break for caregivers is scheduled between 11:30 AM and 1:30 PM.

Firstly, the two clustering approaches AHC and K – Means was applied simultaneously on the set of 156 clients in order to categorize the dataset into 8 smaller clusters based on clients' nearby locations. Using Python, 2 figures below are the visualizing in which 8 clusters were made, each color represents one cluster. After having the member of each cluster, each cluster would be solved respectively by CPLEX to achieve the final optimal solution

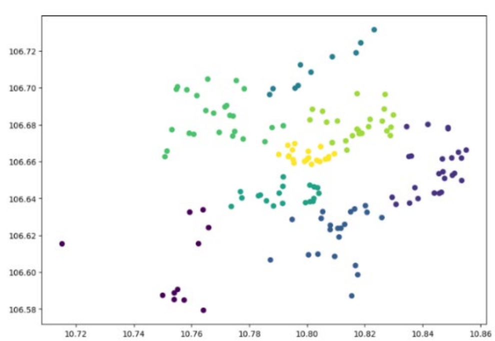

Figure 4. The AHC visualizing plot of 8 clusters

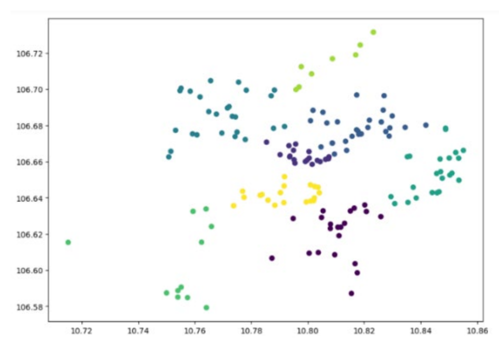

Figure 5. The K – means visualizing plot of 8 clusters

Table 1. Comparison objective values between AHC + Exact method and K-mean +Exact method

AHC+ Exact method			K - Means+ Exact method	
	No. Clusters	No.Nodes	No.Clusters	No.Nodes
	C0	11	C0	21
	C1	25	C1	19
	C2	21	C2	29
	C3	10	C3	26
	C4	19	C4	23

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	C5	25	C5	11
	C6	25	C6	8
	C7	20	C7	19
Objective values (VND)		6996779		7143617.07
Gap (%)	2.1			

In Table 1, the AHC clustering method combined with the Exact Method achieved a total objective value of 6,996,779 VND, whereas the K-Means clustering method combined with the Exact Method yielded a total objective value of 7,143,617.07 VND. The gap between these two methods is 2.1%, indicating that the AHC approach provides a slightly better result in terms of the objective value.

The presented results showcase the optimal routes for eight clusters generated by the AHC method. These routes are aligned with the overarching objective of minimizing the total movements for both home caregivers and personal caregivers, while also minimizing the number of caregivers required to service all designated nodes within each cluster. Within a cluster, we can discern the number of clients encompassed by that specific grouping.

Table 2. Optimal routes for 8 clusters

Clusters	Routing of personal care services	Routing of home care services
C0	R1: 0 – 86 – Lunch break – 7 – 21 – 140 – 0	R2: 0 – 149 – 31– Lunch Break – 69 – 80 – 0 R3: 0 – 3 – Lunch Break – 65 – 78 – 0
C1	R4: 0 – 13 – Lunch break – 76 – 0 R5: 0 – 103 – 146 – 115 – Lunch break – 0 R6: 0 – 136 – 83 – 125 – Lunch break – 18 – 0 R7: 0 – 9 – 24 – 33 – Lunch break – 14 – 49 – 128 – 0	R8: 0 – 123 – 113 – Lunch break – 91 – 119 – 0 R9: 0 – 98 – 114 – Lunch break – 117 – 97 – 120 – 99 – 0
C2	R10: 0 – 1 – 20 – Lunch break – 29 – 79 – 0 R11: 0 – 53 – 17 – Lunch break – 25 – 0 R12: 0 – 54 – Lunch break – 39 – 95 – 52 – 0	R13: 0 – 88 – Lunch break – 57 – 101 – 130 – 133 – 131 – 0 R14: 0 – Lunch break – 58 – 68 – 77 – 82 – 0
C3	R15: 0 – 100 – 26 – 139 – Lunch break – 89 – 0	R16: 0 – Lunch break – 90 – 0 R17: 0 – 116 – 121 – Lunch break – 56 – 28 – 66 – 0
C4	R18: 0 – 46 – 48 – Lunch break – 60 – 132 – 0 R19: 0 – 10 – 55 – 35 – Lunch – 41 – 150 – 0	R20: 0 – 23 – 36 – Lunch break – 64 – 0 R21: 0 – 4 – 71 – 74 – 38 – Lunch break – 67 – 151 – 42 – 0
C5	R22: 0 – 50 – 62 – 106 – 154 – Lunch break – 72 – 85 – 104 – 0 R23: 0 – 2 – 155 – Lunch break – 143 – 16 – 0	R24: 0 – 70 – 152 – 134 – 73 – Lunch break – 87 – 22 – 0 R25: 0 – 144 – 129 – 148 – Lunch break – 43 – 102 – 0 R26: 0 – 19 – 138 – Lunch break – 27 – 0
C6	R27: 0 – 96 – 81– Lunch break – 105 – 92– 0 R28: 0 – 12 – Lunch break – 93 – 0 R29: 0 – 75 – Lunch break – 44 – 0 R30: 0 – 11 – 61 – 84 – Lunch break – 94 – 0	R31: 0 – 107 – 108 – 124 – 141 – Lunch break – 37 – 0 R32: 0 – 126 – Lunch break – 0 R33: 0 – 118 – 110 – 127 – Lunch break – 112 –0 R34: 0 – 109 – 142 – Lunch break – 111 – 0
C7	R35: 0 – 135 – Lunch break – 0 R36: 0 – 147 – Lunch break – 137 – 122 – 145 –0 R37: 0 – 63 – Lunch break – 51 – 0	R38: 0 – 153 – 40 – Lunch break – 34 – 15 – 6 –0 R39: 0 – 156 – Lunch break – 59 – 0 R40: 0 – 30 – 32 – 45 – 47 – Lunch break–5– 8–0

