5th South American Conference on Industrial Engineering and Operations Management
Bogota, Colombia, May 7-9, 2024
Publisher: IEOM Society International, USA DOI: 10.46254/SA05.20240041
Published: May 9, 2024

Optimizing Canine Nutrition: A Linear Programming Approach for Military Dogs' Diet

Oliver J. Saadia Harvy
Student, Industrial Engineering
Department of Chemical, Industrial and Food Engineering
Universidad Iberoamericana
México City, México
oliversaadia01@gmail.com

Arturo Obregón Hochstrasser
Student, Finance
Department of Business Studies
Universidad Iberoamericana
México City, México
arturo.obregon.hoch@gmail.com

Oroselfia Sánchez
Professor, Industrial Engineering
Department of Chemical, Industrial and Food Engineering
Universidad Iberoamericana
México City, México
oroselfia.sanchez@ibero.mx

Abstract

The purpose of this paper is to propose a linear programming model that satisfies hypothetical nutritional requirements that the U.S. Army canine unit might have. The study established specific amounts of protein, carbohydrates, and fat necessary for canines that partake in military activity. It was determined that each daily portion would consist of a base amount of protein, carbohydrates, and fat. However, in order not to compromise the dogs´ health and maintain their peak performance, there had to be a limit set on the daily intakes of macronutrients. Through linear programming optimization sensitivity and dual analysis, our research highlights the economic and sustainable feasibility of producing a plant-based diet for military dogs without compromising the canine's health or military performance. The scope of this research is purely based on a theoretical proposed diet using public data to provide a methodological approach for balancing nutritional quality while maintaining cost efficiency.

Keywords
Optimization, Linear Programming, Diet, Dual, Sensitivity Analysis

1. Introduction
The importance of canine obesity and dietary needs has surged in recent years, bringing pet nutrition into the spotlight. This topic is even more important when referring to service military dogs. The earliest recording of dogs used in

warfare was in 600 B.C. when the king of Lydia deployed dogs to break the invading army of the Cimmerians. As warfare evolved, so did the role of canines in military operations. These highly trained canines play an essential role in military operations, often performing highly demanding tasks that require a specialized diet compared to regular household pets. This research was done to ensure that army dogs are provided with the best possible nutrition as they are an integral part of national security and defense. Proper nutrition is essential for their long-term health and well-being, reducing the risk of medical conditions and extending their service life.
The research starts with a literature review and then we conducted research to gather the best ingredients to use in this diet. We then provide the list of the ingredients to be used, with their macronutrient values and their price in Mexican pesos (MXN). Subsequently, we established the constraints of the minimum and maximum consumption requirements of the macronutrients which will lead the way to construct a linear model program. The model was solved using the Solver software in Microsoft Excel and validated in Python using PuLP library.

1.1 Objectives

The objective of this paper is to find the optimal dog food quantity that should be consumed to minimize costs while satisfying all the nutritional requirements. This problem can be solved using linear programming techniques such as the simplex algorithm. Furthermore, we conducted a sensitivity analysis to assess the impact of varying the prices of decision variables. Additionally, we employed a dual linear programming model to determine the cost implications of raising the quantity of macronutrients in a daily portion.

2. Literature Review

Dogs and cats are often regarded as family members (Marten 2019), but dog and cat animal product consumption release up to 64 million tons of CO_2 (Okin, 2017). Measures of Ecological footprint and dietary paw print are in expansion nowadays. This is because the dogs' diet relies on commercial dry food causing underestimated greenhouse gas emissions. Dogs have a substantial ecological impact, and much of this is tied to their dietary needs (Losey et al., 2022). Meat-based diets require more energy and water than plant-based diets, without considering the environmental impact (Marten 2019). To measure the Ecological and Carbon Paw Prints of dogs (Van Prooijen et al., 2024), it is necessary to study the estimation of the number of calories that a dog requires daily (Marten 2019). However, the pet food industry is unique regarding sustainability because commercial pet food formulations provide excess nutrients (Hughes 1995). That circumstance is still a problem remaining, because of a growing obesity trend among companion animals because they are overconsuming (Marten 2019).

There are more than 163 million dogs and cats that consume animal products which constitute a considerable dietary footprint (Okin, 2017). On the other hand, formal studies about canine daily nutrition are novel in terms of concepts and quantitative techniques. Dogs' daily ingestion varies depending on the activities, rest periods, and breed. K9 are known as Military Working Dogs (MWD) and have activities such as guarding, scouting, and explosive detection (Park et al., 2023).

The estimation of a dog diet based on sustainable and nutritional alignment is not easy to calculate. For example, the relationship between dog food consumption and environmentally sustainable development is not achieved (Marten 2019). This problem has two different aspects related to this study. The first one is the estimation of the number of calories that dogs require daily based on human-made meats instead of current and innovative studies of real quantities (Marten 2019). The second one is that in contrast to human diets, pet food products present a limited combination (Marten 2019). Food production had a higher long-term effect on health as an indicator because more calories were available, which means that dog waste is aligned with human waste (Losey et al., 2022). Meat consumption is increasing in developing nations meaning a greater required land compared with plant crops causing erosion, pesticides, and waste (Okin 2017).

In the same line of observing the interrelation between health indicators and daily diet, Military Dogs are most diagnosed with gastrointestinal disease followed by dental and musculoskeletal diseases (Park et al., 2023). For example, the effects low or high dietary carbohydrate content might have on intestinal glucose (Gal, 2021) are not yet clarified. Regarding macronutrients, some researchers found that protein content in animal-based products is around eleven times higher than that of plant-based products (Swason et al., 2013; Hill,). There is a need to do new calculations including an ingredient list for dog foods and the composition of these ingredients (Okin 2017) to expand the options for dog food. More research is still needed to evaluate the ingredients' animal content and human edibility (Okin 2017).

Besides the previous studies, there are opportunities to improve diet optimization applications using mathematical approaches more than minimizing costs or maximizing nutrients (Donkor, 2023). And interaction between activity level and diet was converted to the class variable is a potential area for improvement (Gal, 2021). There is no clear way to determine the amount of fish-derived energy as a proportion of total animal-derived energy in animal feed (Okin, 2017). In nutrition terms, an optimal diet program can be solved by Linear Programming Modeling, evaluating the budget for the diet and the healthiest distribution of meals in a day (Khoshaim, 2021), or to explore new directions (Cobb et al., 2021).

Table 1. Comparison of Optimization approaches based on literature review.

Model Approach	Main Objective	Authors
Linear Programming	• Mathematical modelling on diet problem for the subways sets menu in Malaysia • Maximization of animal weight gain. • Relation between crop/feeding optimization versus animal Health. • Optimal dietary solutions based on 119 food components using function C as Carbon emissions • Minimize the feed cost for small-scale poultry farms: uses locally available feed ingredients to formulate the boiler feed mix • Optimal daily minimum cost DASH (Dietary Approaches to Stop Hypertension) • Dietary plans that meet nutritional requirements: the project set 33 nutrients and 1.6 kg carbon dioxide equivalents/day as constraints. • Calculation of optimized dietary patterns of four different age groups: children, adolescents, adults, and the elderly	• Khalid, K et al., 2024. • Jiménez et al., 2024 • Erinle et al., 2022 • Zhou, et al., 2022 • Mallick et al., 2020 • Iwujji, et al., 2016 • Dooren et al., 2015 • Brink et al., 2019
Multi-objective Optimization	• User balance 3 important objectives when it comes to getting food recommendations: user preferences, diversity score, nutrition score	• Perera, et al., 2023; Gupta, 2015
Genetic Algorithm	• The diet generation dived into five, namely, breakfast lunch, meal, night tea and dinner.	• Catalán-Salgado, 2014
Particle Swarm Optimization (PSO)	• Eating plan for athletes in training • The best diet optimization result.	• Fister et al., 2016. • Babalola, et al., 2016