6. Sensitivity Analysis

In order to determine the relative influence of parameters on model output and make further improvement for the model, the sensitivity analysis was made follows 4 scenarios below. In each scenario, all parameters are held constant

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except for the parameters being examined. It was hard to test the whole data set of 156 clients, so a small data set of 25 clients and 6 caregivers (4 for homecare services and 2 for personal care services) were picked up to do the sensitivity analysis. The table and figure below are the comparison between the solution of 4 scenarios and the original ones for further conclusions.
Scenario 1: Decrease the lunch break time window [bb, ab] of a caregiver from [11.5, 13.0] to [12.0, 13.0]
Scenario 2: Decrease the client's time window $[ep_i, lp_i]$ from 30 mins to 15 mins only.
Scenario 3: Increase the client's time window $[ep_i, lp_i]$ from 30 mins to 45 mins.
Scenario 4: Increase the client's time window $[ep_i, lp_i]$ from 30 mins to 45 mins as well as Shorten the lunch break time window of a caregiver [bb, ab] from [11.5, 13.0] to [12.0, 13.0].

Figure 6. Comparison between 4 scenario and original case

From the result above, we can see that scenario 3 had a good impact on the objective, especially when it caused the cost paid for daily wage of all caregivers to drop to almost 10%. The more flexible the time window, the more decrease in final solution. On the other hand, scenarios 1 and 2 make the objective increase compared with the origin while scenario 4 did not show much effect on the final results. It can be seen that when I consider the hard time window for client (only 15 mins) in scenario 2, the model required more caregivers to ensure all clients are being served without violating their time windows and skills' requirement. However, in order to afford this case, not only did the travel cost increase 8.87% but also the total caregivers' daily wage increased to nearly 14%, resulting in the 13.74% increase in the final objective.
Therefore, for further recommendation for the improvement, in the case the company wants to decrease the total travel distance and total cost, they should consider the flexible time window to reduce the number of caregivers required. However, in practice, it seems unreasonable since no clients are willing to wait or accept more than 30 mins late. This case somehow decreases the clients' satisfaction. On the contrary, if the company wants to improve customers' satisfaction, they change into the hard time window (no more than 15 mins) rather than flexible time window like the original case. Obviously, it will make their cost increase significantly since more caregivers will be required.

7. Conclusion

This study introduces a Mixed Integer Linear Programming (MILP) model aimed at resolving the Home Health Care Routing and Scheduling Problem for Newborns and Mothers, while accounting for caregivers' lunch breaks. The primary objective of this model is to minimize overall cost. The resulting optimal solutions take into account constraints time windows and client requirements. To validate performance of the proposed approach, a real-world dataset consisting of 156 clients from an HHC provider in Ho Chi Minh City, offering home and personal care services for newborns and mothers, was utilized. To further enhance the model's applicability, two clustering algorithms, namely Agglomerative Hierarchical Clustering (AHC) and K-Means, were employed to divide the data into smaller clusters based on distance. Routings for these clusters are optimized by CLEX. Following the optimal results, AHC emerged as the more effective approach. This methodology can be implemented in various settings where similar routing and scheduling challenges exist.

For future research, it is recommended to extend the proposed model to address scenarios involving unexpected delays in caregivers' travel or situations where clients might need to add more service during service time.