About the global applications that the Diet problems have encountered, we can mention that diet optimization is a vital area to be investigated when attempting to prevent malnutrition and meet nutritional necessities in the heart of scarce knowledge and finance for proper diet (Babalola et al., 2020). And most similar dietary programs were developed after 2000 (Zhou et al., 2022). Now, if we focus on the significant opportunity to study animal nutrition not only based on the human approach, there exist other models such as nonlinear programming, dynamic programming, stochastic programming, or heuristic programming, although used less frequently, can also be relevant (Nguyen et al., 2021). There are a few research published that apply Animal Health requirements to study animal welfare (Jiménez, 2024). Diet plan optimization can be carried out using Linear programming to generate the best diet plan to achieve specific dietary goals by considering the needs, restrictions, and financial constraints (Khalid et al., 2024). In humans, there exists the agreement that Linear Programming can be utilized to translate nutritional requirements based on chosen Dietary Guidelines (Boufous, 2021).

Referencing the study of animal nutrition, which considers requirements for growth and activity, represents an opportunity for exploration, the following table presents several studies conducted specifically for the Diet Problem (see Table 1). All the following research contemplates that the Diet problem is formulated differently depending on the area of this application. And the minimal required amount of each food component is known (Pichugina, 2020).

Given this gap in techniques and sustainability aspects, we propose a Linear Programming Model based on the classic Diet Problem (Van Dooren, 2018; Zheng, 2022). This problem describes a selection of food for the composition of a diet (Tranzola et al., 2022). However, considering the scope of the model, in this study, we evaluated the convenience of sweet potato as an ingredient of a dog's daily diet compared to fish, all while minimizing the cost of meals.

From our literature review, we noted that the classic Diet Model, solved using linear programming, is effective for exploring new dietary plans that meet nutritional requirements for dogs as well.

3. Methods

We solved the classic linear programming model for dietary choice, adhering to the dietary restrictions stipulated, with the objective of procuring the essential nutrients for an active military dog that consumes our product, while minimizing costs. We will achieve this using our primary ingredients: egg, salmon, and sweet potato. Utilizing the simplex method, we will determine the optimal quantity of these ingredients to include in a one-kilogram package. Furthermore, we will make a sensitivity analysis on all three decision variables and a duality analysis to interpret the shadow-price of our model.

This study was conducted through Operational Research Methodology which involves defining the problem, proposing a model, and then solving it followed by final validation to ensure confident results (Hillier et al., 2023). As mentioned in the referenced methodology, we now present the sequence we followed to address this problem (see Figure 1). Due to the complexity of the issue, this study has a methodological scope to solve and analyze and validate the results obtained in the Linear Programming Model. In the following figure, we can observe that the results were neither implemented in animals nor was the effect of the proposed diet analyzed in the real world. The main goal of creating this model and solving it was to conduct research about how Linear Programming approach can be useful to propose a diet that considers macronutrients in a daily portion based on animal requirements rather than human ones. Through a What if? Analysis we applied the Theorical Sensitivity Analysis to analyzed different scenarios that the LP Model respond. Then, we validated the Numerical Model behavior through Python optimization.

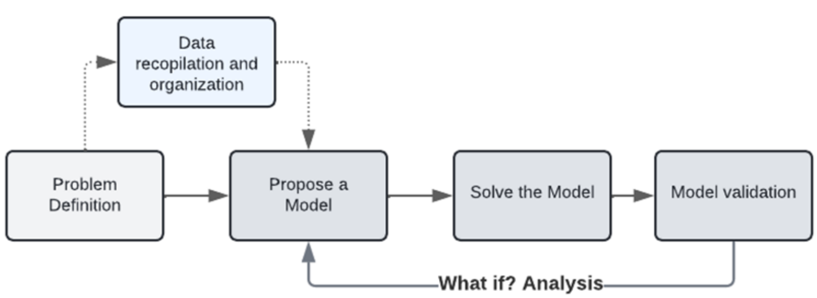

Figure 1. General Methodology Flowchart.

Further the intervention in a specific K9 nutritional program, we propose at first to analyze the substitution of plant-based protein instead of animal protein since a financial perspective. Furthermore, if micronutrients were incorporated, such as vitamins, this model would need to be restructured, verified, and validated once again to ensure reliable results. In the next section we present data sources and proposed parameters that we used in linear modelling.

4. Data Collection

The data for this paper was collected from two main sources, the first of which provided relevant information on the dietary needs for dogs in the United States of America. This source comes from the Association of American Feed Control officials (AAFCO), a private, nonprofit, association in charge of establishing guidelines for ingredient definitions, product labels, feeding trials, and laboratory analyses of the nutrients that go into pet foods. The specific article we based this data collection on (AAFCO, 2014), is focused on pet dog and cat food nutrient profiles from which we gathered our minimum and maximum requirements for the macronutrients. This public data was used to propose the parameters in our mathematical model. For example, this publication suggests that the intake for pet dogs should consist of between 5.5% and 8.5% of fat (AAFCO, 2014). Given the unique demands of a military working dog, we slightly adjusted some of the values to align with what we hypothesized would meet the enhanced requirements of a K9 unit.

The second primary source we used was to determine the price per kg of the ingredients we would be using in the feed we propose. As this paper was elaborated in México, we decided it would be easier to establish the prices of these ingredients in Mexican pesos (MXN). We gathered this information from the Sistema Nacional de Información de Mercados (SNIIM), a public Mexican government website, which provides daily updates on the prices of various food ingredients across the whole country. This resource ensured that the costs of these ingredients were accurate and reflected current market prices.

A dog food company produces pet food sold by the bag. The cost of all the ingredients used and the units they contain of protein, carbohydrates and fat are as follows (see Table 2):

Table 2. Ingredients list and costs.

Ingredients	Cost per kg (MXN)	Protein (per kg)	Carbohydrates (per kg)	Fat (per kg)
Eggs	$46	126 grams	11 grams	96 grams
Salmon	$369	200 grams	0 grams	60 grams
Sweet Potato	$71	2 grams	200 grams	0.3 grams

The U.S. Army requests this food for its canine units, demanding that the food be of the highest quality and that the nutritional values be included on each package. The company conducted extensive research on the best type of food for the army's canine units. This study revealed the amounts of protein, fat, and carbohydrates that an average dog of this type should receive. It was decided that each bag should last for two meals a day for the K9, and each daily portion must contain at least 150g of protein, 300g of carbohydrates and 30g of fat. Similarly, an excess of any of these macronutrients could harm the canine. An excess of fat could impair their military functions, so they have a limit. Per serving, they cannot consume more than 500g of protein, 150g of fat and 750g of carbohydrates.

The company wants to create a linear programming model, and their goal is to find the least expensive combination for their daily portion bags. For this model, there are constraints for protein, fat, and carbohydrates. These constraints are per serving of food. It is important to realize that the restrictions are per serving in a daily portion.

4.1 Description of Linear Programming Model

In this subsection, we propose a Linear Programming Model to estimate the minimum cost for a daily dog food portion. The model is described by the decision variables as follows. Meanwhile, the restrictions of the model represent the minimum and the maximum macronutrients (protein, carbohydrates and fats) requirements. As decision variables we define the following:

- X1: Amount of egg (in kilograms) per daily portion.
- X2: Amount of salmon (in kilograms) per daily portion.

- X3: Amount of sweet potato (in kilograms) per daily portion.

The mathematical form of the defined problem before, takes the following **Primal Linear Model Formulation based on the classic Diet Problem Formulation.** We propose as the Objective Function, the minimum price of a daily diet portion subject to macronutrients requirements. Constrictions one and four represent the protein requirement as minimum and maximum bounds. The same representation for constraints 2 and 5 for Carbs, and 3 and 6 for fat as main nutrient. Those constraints describe the performance of macronutrients in a daily portion for dogs. Meanwhile, the objective function proposes a numerical measure that minimizes the cost of that portion.