References

Afifi, S., Dang, D.-C., & Moukrim, A., Heuristic solutions for the vehicle routing problem with time windows and synchronized visits, *Optimization Letters*, vol. 10, no. 3, pp. 511–525, 2016.

Bahadori-Chinibelagh, S., Fathollahi-Fard, A. M., & Hajiaghaei-Keshteli, M., Two Constructive Algorithms to Address a Multi-Depot Home Healthcare Routing Problem, *IETE Journal of Research*, vol. 68, no. 2, pp. 1108–1114, 2022.

Bazirha, M., Kadrani, A., & Benmansour, R., Optimization of the Stochastic Home Health Care Routing and Scheduling Problem with Multiple Hard Time Windows, *International Journal of Supply and Operations Management*, vol. 9, no. 2, 2022.

Cinar, A., Salman, F. S., & Bozkaya, B., Prioritized single nurse routing and scheduling for home healthcare services, *European Journal of Operational Research*, vol. 289, no. 3, pp. 867–878, 2021.

Decerle, J., Grunder, O., Hajjam El Hassani, A., & Barakat, O., A memetic algorithm for multi-objective optimization of the home health care problem, *Swarm and Evolutionary Computation*, vol. 44, pp. 712–727, 2019.

Demirbilek, M., Branke, J., & Strauss, A. K., Home healthcare routing and scheduling of multiple nurses in a dynamic environment, *Flexible Services and Manufacturing Journal*, vol. 33, no. 1, pp. 253–280, 2021.

Fathollahi-Fard, A. M., Ahmadi, A., Goodarzian, F., & Cheikhrouhou, N., A bi-objective home healthcare routing and scheduling problem considering patients' satisfaction in a fuzzy environment, *Applied Soft Computing*, vol. 93, pp. 106385, 2020.

Fathollahi-Fard, A. M., Hajiaghaei-Keshteli, M., & Tavakkoli-Moghaddam, R., A bi-objective green home health care routing problem, *Journal of Cleaner Production*, vol. 200, pp. 423–443, 2018.

Hosseinpour-Sarkarizi, A., Davari-Ardakani, H., & Izadbakhsh, H., Home Health Care Routing and Scheduling Problem under Uncertainty Considering Patient Preferences and Service Desirability, *Scientia Iranica*, vol. 29, no. 5, pp. 2647-2669, 2022.

Khodabandeh, P., Kayvanfar, V., Rafiee, M., & Werner, F., A Bi-Objective Home Health Care Routing and Scheduling Model with Considering Nurse Downgrading Costs, *International Journal of Environmental Research and Public Health*, vol. 18, no. 3, pp. 900, 2021.

Liu, R., Xie, X., Augusto, V., & Rodriguez, C., Heuristic algorithms for a vehicle routing problem with simultaneous delivery and pickup and time windows in home health care, *European Journal of Operational Research*, vol. 230, no. 3, 475–486, 2013.

Liu, R., Yuan, B., & Jiang, Z., Mathematical model and exact algorithm for the home care worker scheduling and routing problem with lunch break requirements, *International Journal of Production Research*, vol. 55, no. 2, 558–575, 2017.

Moussavi, S. E., Mahdjoub, M., & Grunder, O., A matheuristic approach to the integration of worker assignment and vehicle routing problems: Application to home healthcare scheduling, *Expert Systems with Applications*, vol. 125, pp. 317–332, 2019.

Nasir, J., & Dang, C., Solving a More Flexible Home Health Care Scheduling and Routing Problem with Joint Patient and Nursing Staff Selection, *Sustainability*, vol. 10, no. 2, pp. 148, 2018.

Pahlevani, D., Abbasi, B., Hearne, J. W., & Eberhard, A., A cluster-based algorithm for home health care planning: A case study in Australia, *Transportation Research Part E: Logistics and Transportation Review*, vol. 166, pp. 102878, 2022.

Shi, Y., Boudouh, T., & Grunder, O., A hybrid genetic algorithm for a home health care routing problem with time window and fuzzy demand, *Expert Systems with Applications*, vol. 72, pp. 160–176, 2017.

Xiao, L., Dridi, M., & El Hassani, A. H., Mathematical Model for the Home Health Care Scheduling and Routing Problem with Flexible Lunch Break Requirements, *IFAC-PapersOnLine*, vol. 51, no. 11, pp. 334–339, 2018.

Yang, M., Ni, Y., & Yang, L., A multi-objective consistent home healthcare routing and scheduling problem in an uncertain environment, *Computers & Industrial Engineering*, vol. 160, pp. 107560, 2021.

Yuan, B., Liu, R., & Jiang, Z., A branch-and-price algorithm for the home health care scheduling and routing problem with stochastic service times and skill requirements, *International Journal of Production Research*, vol. 53, no. 24, pp.7450–7464, 2015.

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