Minimize $Z = 46X_1 + 369X_2 + 71X_3$
subject to:

$$126X_1 + 200X_2 + 2X_3 \geq 150$$
$$11X_1 + 200X_3 \geq 300$$
$$96X_1 + 60X_2 + 0.3X_3 \geq 30$$
$$126X_1 + 200X_2 + 2X_3 \leq 500$$
$$11X_1 + 200X_3 \leq 750$$
$$96X_1 + 60X_2 + 0.3X_3 \leq 150$$
$$X_1, X_2, X_3 \geq 0$$

The set of decision variables, that represents egg, salmon and sweet potato was considered as major or equal to cero, as healthy source in a K9 daily portion diet.

5. Results and Discussion

As we mentioned before, the MPL was solved through Solver in Microsoft and validated by PuLP in Python. The obtained results were that the minimal cost for a daily portion of dog food is $155.65, comprising of 1.168 kg of eggs, 0 kg of salmon, and 1.436 kg of sweet potato.

5.1 Numerical Results

In this section we will discuss the different results of the proposed model for a canine daily diet. We will first go through our optimal result for our model and continue with a duality and sensitivity analysis. Through our optimal result analysis, we concluded that the best choice is to produce a vegetarian diet, as it contains no animal protein. As we can see in Table 3, both proteins and carbohydrates are used in their minimum required quantities. Regarding the maximum limit of macronutrients, it is possible to increase protein by 350 grams and carbohydrates by 450 grams, however, this would not be ideal for the canine's health. Similarly, the fat content could still be increased by 37.5 grams, yet this too would not be the healthiest option for the dog. In total, there is a leeway of 46 grams of fat.

Table 3. Linear Model Results.

	OBJECTIVE FUNCTION	155.65						
		X1	**X2**	**X3**	**Final Value**		**RHS**	**SI**
	Minimize	46	369	71				
MINIMUM REQUIRED CONSUMPTION	Protein	126	200	2	150.00	>=	150	0.0
	Carbohydrates	11	0	200	300.00	>=	30	0.0
	Fat	96	60	0.3	112.53	>=	30	-82.5
MAXIMUM REQUIRED	Protein	126	200	2	150.00	<=	500	350.0
	Carbohydrates	11	0	200	300.00	<=	750	450.0

CONSUMPTION	Fat	96	60	0.3	112.53	<=	150	37.5
	Solution	1.168	0.00	1.436				

5.2 What if? Analysis

After obtaining the optimal solution from the Linear Model proposed in this paper, we conducted a 'What if?' Analysis, also known as sensitivity analysis. We propose to examine the following three scenarios using the theoretical formalism of sensitivity analysis, based on the numerical limits suggested by the variability supported by the proposed linear model. This analysis aims to determine:

Scenario 1: How does the consumption of macronutrients and ingredients change when the price of eggs increases?

- Scenario 2: From a nutritional standpoint, is it financially viable to consume very low-priced salmon?
- Scenario 3: Does the cost of the daily portion improve if a better price for sweet potatoes is negotiated with the supplier?

With these three scenarios, using sensitivity analysis, we can gather information on the potential strategies used for each case. As we mentioned before, the macronutrients are represented in Table 3 as (P) for Protein, (C) for Carbohydrates and (F) for fats. Meanwhile, the resource utilization for each scenario can be identified as LHS and for the right-hand side (RHS). For those values that represent slack or surplus for each restriction associated with macronutrients, we can look up in Table 4 the Si values. Finally, the differences between Z values for Scenarios 1, 2 and 3 are included at the end of the same table.

Table 4. Results of What-if? Analysis applied to LPM

Linear Model	**Scenario 1**								**Scenario 2**				**Scenario 3**			
	(a)				**(b)**				**(c)**				**(d)**			
Minimize	**X_1 236**	**X_2 369**	**X_3 71**		**X_1 237**	**X_2 369**	**X_3 71**		**X_1 46**	**X_2 100**	**X_3 71**		**X_1 46**	**X_2 369**	**X_3 50**	
X_1 X_2 X_3	**LHS**		**RHS**	**S_i**	**LHS**		**RHS**	**S_i**	**LHS**		**RHS**	**S_i**	**LHS**		**RHS**	**S_i**
Minimum Required Consumption																
P 126 200 2	150	≥	150	0	150	≥	150	0	150	≥	150	0	150	≥	150	0
C 11 0 200	300		30	0	300		30	0	300		30	0	300		30	0
F 96 60 0.3	112.5		30	-82.5	44.5		30	-14.6	112.5		30	-82.5	112.5		30	-82.5
Maximum Required Consumption																
P 126 200 2	150	≤	500	350	150	≤	500	350	150	≤	500	350	150	≤	500	350
C 11 0 200	300		750	450	300		750	450	300		750	450	300		750	450
F 96 60 0.3	112.5		150	37.5	44.5		150	105.5	112.5		150	37.5	112.5		150	37.5
Optimal solution:	**X_1**	**X_2**	**X_3**	**Z**	**X_1**	**X_2**	**X_3**	**Z**	**X_1**	**X_2**	**X_3**	**Z**	**X_1**	**X_2**	**X_3**	**Z**
	1.2	**0**	**1.4**	**377.5**	**0**	**0.7**	**1.5**	**377.7**	**1.2**	**0**	**1.4**	**155.7**	**1.2**	**0**	**1.4**	**125.5**

About scenario 1, if the price of a kg egg used in a daily portion increased from \$46 to \$236, it would still be a plant-based diet, although this would raise the total cost and fail to meet the goal of minimization. Once the amount exceeds \$236, moving to \$237, it becomes unviable to continue using eggs, and the diet would need to include salmon protein, thereby ceasing to be a vegetarian diet and would become a more expensive and less sustainable option. Table 3, scenario 1a and 1b showcase the models' behavior after increasing the price of a kg of eggs. Table 3 scenario 1a presents the model results after increasing the price of a kg of eggs from $46 to $236. Table 3 scenario 1B indicates the model results after increasing the amount of egg from $236 to $237.

Regarding scenario 2, which analyzes the amount of salmon in a daily portion, we concluded that the best option would be to eliminate salmon. This is due to two reasons: Firstly, as previously mentioned, the product would no longer be vegetarian, making it less sustainable in the long term. Secondly, due to the high cost of salmon compared to egg and sweet potato. Even if the cost per kilo is reduced from 369 to 100 MXN (see Table 3 scenario 2c), the objective function remains the same and involves purchasing salmon of very low quality, which could harm the canine's nutrition and is likely sourced from poorly maintained and unhygienic conditions.
For the quantity of sweet potato used in a daily portion (scenario 3), we concluded that it is the best variable to change. If an agreement is reached with the supplier to reduce the cost per kilogram of sweet potato from $71 to $50, we would be able to lower the objective function from $155.65 to $125.50. The results are illustrated in Table 3 scenario 3d.

5.2 Proposed Improvements

As a proposed improvement, we proposed the development of a dual linear model. The primary purpose of this model is to determine the financial implications associated with either increasing or decreasing the quantity of macronutrients by a gram. This model would enable a better understanding of cost variations tied to adjustments in macronutrient levels. It is important to know this because, if there is a need to produce a more specialized food containing more protein or carbohydrates, we will already understand the extent of the price increase. This model is denoted by the following decision variables:

- Y_1: Cost incurred by adding a gram of protein.
- Y_2: Cost incurred by adding a gram of carbohydrates.
- Y_3: Cost incurred by adding a gram of fat.
- Y_4: Cost incurred by reducing a gram of protein.
- Y_5: Cost incurred by reducing a gram of carbohydrates.
- Y_6: Cost incurred by reducing a gram of protein.

<u>Dual Linear Model Formulation</u>

Objective Function

Maximize $Z' = 150\,Y_1 + 300\,Y_2 + 30\,Y_3 + 500\,Y_4 + 750\,Y_5 + 150\,Y_6$

Subject to:

$$150\,Y_1 + 300\,Y_2 + 30\,Y_3 + 500\,Y_4 + 750\,Y_5 + 150\,Y_6 \leq 46$$
$$200\,Y_1 + 60\,Y_3 - 200\,Y_4 - 60\,Y_6 \leq 369$$
$$2\,Y_1 + 200\,Y_2 + 0.3\,Y_3 - 2\,Y_4 - 2000\,Y_5 - 0.3\,Y_6 \leq 71$$
$$Y_1, Y_2, Y_3, Y_4, Y_5, Y_6 \geq 0$$

As we can see in Table 5, for every gram of protein increased beyond the maximum of 500 grams, the cost will rise by 33 cents. Similarly, for each gram of carbohydrates increased beyond the maximum of 750 grams, the cost will increase by 35 cents.

Table 5. Dual Model Results

	NAME	FINAL VALUE	SHADOW PRICE	R.H.S CONSTRAINT
MINIMUM REQUIRED CONSUMPTION	Protein	150	0	500
	Carbohydrates	300	0	750
	Fat	112.53	0	150
MAXIMUM REQUIRED CONSUMPTION	Protein	150	0.33	150
	Carbohydrates	300	0.35	300
	Fat	112.53	0	30

The optimal solution indicates that the minimum cost for a daily portion is $155.65 MXN. If there is a future need to increase or decrease the quantity of macronutrients in this diet, the shadow prices of the dual model indicate how much the objective function would vary by adding or removing grams of protein, carbohydrates and fat.

5.3 Validation

After solving the Primal Linear Model described in section 4.1, the validation was developed through PuLP optimization in Python obtaining the same Optimal Solution (see Figure 2).

```
[111]: print("Status:", LpStatus[problem.status])

Status: Optimal

[114]: print("Daily diet portion, consists in: \n"+"-"*110)
       for v in problem.variables():
           if v.varValue>=0:
               print(v.name,"=", v.varValue)

Daily diet portion, consists in:
--------------------------------------------------------------------------
Ingredients_eggs = 1.1676861
Ingredients_salmon = 0.0
Ingredients_sweetpotato = 1.4357773

[113]: print ("The Minimum Cost is ${}".format(round(value(problem.objective),2)))

The Minimum Cost is $155.65
```

Figure 2. Optimal Solution validated in PuLP.

6. Discussion

In this paper, we explored different financial and nutritional perspectives of a daily diet for military working dogs. Unlike domestic pets, these dogs engage in more rigorous physical training, therefore they must have a special diet to satisfy their nutritional needs. Using the classical Diet problem in our model, we found that the linear optimization approach proved initially sufficient for researching military K9 needs.

This research highlights that traditional diet frameworks often prioritize human needs over animal requirements, leading to potential health problems such as obesity or cardiovascular problems. Operation Research methods proved valuable in analyzing these needs to minimize waste and reduce CO_2 emissions. The sensibility analysis offered insight into how the variations in ingredient costs impact nutrient consumption. Other approaches could be incorporated to enhance solutions. For example, metaheuristics to achieve better solutions, dynamic programming to research about daily changes in diet, or complexity to analyze the interrelation between sustainable aspects and green suppliers' considerations.

Limitations in our model include a lack of data transparency and few studies that link nutritional needs, CO_2 emissions, health outcomes and intervention for K9 units. Beyond the findings, enhancements, and scope of the Linear Programming approach, we additionally recommend that further works include a veterinary nutritionist specialist to ensure the dietary solutions are effective and scientifically sound.

7. Conclusion

Our project, which was based on the application of a classic linear optimization model known as the diet problem reveals several important insights. The classic diet problem in linear programming is a prime example of how optimization can be used to address nutritional and economic issues. The goal of this problem is to select a combination of foods that satisfy a set of daily nutritional requirements at the lowest possible cost. Originally formulated during World War II by George Stigler, an economist, this model was intended to determine the most cost-effective diet that would meet the nutritional needs of American troops.

Firstly, the optimal solution obtained indicates that a plant-based diet is the most suitable option for dog food, as it fulfills the nutritional requirements without incorporating animal protein, thereby aligning with ethical consumption practices and sustainability. Vitamin A was omitted from the current model due to technical considerations, given its significantly lower values relative to other macronutrients, though its inclusion in future iterations of the model is acknowledged as necessary for enhancing the nutritional quality of the food. In terms of macronutrients, it was concluded that both protein and carbohydrates are utilized in their minimum required amounts, and any significant increase could adversely affect canine health, emphasizing the importance of maintaining nutritional balance.

The sensitivity analysis contributed to a more nuanced understanding of the decision variables. It revealed that the quantity of egg could be increased significantly while maintaining the plant-based nature of the food and without compromising sustainability. However, the associated cost increase and the transition to a non-plant-based diet upon exceeding a certain threshold would render the solution less ideal. Conversely, the removal of salmon from the model proved beneficial both in terms of cost and sustainability, and negotiating a reduction in the cost of sweet potato could lead to a notable improvement in the cost efficiency of the final product.

Lastly, the insights derived from the dual model provided valuable information about the incremental cost of each macronutrient, which is vital to produce a more specialized dog food requiring higher levels of proteins or carbohydrates. This understanding enables us to anticipate and plan effectively for the impact on the product's pricing.

In conclusion, this project has showcased the efficacy of linear programming in developing nutritional solutions for dog foods, capable of being tailored and refined based on cost, nutrition, and sustainability considerations. The model's adaptability and its ability to integrate sensitivity and duality considerations ensure the continuous evolution and enhancement of our product offerings, meeting the needs of both our customers and their pets. This approach not only assures the quality and sustainability of dog food but also highlights our dedication to innovation and excellence in the pet food industry.

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Biographies

Oliver J Saadia Harvy was born on September 17, 2001, in Mexico City. He is currently studying Industrial Engineering at Universidad Iberoamericana and has cultivated a profound interest in the dynamic field of industrial engineering. He has previously gained experience in the automotive industry, having worked at a Ferrari and Audi dealership during the summer. Currently, Oliver is gaining practical experience at Grupo Uniderma, a renowned Mexican company specializing in the importation and distribution of skincare products. His role in this thriving enterprise allows him to apply his academic knowledge to real-world scenarios, blending his engineering skills with the nuances of the skincare industry.

Arturo Obregón Hochstrasser is an emerging young professional in the field of finance, currently a student at the Universidad Iberoamericana in Mexico City. Born on September 27, 1999, in Mexico City, he has always shown a keen interest in numbers and their application in understanding the world. At the age of 24, he is in his eighth semester studying finance, where he has shown a particular affinity for statistics, econometrics, and calculus. Arturo works as a management assistant at Koch Construction, where he has skillfully applied his analytical abilities to enhance processes and contribute to the company's growth. His focus on the practical application of his financial knowledge has been instrumental in solving complex problems and adding value to projects. Looking ahead, Arturo is determined to continue his education and professional development in areas that allow him to deepen his understanding of finance and its impact on economic and social development. His ambition is to become a leading figure in the financial sector, using the tools of statistics and econometrics to unravel economic patterns and contribute to the sustainable development of industry and society.

Oroselfia Sánchez is a Full-time Professor in the Industrial Engineering Department at Universidad Iberoamericana in Mexico City. She obtained her MSc and PhD in Operational Research from the National Autonomous University of Mexico (UNAM), and a Specialization in Quality Systems. Oroselfia has worked in the field in the areas of laboratory management, operational and service processes, and quality management within petroleum and industrial safety systems. Her research focuses on risk and failure analysis techniques aimed at enhancing operational processes.